Near-optimal finite-thrust orbital control near a binary asteroid system

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Introduction

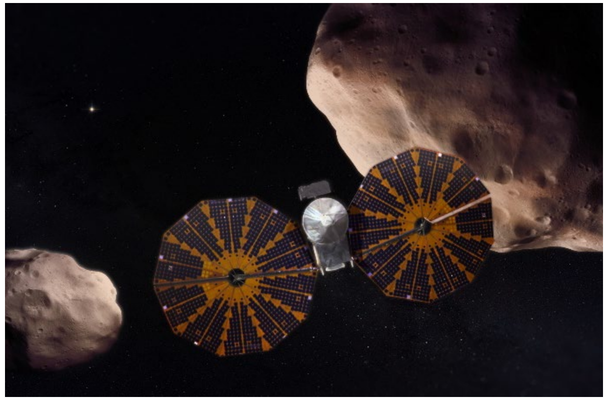
Binary Asteroid System

- Up to 15% of NEAs and MBAs
- Solar System Evolution
- Origin of Life
- Planetary Defense
- Resource Extraction

66391 Moshup

Ongoing & Future Missions

- 21900 Orus—Trojan binary, to be visited by NASA's Lucy
- 65803 Didymos—destination of AIDA, the first mission heading to binary asteroids



Propulsion System

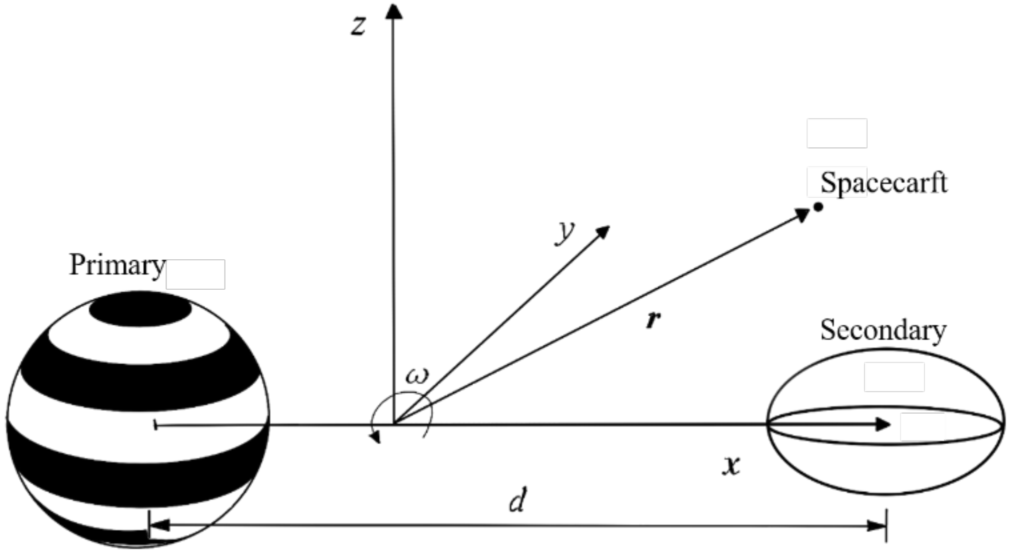
- Impulsive ?
- Low-thrust propulsion

Background

Model Description

Simplifying Assumptions

Doubly Synchronous	Circular Orbit
• **Tidal Lock** of 66391 Moshup • **Spheroidal Shape** of the Primary	• Orbital Eccentricity **e=0.01**



$$\begin{cases} \ddot{x} - 2\omega\dot{y} - \omega^2 x = \dfrac{\partial U}{\partial x} \\ \ddot{y} + 2\omega\dot{x} - \omega^2 y = \dfrac{\partial U}{\partial y} \\ \ddot{z} = \dfrac{\partial U}{\partial z} \end{cases}$$

$$U = U_{sh}(\boldsymbol{r} + \mu\boldsymbol{d}) + U_{ell}[\boldsymbol{r} - (1-\mu)\boldsymbol{d}]$$

Background

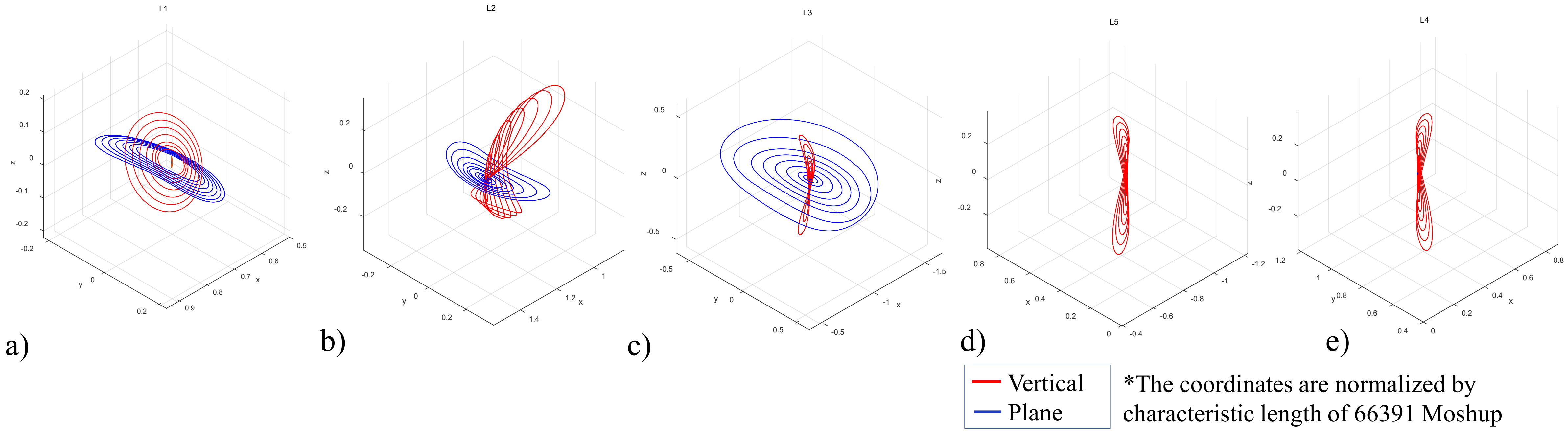
Libration Points

$$\frac{\partial \bar{U}}{\partial x} = \frac{\partial \bar{U}}{\partial y} = \frac{\partial \bar{U}}{\partial z} = 0$$

Table 1 66391 Moshup libration points

*Normalized by characteristic length of 66391 Moshup	x	y	z
L1	7.016×10^{-1}	2.032×10^{-4}	2.325×10^{-4}
L2	1.238	4.553×10^{-5}	3.768×10^{-5}
L3	−1.025	8.417×10^{-3}	4.240×10^{-4}
L4	4.355×10^{-1}	8.727×10^{-1}	1.003×10^{-5}
L5	4.360×10^{-1}	8.720×10^{-1}	2.463×10^{-4}

Libration Point Orbits



*The coordinates are normalized by characteristic length of 66391 Moshup

Bilinear Tangent Guidance

- Traditionally applied to **SPR**
- **Time-optimal** in a **planar** gravitational field

Spacecraft's Motion

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{v} \end{bmatrix}$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{g}(\boldsymbol{r},t) + \frac{F}{m}\boldsymbol{u} \end{bmatrix}$$

Deduction

When minimizing the flight time

$$H = L(t,\boldsymbol{x},\boldsymbol{u}) + \boldsymbol{\lambda}^T f(t,\boldsymbol{x},\boldsymbol{u})$$

Its costate equations

$$\dot{\lambda}_r = -\frac{\partial H}{\partial \boldsymbol{r}} = -\lambda_v^T \left[\frac{\partial \boldsymbol{g}(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\right]$$

$$\dot{\lambda}_v = -\frac{\partial H}{\partial \boldsymbol{v}} = -\lambda_r$$

$$\ddot{\lambda}_v = \boldsymbol{\lambda}_v^T \left[\frac{\partial \boldsymbol{g}(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\right]$$

$$\hat{\boldsymbol{u}}^* = -\frac{\boldsymbol{\lambda}_v}{\lambda_v}$$

A planar gravitational field:

$$\ddot{\lambda}_v = \lambda_v^T \left[\frac{\partial g(\boldsymbol{r},t)}{\partial \boldsymbol{r}}\right] = 0$$

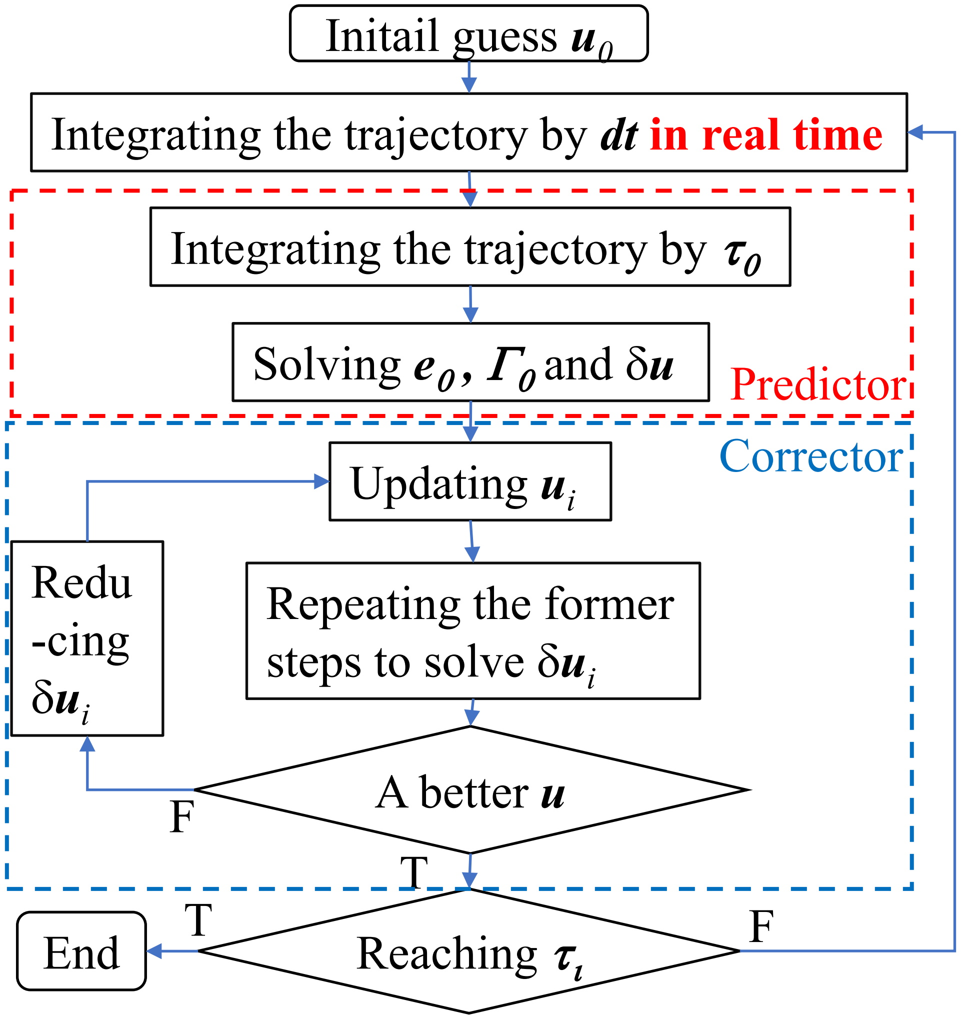
$\boldsymbol{\lambda}_v$ be integrated twice

$$\lambda_v = \lambda_v(t_0) - \lambda_r(t_0)t \Rightarrow \boldsymbol{\lambda}_v = \boldsymbol{\alpha} + \boldsymbol{\beta}t$$

$$\hat{\boldsymbol{u}}^* = -\frac{\boldsymbol{\lambda}_v}{\lambda_v} = \frac{\alpha + \beta t}{|\alpha + \beta t|}$$

Bilinear Tangent Guidance

The Predictor-Corrector



Control Profiles $\boldsymbol{u}_0$

$$u_0 = \begin{bmatrix} \boldsymbol{\alpha}_0 \\ \boldsymbol{\beta}_0 \\ \tau_0 \end{bmatrix}$$

Solving Errors

Error vector e

$$e_{cor}(\boldsymbol{x},\boldsymbol{u},t) = \begin{bmatrix} (\boldsymbol{r}(t_f)-\boldsymbol{r}_t)\times k_r(\tau - t) \\ (\boldsymbol{v}(t_f)-\boldsymbol{v}_t)\tau \times k_v t \end{bmatrix}$$

Jacobian Γ

$$\boldsymbol{\Gamma}(t_0,t_f) = \begin{bmatrix} \frac{\partial \boldsymbol{r}(t_f)}{\partial \boldsymbol{\alpha}} & \frac{\partial \boldsymbol{r}(t_f)}{\partial \boldsymbol{\beta}} & \frac{\partial \boldsymbol{r}(t_f)}{\partial \tau} \\ \frac{\partial \boldsymbol{v}(t_f)}{\partial \boldsymbol{\alpha}} & \frac{\partial \boldsymbol{v}(t_f)}{\partial \boldsymbol{\beta}} & \frac{\partial \boldsymbol{r}(t_f)}{\partial \tau} \end{bmatrix}$$

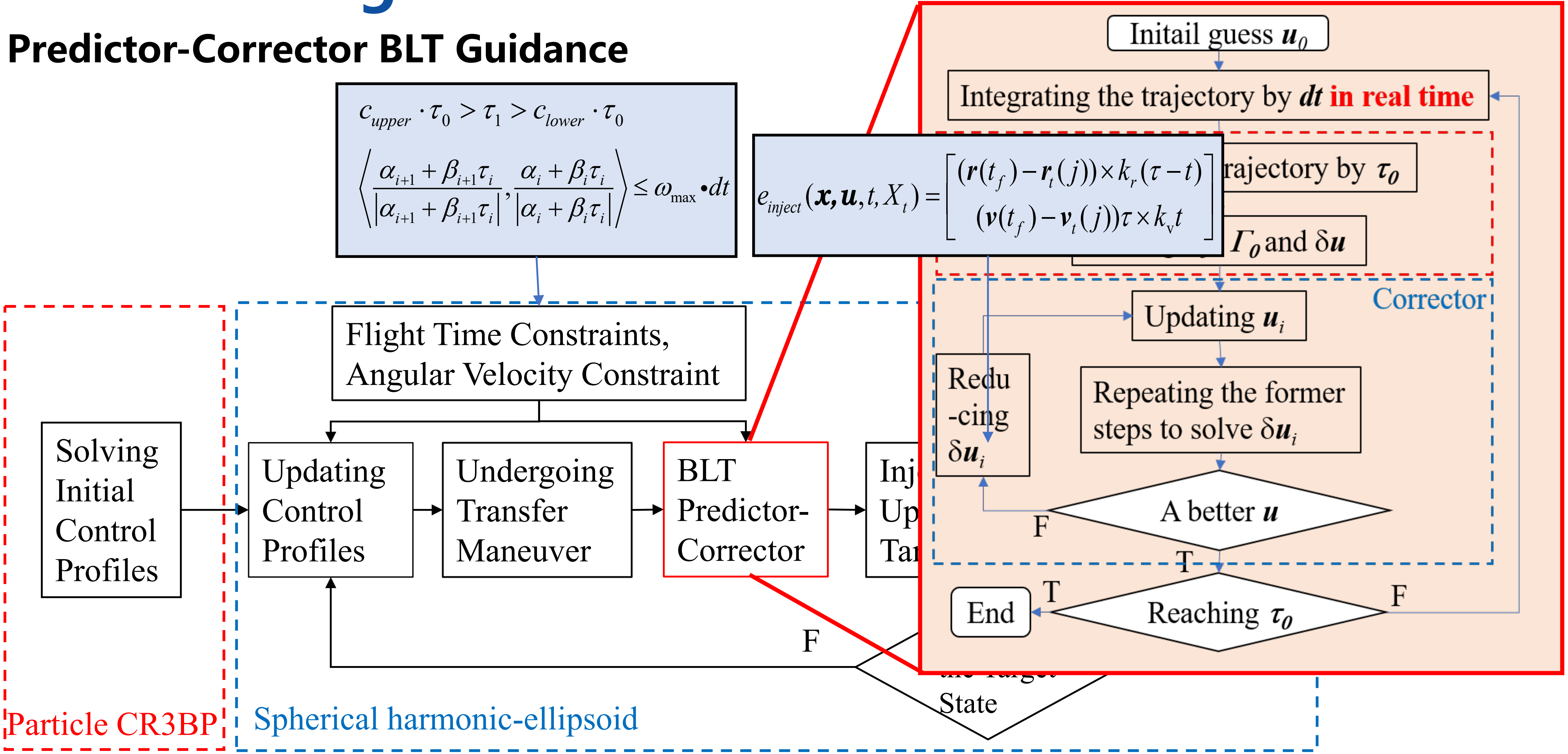
Newton-Raphson Method

$$\boldsymbol{u}^{j+1} = \boldsymbol{u}^j - \boldsymbol{\Gamma}(\boldsymbol{u}^j)^T[\boldsymbol{\Gamma}(\boldsymbol{u}^j)\bullet\boldsymbol{\Gamma}(\boldsymbol{u}^j)^T]^{-1}\boldsymbol{e}(\boldsymbol{u}^j)$$

$$\Gamma\delta\boldsymbol{u} = -\boldsymbol{e}$$

Bilinear Tangent Guidance

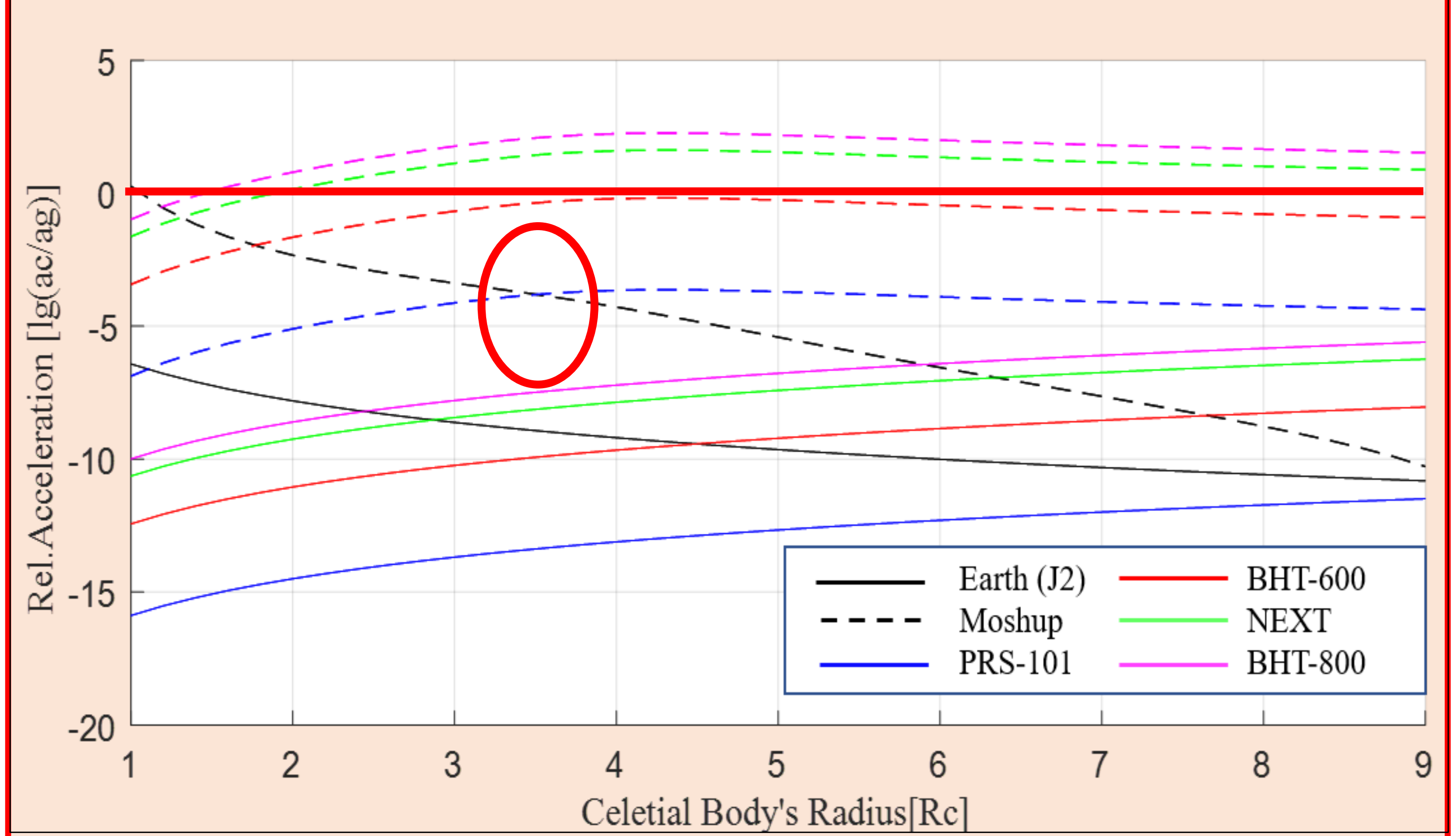
Predictor-Corrector BLT Guidance



Transfer Trajectory Simulation

Finite thrust near binary systems

Relative accelerations of finite-thrust systems near Earth and 66391 Moshup



Performance of Finite thrusters

Thrusters	Type	F[mN]	I_{sp} [s]
PRS-101	PPT	1.48	1350
BHT-600	Hall	39	1500
NEXT	Ion	236	4190
BHT-8000	Hall	449	2210

- *The denominator of the ratio is contributed by the corresponding planetry central gravity(Earth and 66391 Mosup’s Primary)

Transfer Trajectory Simulation

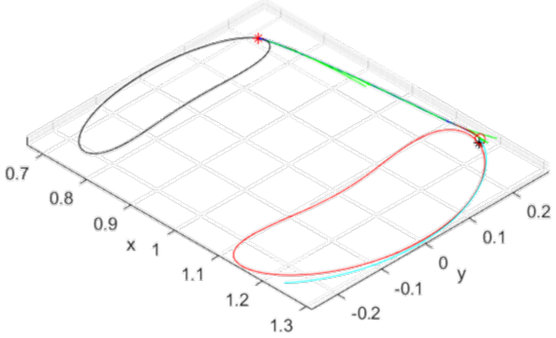
Transfer trajectories between different libration point orbits

Spacecraft's Parameter in Simulation

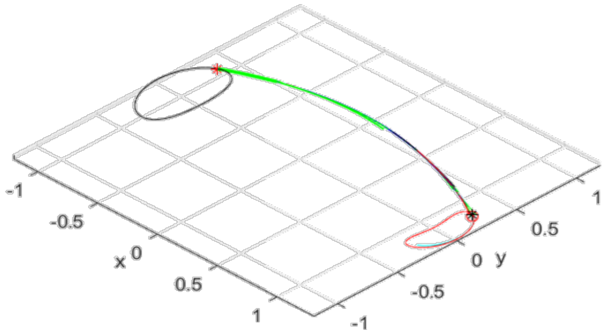
Mass[kg]	Prop.Sys.	F[mN]	Isp[s]
2.36e3	NEXT(Ion)	236	4190

— Initial Orbit
— Final Orbit
— Transfer Trajectory
— Predictions by BLT
— Thrust-off Coasting

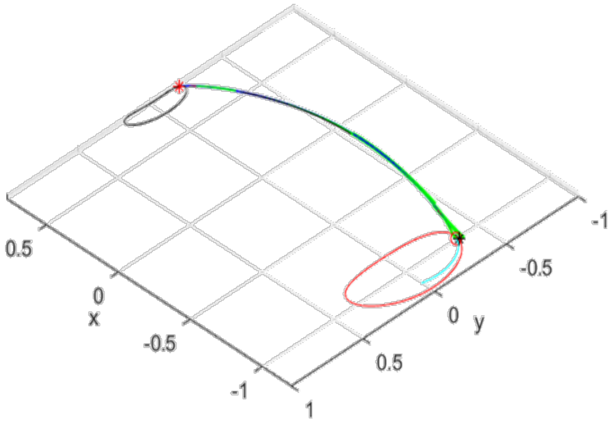
L1 Planar to L2 Planar



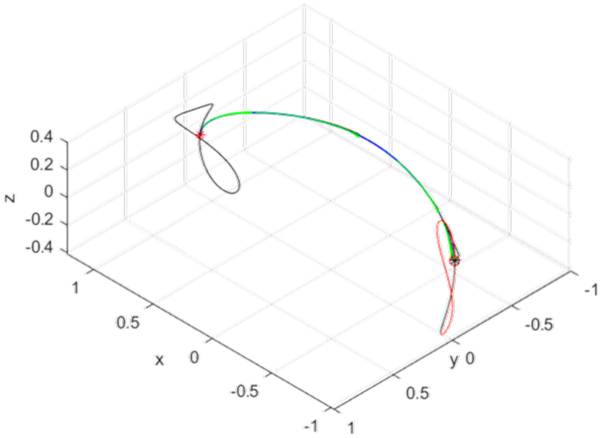
L3 Planar to L2 Planar



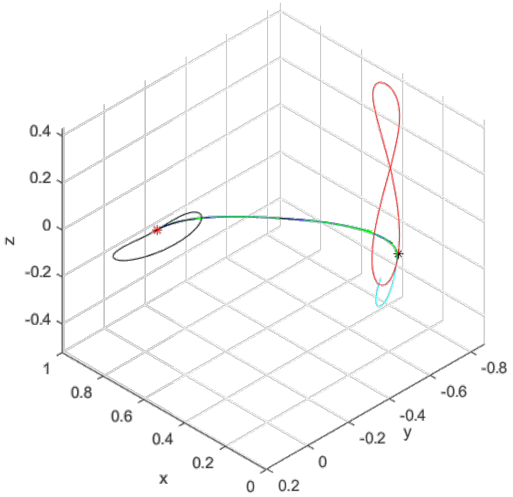
L1 Planar to L3 Planar



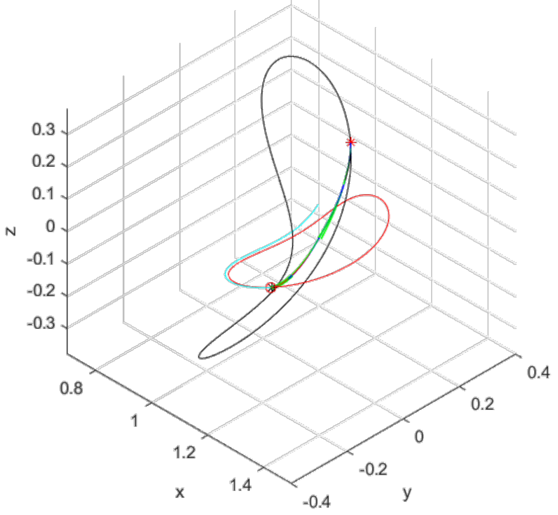
L2 Vert. to L3 Vert.



L1 Planar to L3 Vert.



L2 Vert. to L2 Planar



Transfer Trajectory Simulation

Transfer trajectories between different libration point orbits

Low-thrust performance using BLT

Initial Orbit	Initial C_J	Target Orbit	Target C_J	Pos. Error $\boldsymbol{R}_e$[m]	Vel. Error $\boldsymbol{V}_e$[m/s]	Flight time t_f[min]	Fuel Δm[g]
L1 Plane	0.2117	L2 Plane	0.2110	18.97	0.0013	103.26	34.90
L1 Plane	0.2119	L3 Plane	0.1967	37.03	0.0022	231.36	78.19
L3 Plane	0.1867	L2 Plane	0.2101	24.02	0.0046	261.54	88.38
L2 Vert.	0.1870	L3 Vert.	0.1766	12.53	0.0009	248.50	83.98
L1 Plane	0.2100	L5 Vert.	0.1797	18.91	0.0028	150.55	50.88
L2 Vert.	0.1866	L2 Plane	0.2101	4.225	0.0036	84.02	28.39

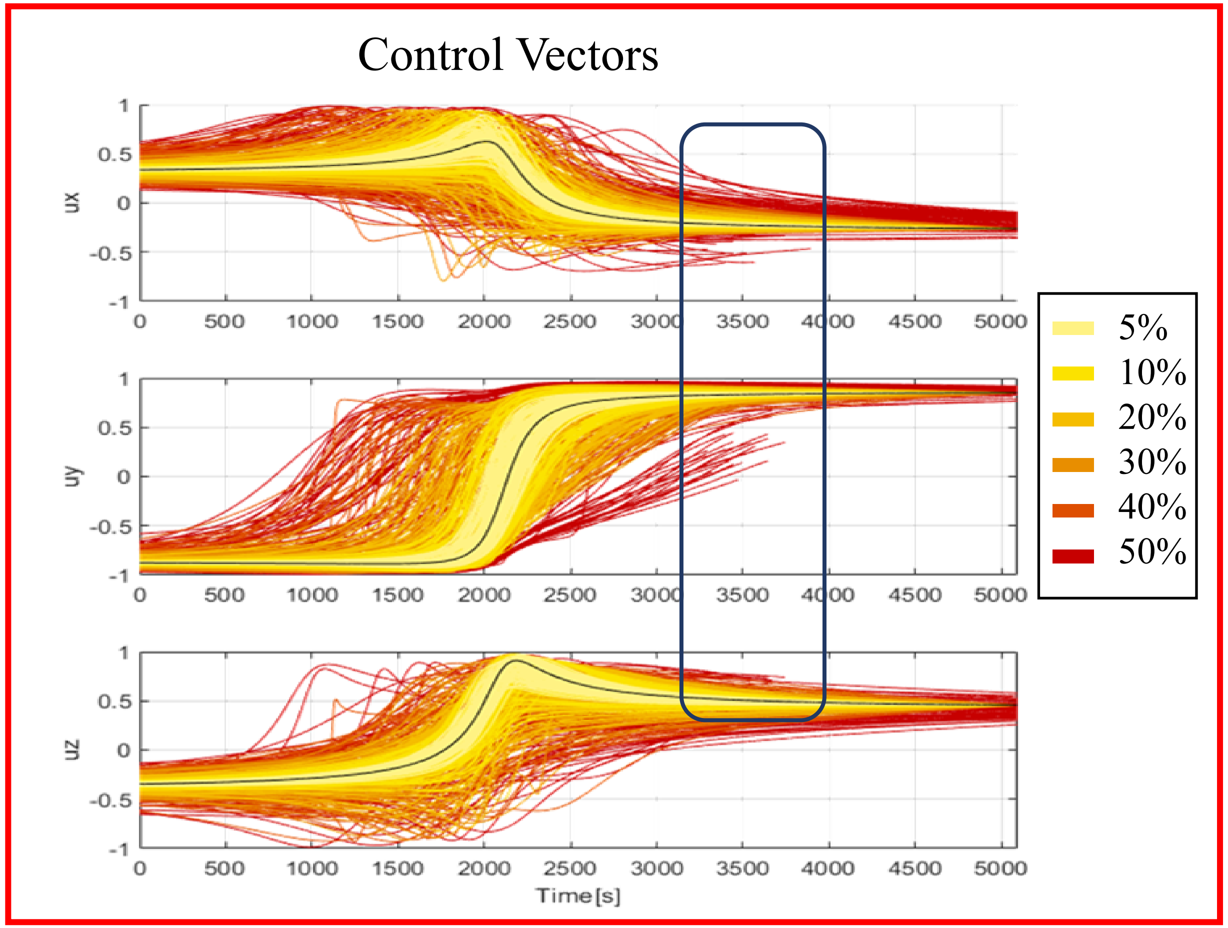
- Max maneuvers cost **88.38g**
- Fuel & time expenditure **roughly linear** to distance
- Position and velocity errors in the **same order of magnitude**

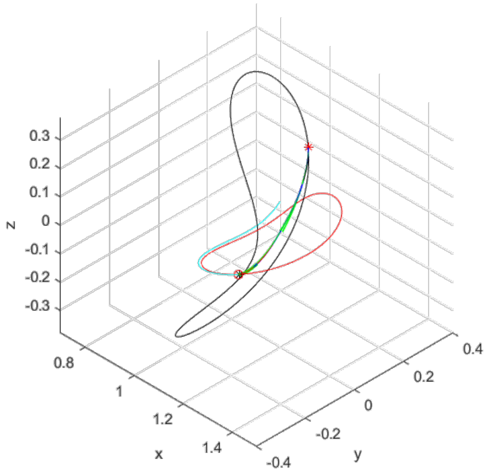
- **Fuel saving**
- **The accuracy and fuel usage insensitive to flight process or transfer conditions**
- **Very low computational overhead**

Transfer Trajectory Simulation

Corrected trajectories under perturbations

Perturbation simulations (L2 Vert. to Planar, perturbed N=500 Monte-Carlo)



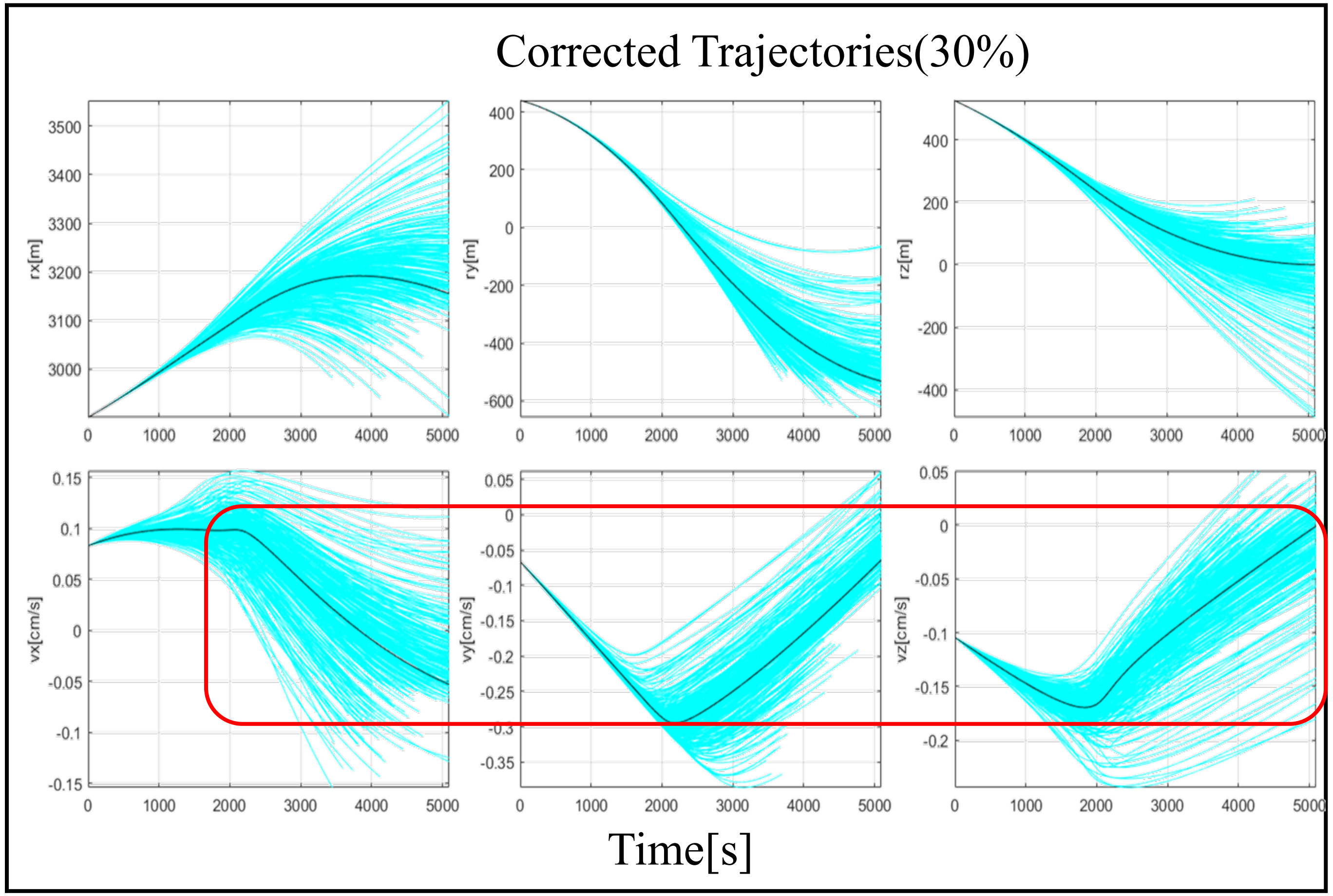


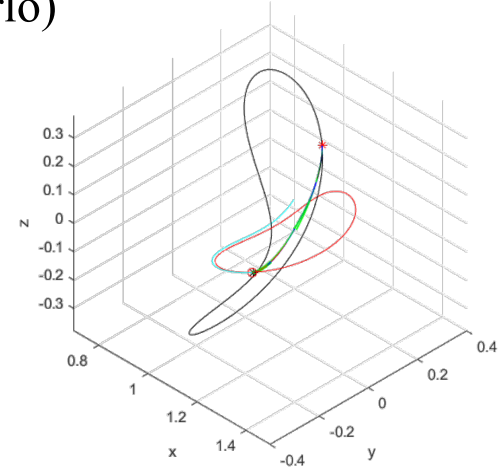
- Unconverged examples ended early
- Convergence limit within 30% perturbation

Transfer Trajectory Simulation

Corrected trajectories under perturbations

Perturbation simulations (L2 Vert. to Planar, 30% perturbed N=500 Monte-Carlo)

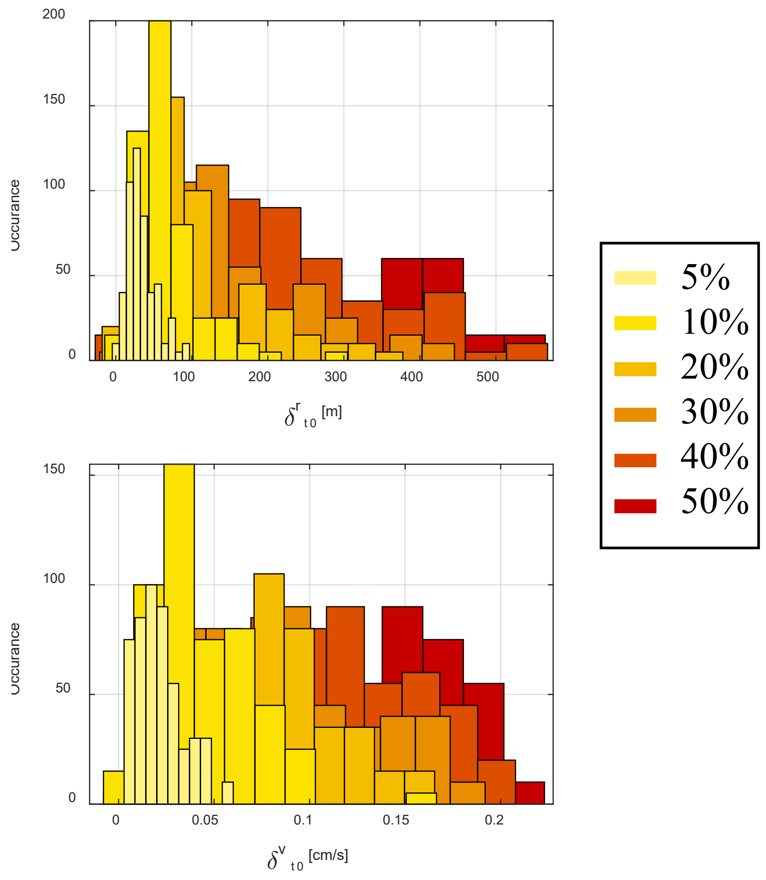




- Convergence limit within 30% perturbation
- Unconverged examples ended early
- Better velocity corrections

Transfer Trajectory Simulation

Corrected trajectories under perturbations



Perturbated Trajectory Statistics

Perturbation [%]	Converged Trajectories [%]	Mean Pos. Error[m]*	Mean Vel. Error[m/s]
5	99.4	26.42	2.24e-4
10	92.8	78.24	4.96e-4
20	86.2	106.33	9.35e-4
30	79.6	227.71	1.55e-3
40	64.8	386.12	1.96e-3
50	57.2	412.45	1.99e-3

* Pos. for Final Positions, Vel. for Final Velocity

- Convergence limit within 30% perturbation
- Unconverged examples ended early
- Better velocity corrections
- Perturbations within 10% ensure 100% convergence
- Errors climb up steeply when exceeding convergence limit

Conclusions

BLT Predictor-Corrector near binary systems

- **Wide Range of Control Authority**

  Insensitive to transfer conditions
- **Low Fuel Cost**

  ~100g fuel for most libration point orbit transfers
- **Accuracy and Low Computational Overhead**

- **Nature of BLT Guidance: state-to-state transition**

  Eliminating deviations from the reference trajectories

  Altering the control profiles instead of ‘steering’

Acknowledgements

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