Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	MIDTERM		Semester/Academic year	1	2023-2024
			Date	23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1811

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

For questions 1–2, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$

$$\omega_1 \; x_1 + x_2 \leq 7,$$

$$\omega_2 \; x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

1. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1, \;\; \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2, \;\; 0)$.
B. $\widehat{\boldsymbol{\omega}}_U = (1, \;\; 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4, \;\; 6)$.
C. $\widehat{\boldsymbol{\omega}}_U = (1, \;\; 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.
D. Other answer.

2. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2, \;\; 0)$.
B. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0, \;\; 2)$
C. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0, \;\; 4)$.
D. Other answer.

3. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

D. None of the other answers are correct.

4. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 6 B. 5 C. 7 D. 8

5. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 48.47 B. 31.75 C. 19.58 D. 40.44

6. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 389.6. B. 112.68. C. 1280.93. D. 1312.68.

7. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 20. B. 24. C. 84. D. 18.

8. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 11 B. 17 C. 18 D. 14

9. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 6 packets of VitaPlus and 8 packets of BeHealthy
B. Has no optimal solution
C. 0 packets of VitaPlus and 16 packets of BeHealthy
D. 18 packets of VitaPlus and 0 packets of BeHealthy

10. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$x_1 + x_2 - x_3 \geq 10$$
$$x_1 - 2x_2 + x_4 \leq 0$$
$$x_i \geq 0.$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and IV. B. I and III. C. Only IV. D. II and IV.

11. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 79.1112M B. 420.7616M C. 258.0852M D. 853.9739M

12. (L.O.2.1) Which of the following is false?

A. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.

B. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

C. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

D. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

13. (L.O.2.1) Which of the following is false?

A. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.

B. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).

C. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.

D. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

14. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

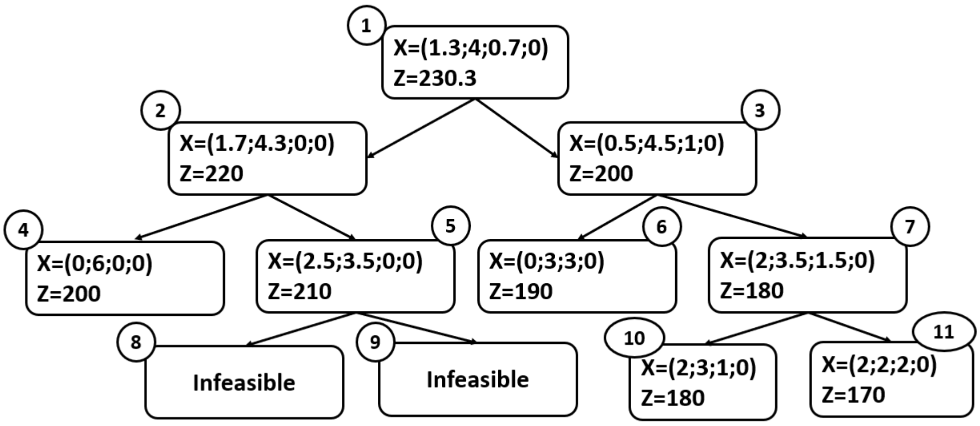
$$x + 4y \le 20, x + y \ge 8, 5x + y \le 32, \text{ and } x, y \ge 0.$$

A. 244/19 B. 284/19 C. 301/19 D. Infeasible

15. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 18 B. 27 C. 14 D. 5

16. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 9 B. 6 C. 4 D. 11

17. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 2 years later. B. 25 months later. C. 26 months later. D. 27 months later.

18. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 52 and 21. B. 48 and 24. C. 37 and 32. D. 45 and 50.

19. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 19 B. 15 C. 16 D. 20

20. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 60% and 40%. B. 90% and 10%. C. 66.67% and 33.33%. D. Other answer.

For questions 21–23, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \leq z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j ($j = 1, \ldots, m$) is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \ldots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \ldots, 3$.

21. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
B. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$
C. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$

22. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}} \; (Z_2 = \; \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T \; \boldsymbol{z}, \\ 0 \leq \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \geq 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 540$
B. $Z_2 = 400$
C. $Z_2 = 280$
D. $Z_2 = 130$

23. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \; \boldsymbol{b}^T \cdot \boldsymbol{x} + \; Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1700$
B. $G_{\min} = 1980$
C. $G_{\min} = 2240$
D. $G_{\min} = 3240$

24. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 355M

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 345M

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

D. None of the other answers are correct.

25. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Linearity. B. Additivity. C. Continuity. D. Finiteness.

. . . . END OF EXAM . . . .

Solution 1811

1. B.
2. D.
3. A.
4. A.
5. D.
6. C.
7. C.
8. B.
9. A.
10. C.
11. A.
12. C.
13. C.
14. A.
15. A.
16. A.
17. C.
18. B.
19. C.
20. C.
21. B.
22. C.
23. B.
24. A.
25. C.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY **FACULTY OF CSE**	**MIDTERM**	Semester/Academic year	1	2023-2024
		Date		23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1812

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 853.9739M B. 79.1112M C. 420.7616M D. 258.0852M

2. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. Infeasible B. 244/19 C. 284/19 D. 301/19

3. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 18 packets of VitaPlus and 0 packets of BeHealthy
B. 6 packets of VitaPlus and 8 packets of BeHealthy
C. Has no optimal solution
D. 0 packets of VitaPlus and 16 packets of BeHealthy

4. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 20 B. 19 C. 15 D. 16

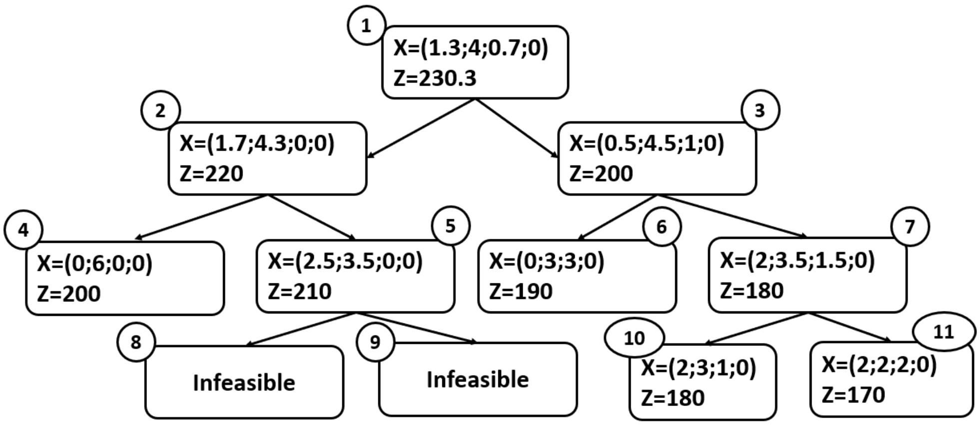
5. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 8 B. 6 C. 5 D. 7

6. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 11 B. 9 C. 6 D. 4

7. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. None of the other answers are correct.

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 355M

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 345M

D. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

For questions 8–10, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector D** $= (D_1, D_2)^T$, meaning

$$0 \leq z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j $(j = 1, \ldots, m)$ is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).

The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\ z_i, \quad j = 1, \dots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\ \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \dots, 3$.

8. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$
B. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
C. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$

9. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}}\ (Z_2 = \ \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\ \boldsymbol{z}, \\ 0 \le \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \ge 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 130$
B. $Z_2 = 540$
C. $Z_2 = 400$
D. $Z_2 = 280$

10. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \ \boldsymbol{b}^T \cdot \boldsymbol{x} + \ Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 3240$
B. $G_{\min} = 1700$
C. $G_{\min} = 1980$
D. $G_{\min} = 2240$

11. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 18.
B. 20.
C. 24.
D. 84.

12. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 5
B. 18
C. 27
D. 14

13. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 40.44 B. 48.47 C. 31.75 D. 19.58

14. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\geq 10 \\ x_1 - 2x_2 + x_4 &\leq 0 \\ x_i &\geq 0. \end{aligned}$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. II and IV. B. I and IV. C. I and III. D. Only IV.

15. (L.O.2.1) Which of the following is false?

A. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.
B. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.
C. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).
D. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.

16. (L.O.2.1) Which of the following is false?

A. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.
B. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.
C. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.
D. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

17. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 27 months later. B. 2 years later. C. 25 months later. D. 26 months later.

18. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. Other answer. B. 60% and 40%. C. 90% and 10%. D. 66.67% and 33.33%.

19. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 1312.68. B. 389.6. C. 112.68. D. 1280.93.

For questions 20–21, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$
$$\omega_1\ x_1 + x_2 \leq 7,$$
$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

20. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. `Other answer.`
B. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ `and` $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.
C. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ `and` $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.
D. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ `and` $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.

21. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. `Other answer.`
B. $Z_m = \frac{50}{10}$ `and` $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.
C. $Z_m = 4$ `and` $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$
D. $Z_m = 3$ `and` $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.

22. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 45 and 50. B. 52 and 21. C. 48 and 24. D. 37 and 32.

23. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A. None of the other answers are correct.

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

D.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

24. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Finiteness. B. Linearity. C. Additivity. D. Continuity.

25. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 14 B. 11 C. 17 D. 18

....END OF EXAM..

Solution 1812

1. B.
2. B.
3. B.
4. D.
5. B.
6. B.
7. B.
8. C.
9. D.
10. C.
11. D.
12. B.
13. A.
14. D.
15. D.
16. D.
17. D.
18. D.
19. D.
20. C.
21. A.
22. C.
23. B.
24. D.
25. C.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	MIDTERM	Semester/Academic year	1	2023-2024
		Date		23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1813

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. 244/19 B. Infeasible C. 284/19 D. 301/19

2. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 79.1112M B. 853.9739M C. 420.7616M D. 258.0852M

3. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 6 B. 8 C. 5 D. 7

4. (L.O.2.1) Which of the following is false?

A. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.
B. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.
C. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).
D. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.

5. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 48.47 B. 40.44 C. 31.75 D. 19.58

6. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 52 and 21. B. 45 and 50. C. 48 and 24. D. 37 and 32.

7. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\geq 10 \\ x_1 - 2x_2 + x_4 &\leq 0 \\ x_i &\geq 0. \end{aligned}$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and IV. B. II and IV. C. I and III. D. Only IV.

8. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 60% and 40%. B. Other answer. C. 90% and 10%. D. 66.67% and 33.33%.

9. (L.O.2.1) Which of the following is false?

A. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.
B. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.
C. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.
D. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

For questions 10–12, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \leq z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j $(j = 1, \ldots, m)$ is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \ldots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \ldots, 3$.

10. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
B. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$
C. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$

11. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}}\ (Z_2 = \ \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\ \boldsymbol{z}, \\ 0 \le \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \ge 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 540$
B. $Z_2 = 130$
C. $Z_2 = 400$
D. $Z_2 = 280$

12. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \ \boldsymbol{b}^T \cdot \boldsymbol{x} + \ Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1700$
B. $G_{\min} = 3240$
C. $G_{\min} = 1980$
D. $G_{\min} = 2240$

13. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 11 B. 14 C. 17 D. 18

14. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 18 B. 5 C. 27 D. 14

15. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 20. B. 18. C. 24. D. 84.

16. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 6 packets of VitaPlus and 8 packets of BeHealthy
B. 18 packets of VitaPlus and 0 packets of BeHealthy
C. Has no optimal solution
D. 0 packets of VitaPlus and 16 packets of BeHealthy

For questions 17–18, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$

$$\omega_1\ x_1 + x_2 \leq 7,$$

$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \textbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \textbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

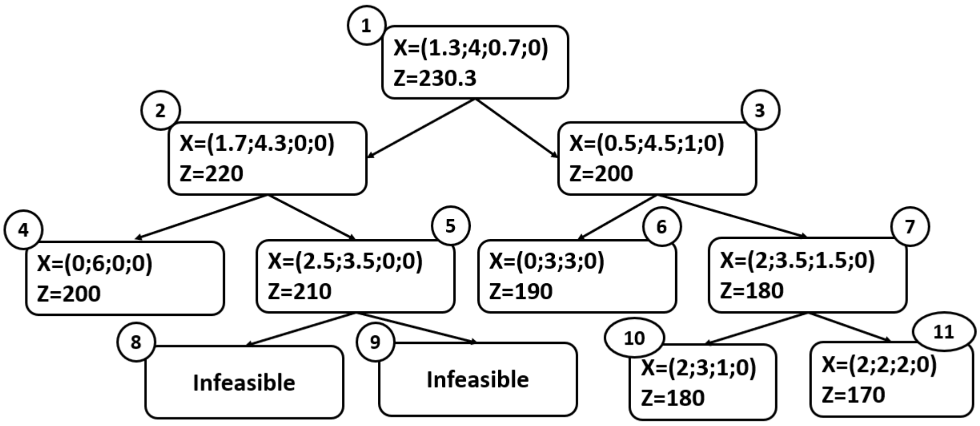
17. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.
B. Other answer.
C. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.
D. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.

18. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.
B. Other answer.
C. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$
D. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.

19. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 9 B. 11 C. 6 D. 4

20. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 2 years later. B. 27 months later. C. 25 months later. D. 26 months later.

21. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

B. None of the other answers are correct.

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

D.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

22. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 355M

B. None of the other answers are correct.

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 345M

D. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

23. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \le 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 19 B. 20 C. 15 D. 16

24. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 389.6. B. 1312.68. C. 112.68. D. 1280.93.

25. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Linearity. B. Finiteness. C. Additivity. D. Continuity.

....END OF EXAM..

Solution 1813

1. A.
2. A.
3. A.
4. D.
5. B.
6. C.
7. D.
8. D.
9. D.
10. C.
11. D.
12. C.
13. C.
14. A.
15. D.
16. A.
17. C.
18. B.
19. A.
20. D.
21. A.
22. A.
23. D.
24. D.
25. D.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	**MIDTERM**		Semester/Academic year	1	2023-2024
			Date		23/12/2023
	Course title	Mathematical Modeling			
	Course ID	CO2011			
	Duration	80 mins	Question sheet code		1814

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 20. B. 84. C. 24. D. 18.

2. (L.O.2.1) Which of the following is false?

A. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.
B. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.
C. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).
D. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

3. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 389.6. B. 1280.93. C. 112.68. D. 1312.68.

4. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 79.1112M B. 258.0852M C. 420.7616M D. 853.9739M

5. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 18 B. 14 C. 27 D. 5

6. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 6 packets of VitaPlus and 8 packets of BeHealthy
B. 0 packets of VitaPlus and 16 packets of BeHealthy
C. Has no optimal solution
D. 18 packets of VitaPlus and 0 packets of BeHealthy

7. (L.O.2.1) Which of the following is false?

A. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.

B. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

C. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

D. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

8. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 60% and 40%. B. 66.67% and 33.33%. C. 90% and 10%. D. Other answer.

For questions 9–10, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$

$$\omega_1\ x_1 + x_2 \leq 7,$$

$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

9. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.

B. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.

C. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.

D. Other answer.

10. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.

B. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.

C. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$

D. Other answer.

11. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Linearity. B. Continuity. C. Additivity. D. Finiteness.

For questions 12–14, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$

be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \leq z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j $(j = 1, \ldots, m)$ is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \ldots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \ldots, 3$.

12. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
B. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
C. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$

13. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}}\ (Z_2 = \ \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\ \boldsymbol{z}, \\ 0 \leq \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \geq 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 540$
B. $Z_2 = 280$
C. $Z_2 = 400$
D. $Z_2 = 130$

14. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \ \boldsymbol{b}^T \cdot \boldsymbol{x} + \ Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1700$
B. $G_{\min} = 2240$
C. $G_{\min} = 1980$
D. $G_{\min} = 3240$

15. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

D. None of the other answers are correct.

16. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$x_1 + x_2 - x_3 \geq 10$$
$$x_1 - 2x_2 + x_4 \leq 0$$
$$x_i \geq 0.$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

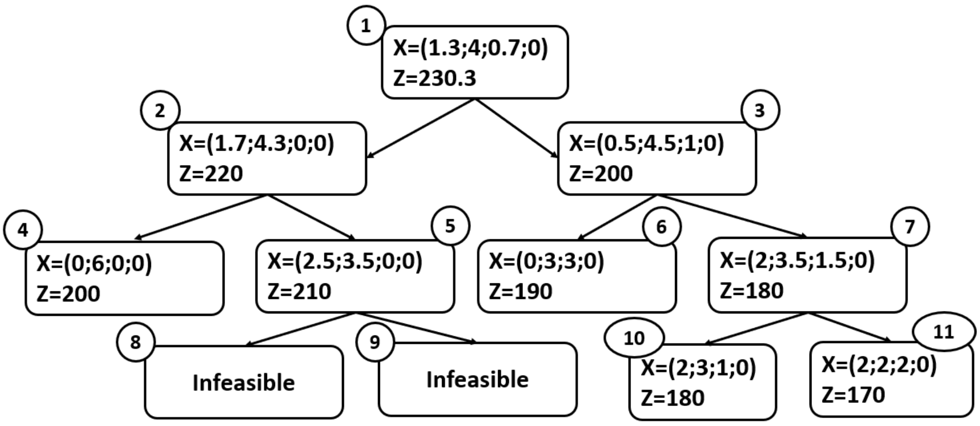
II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and IV. B. Only IV. C. I and III. D. II and IV.

17. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 9 B. 4 C. 6 D. 11

18. (L.O.3.2)

Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 6 B. 7 C. 5 D. 8

19. (L.O.3.1)

When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 52 and 21. B. 37 and 32. C. 48 and 24. D. 45 and 50.

20. (L.O.3.2)

Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 48.47 B. 19.58 C. 31.75 D. 40.44

21. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 2 years later. B. 26 months later. C. 25 months later. D. 27 months later.

22. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 11 B. 18 C. 17 D. 14

23. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 355M

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 345M

D. None of the other answers are correct.

24. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 19 B. 16 C. 15 D. 20

25. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. 244/19 B. 301/19 C. 284/19 D. Infeasible

....END OF EXAM..

Solution 1814

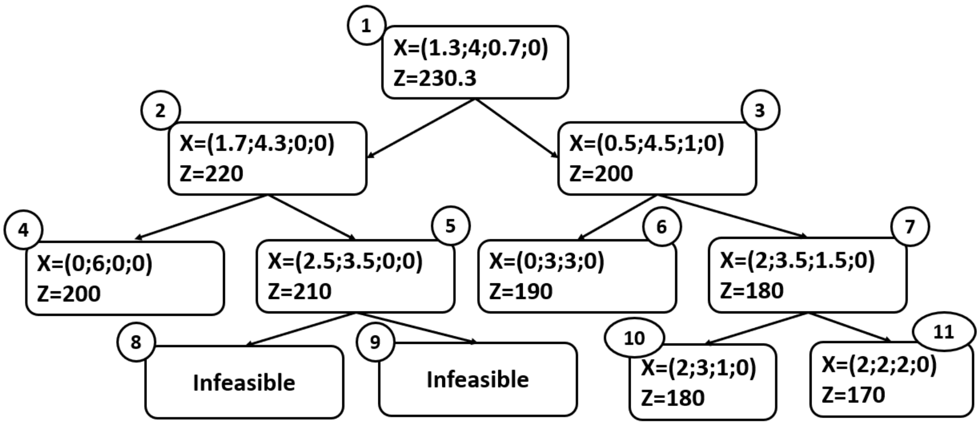
1. B.
2. B.
3. B.
4. A.
5. A.
6. A.
7. B.
8. B.
9. C.
10. D.
11. B.
12. C.
13. B.
14. C.
15. A.
16. B.
17. A.
18. A.
19. C.
20. D.
21. B.
22. C.
23. A.
24. B.
25. A.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	**MIDTERM**		Semester/Academic year	1 | 2023-2024
			Date	23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1815

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 11 B. 9 C. 4 D. 6

2. (L.O.3.1)
Find
$$\max(x + 2y)$$
subject to
$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. Infeasible B. 244/19 C. 301/19 D. 284/19

3. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 40.44 B. 48.47 C. 19.58 D. 31.75

4. (L.O.3.1)
Find
$$\max(5x + 3y + 2z + 7u + 4v)$$
subject to
$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 20 B. 19 C. 16 D. 15

5. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 27 months later. B. 2 years later. C. 26 months later. D. 25 months later.

6. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$x_1 + x_2 - x_3 \geq 10$$
$$x_1 - 2x_2 + x_4 \leq 0$$
$$x_i \geq 0.$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. II and IV. B. I and IV. C. Only IV. D. I and III.

For questions 7–8, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$
$$\omega_1\ x_1 + x_2 \leq 7,$$
$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

7. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. Other answer.
B. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.
C. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.
D. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.

8. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. Other answer.
B. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.
C. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.
D. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$

9. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 853.9739M B. 79.1112M C. 258.0852M D. 420.7616M

10. (L.O.2.1) Which of the following is false?

A. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

B. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.

C. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

D. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

11. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 1312.68. B. 389.6. C. 1280.93. D. 112.68.

12. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A. None of the other answers are correct.

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

D.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

13. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 5 B. 18 C. 14 D. 27

14. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 18. B. 20. C. 84. D. 24.

15. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Finiteness. B. Linearity. C. Continuity. D. Additivity.

16. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. Other answer. B. 60% and 40%. C. 66.67% and 33.33%. D. 90% and 10%.

17. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 14 B. 11 C. 18 D. 17

18. (L.O.2.1) Which of the following is false?

A. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

B. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.

C. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.

D. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).

19. (L.O.3.1)

When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 45 and 50. B. 52 and 21. C. 37 and 32. D. 48 and 24.

20. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 18 packets of VitaPlus and 0 packets of BeHealthy
B. 6 packets of VitaPlus and 8 packets of BeHealthy
C. 0 packets of VitaPlus and 16 packets of BeHealthy
D. Has no optimal solution

21. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. None of the other answers are correct.

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 355M

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

D. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 345M

For questions 22–24, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \le z_i = d_i, \quad i = 1, \dots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j ($j = 1, \dots, m$) is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \dots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \dots, 3$.

22. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$
B. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
C. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$

23. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}} \ (Z_2 = \ \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T \ \boldsymbol{z}, \\ 0 \le \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \ge 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 130$
B. $Z_2 = 540$
C. $Z_2 = 280$
D. $Z_2 = 400$

24. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \ \boldsymbol{b}^T \cdot \boldsymbol{x} + \ Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 3240$
B. $G_{\min} = 1700$
C. $G_{\min} = 2240$
D. $G_{\min} = 1980$

25. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \ge 0$ is the population size at time $t \ge t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 8
B. 6
C. 7
D. 5

....END OF EXAM..

Solution 1815

1. B.
2. B.
3. A.
4. C.
5. C.
6. C.
7. D.
8. A.
9. B.
10. C.
11. C.
12. B.
13. B.
14. C.
15. C.
16. C.
17. D.
18. C.
19. D.
20. B.
21. B.
22. D.
23. C.
24. D.
25. B.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	MIDTERM	Semester/Academic year	1	2023-2024
		Date		23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1816

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 52 and 21. B. 45 and 50. C. 37 and 32. D. 48 and 24.

2. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 60% and 40%. B. Other answer. C. 66.67% and 33.33%. D. 90% and 10%.

3. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

B. None of the other answers are correct.

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

D.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

4. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 20. B. 18. C. 84. D. 24.

5. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 355M

B. None of the other answers are correct.

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

D. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 345M

6. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 18 B. 5 C. 14 D. 27

7. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \le 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 11 B. 14 C. 18 D. 17

8. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Linearity. B. Finiteness. C. Continuity. D. Additivity.

For questions 9–11, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \le z_i = d_i, \quad i = 1, \dots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j ($j = 1, \dots, m$) is the numbers of parts to be ordered before

production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).

The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \dots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \dots, 3$.

9. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
B. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$
C. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$

10. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}}\ (Z_2 = \ \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\ \boldsymbol{z}, \\ 0 \le \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \ge 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.

This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 540$
B. $Z_2 = 130$
C. $Z_2 = 280$
D. $Z_2 = 400$

11. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \ \boldsymbol{b}^T \cdot \boldsymbol{x} + \ Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1700$
B. $G_{\min} = 3240$
C. $G_{\min} = 2240$
D. $G_{\min} = 1980$

12. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 389.6.
B. 1312.68.
C. 1280.93.
D. 112.68.

13. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\geq 10 \\ x_1 - 2x_2 + x_4 &\leq 0 \\ x_i &\geq 0. \end{aligned}$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and IV. B. II and IV. C. Only IV. D. I and III.

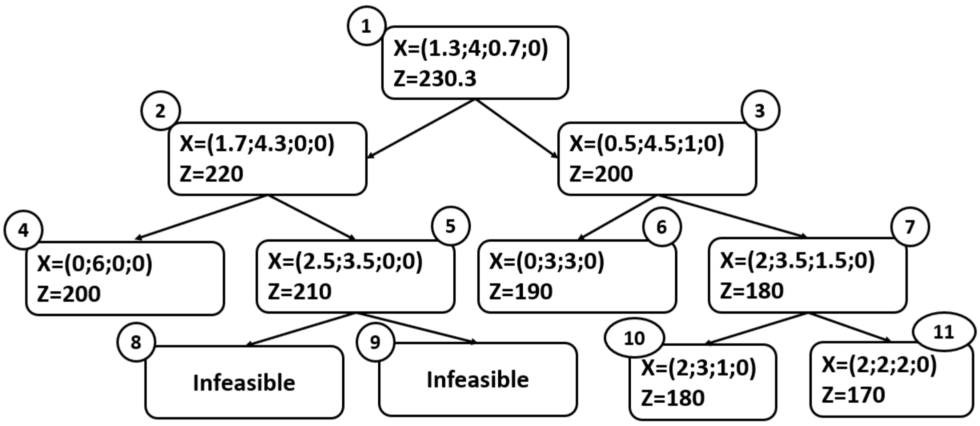
14. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 6 B. 8 C. 7 D. 5

15. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 9 B. 11 C. 4 D. 6

16. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 2 years later. B. 27 months later. C. 26 months later. D. 25 months later.

17. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. 244/19 B. Infeasible C. 301/19 D. 284/19

18. (L.O.2.1) Which of the following is false?

A. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.

B. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

C. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

D. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

For questions 19–20, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$

$$\omega_1\ x_1 + x_2 \leq 7,$$

$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \textbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \textbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

19. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.

B. Other answer.

C. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.

D. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.

20. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.

B. Other answer.

C. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.

D. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$

21. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs $3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs $4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 6 packets of VitaPlus and 8 packets of BeHealthy

B. 18 packets of VitaPlus and 0 packets of BeHealthy

C. 0 packets of VitaPlus and 16 packets of BeHealthy

D. Has no optimal solution

22. (L.O.2.1) Which of the following is false?

A. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.

B. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

C. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.

D. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).

23. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 48.47 B. 40.44 C. 19.58 D. 31.75

24. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 79.1112M B. 853.9739M C. 258.0852M D. 420.7616M

25. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 19 B. 20 C. 16 D. 15

....END OF EXAM...

Solution 1816

1. D.
2. C.
3. A.
4. C.
5. A.
6. A.
7. D.
8. C.
9. D.
10. C.
11. D.
12. C.
13. C.
14. A.
15. A.
16. C.
17. A.
18. C.
19. D.
20. B.
21. A.
22. C.
23. B.
24. A.
25. C.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY **FACULTY OF CSE**	**MIDTERM**	Semester/Academic year	1	2023-2024
		Date		23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1817

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 6 B. 7 C. 8 D. 5

2. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 2 years later. B. 26 months later. C. 27 months later. D. 25 months later.

3. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 18 B. 14 C. 5 D. 27

4. (L.O.2.1) Which of the following is false?

A. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.

B. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.

C. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

D. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.

5. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 60% and 40%. B. 66.67% and 33.33%. C. Other answer. D. 90% and 10%.

6. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 20. B. 84. C. 18. D. 24.

7. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 52 and 21. B. 37 and 32. C. 45 and 50. D. 48 and 24.

8. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 19 B. 16 C. 20 D. 15

9. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

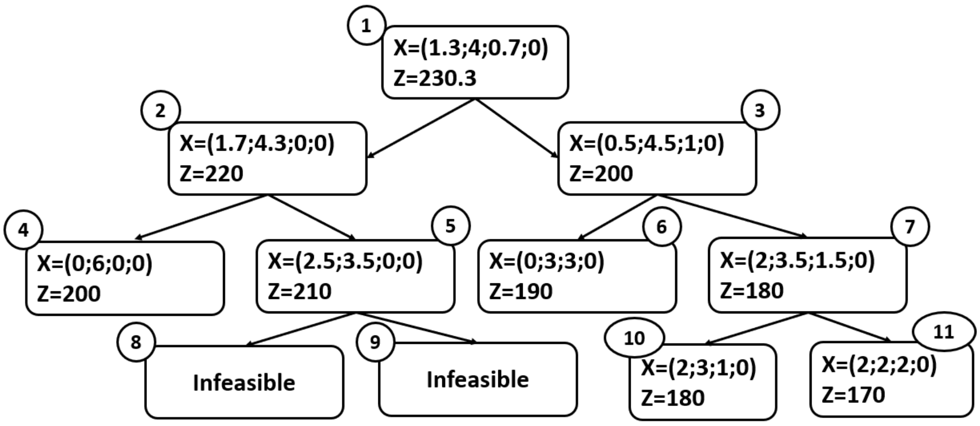
$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. 244/19 B. 301/19 C. Infeasible D. 284/19

10. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 79.1112M B. 258.0852M C. 853.9739M D. 420.7616M

11. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 9 B. 4 C. 11 D. 6

12. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

C. None of the other answers are correct.

D.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

13. (L.O.2.1) A person saves \$100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 389.6. B. 1280.93. C. 1312.68. D. 112.68.

14. (L.O.3.2)

Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 48.47 B. 19.58 C. 40.44 D. 31.75

15. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 355M

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ 1 \le B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

C. None of the other answers are correct.

D. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \le 9 \\ B_8, B_{10}, B_{50}, B_{56} \le 4 \\ 0 \le B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \le 370 \end{cases}$; min cost: 345M

For questions 16–17, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \ge 0, x_2 \ge 0,$$
$$\omega_1\ x_1 + x_2 \le 7,$$
$$\omega_2\ x_1 + x_2 \ge 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

16. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1,\ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ 0)$.

B. $\widehat{\boldsymbol{\omega}}_U = (1,\ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.

C. Other answer.

D. $\widehat{\boldsymbol{\omega}}_U = (1,\ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ 6)$.

17. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ 0)$.

B. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 4)$.

C. Other answer.

D. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ 2)$

For questions 18–20, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector** $\mathbf{D} = (D_1, D_2)^T$, meaning

$$0 \le z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \textbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \textbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j $(j = 1, \ldots, m)$ is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \ldots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \ldots, 3$.

18. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
B. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
C. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$
D. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$

19. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}} \; (Z_2 = \; \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z}, \\ 0 \leq \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \geq 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 540$
B. $Z_2 = 280$
C. $Z_2 = 130$
D. $Z_2 = 400$

20. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \; \boldsymbol{b}^T \cdot \boldsymbol{x} + \; Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1700$
B. $G_{\min} = 2240$
C. $G_{\min} = 3240$
D. $G_{\min} = 1980$

21. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Linearity.
B. Continuity.
C. Finiteness.
D. Additivity.

22. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\geq 10 \\ x_1 - 2x_2 + x_4 &\leq 0 \\ x_i &\geq 0. \end{aligned}$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and IV. B. Only IV. C. II and IV. D. I and III.

23. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 11 B. 18 C. 14 D. 17

24. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. 6 packets of VitaPlus and 8 packets of BeHealthy
B. 0 packets of VitaPlus and 16 packets of BeHealthy
C. 18 packets of VitaPlus and 0 packets of BeHealthy
D. Has no optimal solution

25. (L.O.2.1) Which of the following is false?

A. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.
B. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.
C. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.
D. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).

....END OF EXAM..

Solution 1817

1. A.
2. B.
3. A.
4. B.
5. B.
6. B.
7. D.
8. B.
9. A.
10. A.
11. A.
12. A.
13. B.
14. C.
15. A.
16. D.
17. C.
18. D.
19. B.
20. D.
21. B.
22. B.
23. D.
24. A.
25. B.

Lecturer:	*December 1st, 2023*	**Approved by:**	*December 1st, 2023*
(Signature and Fullname)		*(Signature and Fullname)*	

UNIVERSITY OF TECHNOLOGY FACULTY OF CSE	**MIDTERM**	Semester/Academic year	1	2023-2024
		Date		23/12/2023
	Course title	Mathematical Modeling		
	Course ID	CO2011		
	Duration	80 mins	Question sheet code	1818

Notes: - Students do not use course materials except one A4 hand-written sheet.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.2.1) Consider the linear programming problem below.
Optimize $F = 5x_1 - 4x_2$ subject to

$$x_1 + x_2 - x_3 \geq 10$$
$$x_1 - 2x_2 + x_4 \leq 0$$
$$x_i \geq 0.$$

Which of the following statements is true?

I. F must have a minimum on the given feasible region.

II. F must have a maximum on the given feasible region.

III. The feasible region is bounded.

IV. The feasible region is unbounded.

A. I and III. B. I and IV. C. Only IV. D. II and IV.

For questions 2–4, we use the following assumption.

A start-up company considers a production plan of $n = 2$ types of laptops: A (type 1)= Ultra-book and B (type 2) = Workstation, with breakthrough tech (like quantum chip) from 2025. Denote production vector $\boldsymbol{z} = (z_1, z_2)^T$ be the number of laptop units produced for types A and B. Due to both uncertainty (risk) and high demand of products we require the production quantities $\boldsymbol{z}$ must exactly fulfill a **random demand vector $\mathbf{D}$** $= (D_1, D_2)^T$, meaning

$$0 \leq z_i = d_i, \quad i = 1, \ldots, 2$$

where d_i are observed values of variables D_1, D_2. We define 1000 items for one unit of each variable in model, and assume that demand $D_1 \sim \mathbf{Bin}(10, \frac{1}{2})$ and $D_2 \sim \mathbf{Bin}(6, \frac{1}{3})$, both are binomial variables.

We next express the uncertainty of production by a two-stage Stochastic Linear Program (2-SLP).
Suppose the laptops need $m = 3$ basic parts (e.g. CPU, RAM and Graphic card) to produce, so let $m = 3$ decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T$ in the first stage, here x_j $(j = 1, \ldots, m)$ is the numbers of parts to be ordered before production of laptops type A and B above. Specifically x_1 is the number of CPUs, x_2 is the number of RAM and x_3 is the number of Graphic cards (in units of 1000 items).
The number of parts y_j left in inventory depends on the number of ordered parts x_j by equation

$$y_j = x_j - \sum_{i=1}^{n} a_{ji}\, z_i, \quad j = 1, \ldots, m$$

or in matrix form

$$\boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T\, \boldsymbol{z},$$

where $\boldsymbol{y} = (y_1, y_2, y_3)^T$, $\mathbf{A} = [a_{ij}]$ is the coefficient matrix (of production demand) with dimension $n \times m = 2 \times 3$, with constant entries on the first row are $a_{1j} = 2$ and on the second row are $a_{2j} = 1$ for $j = 1, \ldots, 3$.

2. Assume the company uses the **No waiting** approach (in the 1st stage), we fix decision variables $\boldsymbol{x} = (x_1, x_2, x_3)^T = (12, 14, 17)$ beforehand, and assume the production would follow `Unbiased` scenario (the mean scenario), when production vector $\boldsymbol{z}$ is the mean of demand $\mathbf{D} = (D_1, D_2)$. The *production* $\boldsymbol{z}$ and the *inventory vector* $\boldsymbol{y} = (y_1, y_2, y_3)^T$ [the vector of parts y_j left in inventory] are respectively found as

A. $z_1 = 5, z_2 = 2$ and $y_1 = 0, y_2 = 2; y_3 = 5$
B. $z_1 = 5, z_2 = 2$ and $y_1 = 5, y_2 = 2; y_3 = 0$
C. $z_1 = 2, z_2 = 5$ and $y_1 = 0, y_2 = 2; y_3 = 5$
D. $z_1 = 2, z_2 = 3$ and $y_1 = 2, y_2 = 5; y_3 = 8$

3. In the 2nd phase of our 2-stage Stochastic Program, we define model

$$\begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}} \; (Z_2 = \; \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y}) \\ \text{with} \quad \boldsymbol{c} = (c_i) \text{ are production cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - \mathbf{A}^T \; \boldsymbol{z}, \\ 0 \leq \boldsymbol{z} = \boldsymbol{d}, \quad \boldsymbol{y} \geq 0. \end{cases} \tag{1}$$

where vector $\boldsymbol{c} = (c_1, c_2)^T$ keeps costs to make each laptop of product type 1 and type 2, vector $\boldsymbol{s} = (s_1, s_2, s_3)^T$ keeps the salvage value per unit of part j not used (after the demand is known), $j = 1, 2, 3$.
This model (of only the 2nd stage of our SP) practically accepted that production meets demand, $\boldsymbol{z} = \boldsymbol{d}$. The objective $Z_2 = Q(\boldsymbol{z}, \boldsymbol{y}, \boldsymbol{d}) = Q(\boldsymbol{x}, \boldsymbol{d})$ obviously depends on both pre-determined decision $\boldsymbol{x}$ in stage 1 and also random demand $\boldsymbol{z} = \boldsymbol{d}$ sorted out by binomial demand $\boldsymbol{D}$ (the mean scenario) in the above question. We plan production with production costs $\boldsymbol{c} = (c_1, c_2) = (70, 30)^T$ (in USD) and salvage values $\boldsymbol{s} = (25, 15, 20)^T$ (in USD). The value of objective function Z_2 (unit in 1000 USD) is

A. $Z_2 = 400$
B. $Z_2 = 540$
C. $Z_2 = 280$
D. $Z_2 = 130$

4. Our whole model (of the two-stage Stochastic Linear Program) following the **Scenario approach** now is determined from the following optimization problem

$$\min(G = g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = \; \boldsymbol{b}^T \cdot \boldsymbol{x} + \; Z_2), \tag{2}$$

where $\boldsymbol{b}^T = (b_1, b_2, b_3) = 2 \cdot \boldsymbol{s}^T$ built by pre-order cost b_j per unit of part j (before the demand is known), and $Z_2 = Q(\boldsymbol{x}, \boldsymbol{d})$ defined in Equation (1). If we still plan production with ordering decision $\boldsymbol{x} = (12, 14, 17)$, the salvage values $\boldsymbol{s} = (25, 15, 20)^T$, the random demand $\boldsymbol{z} = \boldsymbol{d}$ (chosen by the Unbiased scenario of binomial demand $\boldsymbol{D}$), then the optimal value of G is

A. $G_{\min} = 1980$
B. $G_{\min} = 1700$
C. $G_{\min} = 2240$
D. $G_{\min} = 3240$

5. (L.O.3.1) A dietician recommends that a particular individual must consume a minimum of 18 units of calcium, 16 units of iron, and 14 units of zinc each week. The person would like to make sure that she complies with the diet by buying some food supplements containing all the nutrients she needs from her local health shop, which sells packets of 'VitaPlus' and 'BeHealthy'. She would like to choose a viable combination of these supplements at a minimal cost. VitaPlus costs \$3 a packet and contains 1 unit of calcium, 4 units of iron, and 1 unit of zinc. A packet of BeHealthy costs \$4 and contains 1.5 units of calcium, 1 unit of iron, and 1 unit of zinc. What is the number of packets of VitaPlus and BeHealthy that need to be bought?

A. Has no optimal solution
B. 6 packets of VitaPlus and 8 packets of BeHealthy
C. 0 packets of VitaPlus and 16 packets of BeHealthy
D. 18 packets of VitaPlus and 0 packets of BeHealthy

6. (L.O.3.2)
Nam intends to start saving funds for his daughter's college education. He is considering depositing a certain amount of money into an account with a 6.23% annual interest rate. The goal is to accumulate 120 million VND after 18 years. What is the closest amount (in million VND) he should deposit? Choose the most correct answer.

A. 31.75
B. 48.47
C. 19.58
D. 40.44

For questions 7–8, we use the following assumption.

Consider the following optimization problem

$$\text{Minimize } (Z = x_1 + x_2),$$

subject to

$$x_1 \geq 0, x_2 \geq 0,$$
$$\omega_1\ x_1 + x_2 \leq 7,$$
$$\omega_2\ x_1 + x_2 \geq 4$$

where the last two conditions depend on random parameters $\omega_1 \sim \mathbf{Unif}(-2, 4)$ (uniform random variable) and $\omega_2 \sim \mathbf{Bin}(6, \frac{1}{2})$ (binomial random variable). Put vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$.
In the `Guessing at uncertainty` method we might guess reasonable values of $\boldsymbol{\omega}$ in a few ways namely *Unbiased* (choosing the mean values), *Pessimistic* (choosing the min values) and *Optimistic* (choosing the max values).

7. The values of vector $\boldsymbol{\omega}$ when using the `Unbiased method` and `Optimistic` method, denoted by $\widehat{\boldsymbol{\omega}}_U$ and $\widehat{\boldsymbol{\omega}}_O$, respectively are

A. $\widehat{\boldsymbol{\omega}}_U = (1,\ \ 3)$ and $\widehat{\boldsymbol{\omega}}_O = (4,\ \ 6)$.
B. $\widehat{\boldsymbol{\omega}}_U = (1,\ \ \frac{2}{3})$ and $\widehat{\boldsymbol{\omega}}_O = (2,\ \ 0)$.
C. $\widehat{\boldsymbol{\omega}}_U = (1,\ \ 0)$ and $\widehat{\boldsymbol{\omega}}_O = (4, 3)$.
D. Other answer.

8. If we use the `Pessimistic` way then the optimal value Z_m and optimal point $\widehat{\boldsymbol{x}}_{Opt}$ respectively are

A. $Z_m = 4$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ \ 2)$
B. $Z_m = \frac{50}{10}$ and $\widehat{\boldsymbol{x}}_{Opt} = (2,\ \ 0)$.
C. $Z_m = 3$ and $\widehat{\boldsymbol{x}}_{Opt} = (0,\ \ 4)$.
D. Other answer.

9. (L.O.3.2)
Assume the growth of a fish population in a lake can be modeled using

$$\frac{dN}{dt} = 0.25N\left(1 - \frac{N}{100}\right),$$

where $N = N(t) \geq 0$ is the population size at time $t \geq t_0 = 0$ (in month) with initial size 30 fishes at t_0. In how many months the fish population size reaches approximately 65.76 fishes? Choose the most correct answer.

A. 5
B. 6
C. 7
D. 8

10. (L.O.2.1) A person saves $100 in a bank account at the beginning of each month. The bank offers a return of 12% compounded monthly. Determine the total amount saved after 12 months.

A. 112.68.
B. 389.6.
C. 1280.93.
D. 1312.68.

11. (L.O.3.1)
When designing a car park with an area of 7200 square feet to optimize the use of available space, the plan includes allocating some spaces for small cars and the remainder for large cars. Each small car space is designated 90 square feet, while each large car space is allocated 120 square feet. It is a requirement that every car occupies a space of the appropriate size. The anticipated ratio of small cars to large cars wishing to park at any given time is expected to be neither less than 2:3 nor greater than 2:1. To maximize the number of parked cars, the optimal numbers of spaces for small and large cars respectively are

A. 48 and 24.
B. 52 and 21.
C. 37 and 32.
D. 45 and 50.

12. (L.O.2.1) Which of the following is false?

A. Rounding non-integer solution values up to the nearest integer value can result in an infeasible solution.
B. When solving a minimization problem, the least attainable loss linked to the relaxed solution (LP-relaxation) is no larger than the optimal value connected with the integer solution.
C. Suppose we have a non-integer optimal solution for a Linear Programming problem. We round off the non-integer optimal solution to an integer solution. Now this integer solution may not be an optimal solution for the corresponding Integer Linear Programming problem because the integer solution needs to satisfy additional constraints.
D. The number of nodes considered in a branch and bound tree for maximization integer programming problems is always minimized by going to the node with the largest upper bound.

13. (L.O.3.2) A transportation company currently has 9 buses for 840 customers. The company estimates that a bus can serve at most 100 customers. Knowing that the customers' number is growing continuously at a fixed rate and will be doubled after 5 years. What is the number of buses the company will need to add after 100 months?

A. 27 B. 18 C. 14 D. 5

14. (L.O.2.1) The *maximum* number of potential extreme points for a linear programming problem with 3 non-negativity constraints on variables and 6 other (main) constraints is

A. 24. B. 20. C. 84. D. 18.

15. (L.O.2.1) Assume a population of fish grows exponentially. A pond is stocked initially with 500 fish. After 6 months, there are 1000 fish in the pond. The owner will allow his friends and neighbors to fish on his pond after the fish population reaches 10,000. When will the owner's friends be allowed to fish?

A. 25 months later. B. 2 years later. C. 26 months later. D. 27 months later.

16. (L.O.3.1) A transportation company wants to manage a 370M budget project to go from the centre of Ho Chi Minh City to various universities in Thu Duc. There are 4 routes that go directly to the universities: 8, 10, 50, and 56; and the cost of assigning a bus to each route is 40M, 35M, 45M, and 50M respectively. The company has 9 buses and each route needs at least 1 bus and can have at most 4 buses while all the buses should be assigned to maximize the utility.
Which of the following systems is the constraint for the LP model that represents the problem of minimizing the cost for the project? Furthermore, what is the min cost of the LP problem?

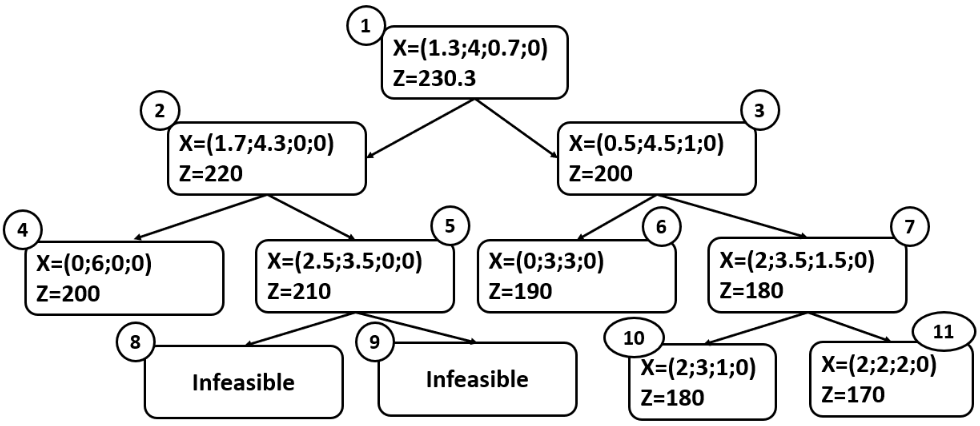
A. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 345M

B. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} = 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} \leq 370 \end{cases}$; min cost: 355M

C. $\begin{cases} B_8 + B_{10} + B_{50} + B_{56} \leq 9 \\ 1 \leq B_8, B_{10}, B_{50}, B_{56} \leq 4 \\ 0 \leq B_8, B_{10}, B_{50}, B_{56} \\ 40B_8 + 35B_{10} + 45B_{50} + 50B_{56} = 370 \end{cases}$; min cost: 370M

D. None of the other answers are correct.

17. (L.O.3.1) The following binary tree represents the branch and bound process of an ILP problem.



Which of the following nodes (called by number) is the stopping point of the branch and bound process if we follow a breadth-first approach?

A. 6 B. 9 C. 4 D. 11

18. (L.O.2.1) Consider a linear programming

$$\max_{x_i}(5x_1 + 4x_2 + 6x_3 + 8x_4)$$

$$s.t. \qquad 4x_1 + 3x_2 + 7x_3 + 3x_4 \leq 11$$

$$x_i \in \{0, 1\}$$

The optimal value is

A. 17 B. 11 C. 18 D. 14

19. (L.O.2.1) Which of the following is false?

A. For LP problems, if multiple optimal solutions exist, then an optimal solution at an extreme point in the feasible domain may have an adjacent extreme point in the feasible domain that also is optimal (the same value of the objective function).
B. For minimization LP problems, if the objective function evaluated at an extreme point in the feasible domain is no larger than its value at every adjacent extreme point in the feasible domain, then that solution is optimal.
C. For LP problems, only extreme points in the feasible domain can be optimal solutions, so the number of optimal solutions cannot exceed the number of extreme points in the feasible domain.
D. The simplex method's minimum ratio rule for choosing the leaving basic variable is used because making another choice with a larger ratio would yield a basic solution that is not feasible.

20. (L.O.3.1)
Find

$$\max(5x + 3y + 2z + 7u + 4v)$$

subject to

$$2x + 8y + 4z + 2u + 5v \leq 10 \text{ and } x, y, z, u, v \in \{0, 1\}.$$

A. 15 B. 19 C. 16 D. 20

21. (L.O.2.1) Which property of Linear Programming is violated in Integer Linear Programming among the following options?

A. Additivity. B. Linearity. C. Continuity. D. Finiteness.

22. (L.O.3.1)
Find

$$\max(x + 2y)$$

subject to

$$x + 4y \leq 20, x + y \geq 8, 5x + y \leq 32, \text{ and } x, y \geq 0.$$

A. 284/19 B. 244/19 C. 301/19 D. Infeasible

23. (L.O.3.2) A transportation company has a starting revenue and cost of 230M, and 370M respectively in 2020. Knowing that every year the revenue will grow by 30% and the cost will grow by 10%. What is the total profit of the company at the end of 2025?

A. 420.7616M B. 79.1112M C. 258.0852M D. 853.9739M

24. (L.O.3.1) Given the starting Tableau for the simplex method to maximize $10x_1 + 50x_2 + 30x_3 + 60x_4$

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
	1	1	1	1	0	0	0	0	0	5
	35	40	45	50	0	0	0	0	0	200
	1	0	0	0	0	1	0	0	0	3
	0	1	0	0	0	0	1	0	0	3
	1	0	1	0	0	0	0	1	0	3
	0	1	0	1	0	0	0	0	1	3
max	-10	-50	-30	-60	0	0	0	0	0	0

Which of the following can be a final tableau when applying the simplex method to solve the problem?

A.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_3	3	0	1	0	-0.2	0	-2	0	0	1.3
x_2	0.7	1	0	0	0	0	1	0	0	3
s_2	0.3	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0	0	0	0	1	0	3
x_4	0.3	0	0	1	0.07	0	-0.3	0	0	0.7
max	40	0	0	0	6	0	50	0	0	230.3

B.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0.3	0	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0	0	0	1	0	0	3
x_4	0	0	0.7	1	0.07	0	-0.3	0	0	0.7
s_4	0	0	1	0	0	0	0	1	0	3
s_2	0	0	-0.3	0	0.07	1	0.7	0	0	1.7
s_5	0	0	-0.7	0	-0.07	0	0.3	0	1	2.3
max	0	0	13.3	0	3.3	0	23.3	0	0	203.3

C.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
x_1	1	0	0	0.3	-0.07	0	-0.7	0	0	1.3
x_2	0	1	0	0.7	0	0	1	0	0	3
x_3	0	0	1	1	0.07	0	-0.3	0	0	0.7
s_2	0	0	0	0	0.07	1	0.7	0	0	1.7
s_5	0	0	0	0	-0.07	0	0.3	0	1	2.3
s_4	0	0	0	0.7	0	0	0	1	0	3
max	0	0	0	11.3	3.3	0	23.3	0	0	183.17

D. None of the other answers are correct.

25. (L.O.2.4) Consider a regional airport supported by two major airlines: Vietnam Airlines and Vietjet Air, each flying out to respective hubs. We surveyed the weekly local business travelers and found that 80% of those who traveled on Vietnam Airlines traveled again on Vietnam Airlines, and 20% switched to fly Vietjet Air. Of those who traveled on Vietjet Air, 60% traveled again on Vietjet Air, but 40% switched to Vietnam Airlines. We assume these tendencies continue weekly and that no additional local business travelers enter or leave the system. Then, in the long term, the proportions of passengers will have traveled on Vietnam Airlines and VietJet Air are:

A. 90% and 10%. B. 60% and 40%. C. 66.67% and 33.33%. D. Other answer.

....END OF EXAM..

Solution 1818

1. C.
2. A.
3. C.
4. A.
5. B.
6. D.
7. A.
8. D.
9. B.
10. C.
11. A.
12. C.
13. B.
14. C.
15. C.
16. B.
17. B.
18. A.
19. C.
20. C.
21. C.
22. B.
23. B.
24. B.
25. C.