Trajectory Design of Reusable Transport Stations Between the Earth and Mars: The Solution of Problem A in CTOC12

Abstract The problem A of 12th China Trajectory Optimization Competition (CTOC) is about Martian migration via reusable transport stations. The mission requires a global trajectory design, which includes departure from the distant retrograde orbit (DRO), encountering with the Earth and Mars in sequence, returning and insertion to the DRO finally. This paper introduces the method and results of Beihang University (BUAA) and the Second Academy of China Aerospace Science and Industry Corporation, consisting of the design of interplanetary transfers, as well as escapes and captures of the Earth-Moon system, the planning of massive missions and the optimization of numbers of stations. The feasible windows of the interplanetary transfer are obtained by developing a definitely-matching gravity-assist optimization method. The low-energy escapes and captures in the Earth-Moon system is achieved via the establishing and solving of a time-phase decoupling nonlinear programming model and the design of phase adjustment maneuvers. An algorithm is proposed to select a combination of transport missions which satisfies constraints and enables maximum number of immigrants. The minimum quantity of stations is obtained by programming. The final result enables a 4320 people migration to Mars by 24 transport stations during 20 years, ranking third out of all teams participating in the competition.

Key words Martian migration, Trajectory design, Interplanetary, The Earth-Moon system, Optimization

引言

Introduction

在本题目中，空间站从月球远距离逆行轨道(Lunar Distant Retrograde Orbit，简称 DRO)出发，经过地球装载移民、地火转移、火地转移，最终重返 DRO 完成整个飞行流程。DRO 是地月空间一类稳定的三体周期轨道，在地月系统探索[2-3]、近地小行星捕获[4]以及行星际探测[5]等方面具有重要研究价值。文献[5]中分析了 DRO 作为燃料补给站对火星探测任务的作用，并设计了由 DRO 出发经地球飞掠到达火星的转移轨道。考虑到题目中存在着较强的燃料约束，实现低能转移或成为本题目的解题关键。在地月系统内，应用流形、月球借力(Lunar Gravity Assist，简称 LGA)、弱稳定边界(Weak Stability Boundary，简称 WSB)等动力学机制可以有效地降低轨道转移所需燃料。文献[6-7]分别在日地系与地月系中研究了应用平动点流形实现 DRO 低能入轨的方法。文献[8]实现了包括月球借力在内的多种由 DRO 出发到地球的轨道转移，进一步从能量角度讨论了月球借力对转移轨道的影响。文献[9]通过轨道拼接的方法计算得到了大量 DRO 低能转移轨道，其轨道形式包含多次有推力月球借力以及弱稳定边界等。在地月系统外，行星借力是目前被广泛应用的星际探测技术。文献[10]中利用引力辅助技术设计了火星和远日行星探测的行星际转移轨道，文献[11]基于引力辅助分析主带小行星的可达性问题，引力辅助还可与小推力优化结合使用设计星际飞行轨道[12]。

In this topic, the space station starts from the remote retrograde orbit of the moon (Lunar Distant Retrograde Orbit, DRO), through the earth loading migration, earth-fire transfer, fire-earth transfer, and finally returns to DRO to complete the entire flight process. DRO is a kind of stable three-body periodic orbit in the Earth-Moon space, which is of great research value in the exploration of the Earth-Moon system[2-3], the capture of near-Earth asteroids[4]and the interplanetary exploration[5]. In reference [5], the role of DRO as a refueling station for Mars exploration missions was analyzed, and a transfer orbit from DRO to Mars via Earth flyby was designed. Considering that there is a strong fuel constraint in the problem, the realization of low-energy transfer may be the key to solve the problem. In the Earth-Moon system, manifold, Lunar Gravity Assist (LGA), Weak Stability Boundary (WSB) and other dynamic mechanisms can effectively reduce the fuel required for orbit transfer. References [6-7] have studied the method of realizing DRO low-energy orbit injection by using the libration point manifold in the Sun-Earth system and the Earth-Moon system respectively. Reference [8] realized a variety of orbital transfer from DRO to the earth, including lunar gravity-assist, and further discussed the influence of lunar gravity-assist on the transfer orbit from the energy point of view. In reference [9], a large number of DRO low-energy transfer orbits are calculated by the method of orbit splicing, and their orbit forms include multiple thrust-lunar gravity-assist and weak stability boundaries. Outside the Earth-Moon system, planetary force borrowing is a widely used interstellar exploration technology at present. In reference [10], the interplanetary transfer orbit for Mars and aphelion planet exploration was designed by using the gravity assistance technology. In reference [11], the accessibility of main-belt asteroids was analyzed based on the gravity assistance. The gravity assistance can also be combined with the low-thrust optimization to design the interplanetary flight orbit[12].

上述方法可以为题目中特定阶段的轨道设计提供依据，但由于题目涉及若干约束，因此需要在满足条件的前提下整体解决多变量优化问题。

The above method can provide the basis for the trajectory design in the specific stage of the problem, but because the problem involves several constraints, it is necessary to solve the multi-variable optimization problem as a whole on the premise of meeting the conditions.

本题目结合地月空间限制性四体问题与行星际二体问题，为参赛队提出了一个复杂深空轨道动力学环境下非线性和多约束的任务规划与轨道优化问题。作者代表北京航空航天大学宇航学院与航天科工二院参加了本次竞赛，并获得甲组季军。本文将给出轨道设计与优化的思路方法，并展示相关设计结果。

This topic combines the restricted four-body problem in Earth-Moon space with the two-body problem in interplanetary space, and proposes a nonlinear and multi-constrained mission planning and orbit optimization problem for the participating teams in the complex deep space orbit dynamics environment. The author participated in the competition on behalf of the School of Astronautics of Beijing University of Aeronautics and Astronautics and the Second Academy of Aerospace Science and Technology, and won the third place in Group A. This paper will give the ideas and methods of track design and optimization, and show the relevant design results.

1. 题目分析

Analysis of the topic

1.1. 问题概述

Overview of the problem

利用可重复使用地火运输空间站进行火星移民任务。任务开始于 2025 年 1 月 1 日，持续 20 年。在这期间，使用不超过 50 个地火运输空间站，将尽量多的移民从地球送往火星。

Mars migration missions using reusable

earth-fire transport space stations. The mission begins on January 1, 2025 and lasts for 20 years. During this period, no more than 50 Earthfire transport stations will be used to send as many immigrants as possible from Earth to Mars.

每个空间站从部署在月球 DRO 上无机动能力的服务站启航，依次前往地球与载人飞船对接；其后空间站载人飞船组合体离开地月空间飞往火星；在抵达火星时，空间站释放载人飞船，随后借助火星引力辅助返回地月空间；与 DRO 服务站完成对接补充燃料和维护后，称为完成一次运输任务。

Each space station sets sail from a non-maneuverable service station deployed on the lunar DRO and goes to the Earth in turn to dock with the manned spacecraft. After that, the manned spacecraft assembly of the space station left the Earth-Moon space and flew to Mars. When arriving at Mars, the space station releases the manned spacecraft, and then returns to the Earth-Moon space with the help of Mars gravity. After docking with the DRO service station for refueling and maintenance, it is said that a transportation task has been completed.

任务设计共设置三项指标，第一项指标是最大化火星移民人数；第二项指标是最小化空间站使用数量；第三项指标是最小化全部地火转移运输任务施加的总速度脉冲。

There are three indicators in the mission design. The first indicator is to maximize the number of Mars immigrants. The second indicator is to minimize the number of space stations used; The third objective is to minimize the total velocity pulse imposed by all the earth-fire transfer transportation tasks.

任务约束包括时间约束、速度脉冲约束、位置约束、飞越高度与速度约束几个方面。

Task constraints include time constraints, velocity pulse constraints, position constraints, flight height and velocity constraints.

1.2. 动力学模型

Dynamics model

空间站飞行动力学模型按飞行区域划分而异。定义地月空间为以地球为中心，距离地心 500 万公里范围内的空间。在地月空间内部，采用日—地—月—空间站限制性四体动力学模型，在地心赤道惯性坐标系（ECI）中描述，其方程为

The space station flight dynamics model is different according to the flight area division. Earth-moon space is defined as the space centered on the earth and within 5 million kilometers from the center of the earth. In the interior of the Earth-Moon space, the Sun-Earth-Moon-Space Station restricted four-body dynamic model is adopted, which is described in the geocentric equatorial inertial coordinate system (ECI), and its equation is

$$\begin{aligned}\ddot{\boldsymbol{r}} &= -\frac{\mu_{\mathrm{E}}}{r^3}\boldsymbol{r} - \mu_{\mathrm{m}}\left(\frac{\boldsymbol{r}_{\mathrm{m}}}{r_{\mathrm{m}}^3} + \frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{m}}}{\left\|r-r_{\mathrm{m}}\right\|^3}\right) \\ &\quad - \mu_{\mathrm{S}}\left(\frac{\boldsymbol{r}_{\mathrm{S}}}{r_{\mathrm{S}}^3} + \frac{\boldsymbol{r}-\boldsymbol{r}_{\mathrm{S}}}{\left\|r-r_{\mathrm{S}}\right\|^3}\right)\end{aligned} \tag{1}$$

式中，μ_{E}，μ_{m} 和 μ_{S} 分别为地球、月球、太阳的引力常数，$\boldsymbol{r}$，$\boldsymbol{r}_{\mathrm{m}}$ 和 $\boldsymbol{r}_{\mathrm{S}}$ 分别为空间站、月球和太阳在 ECI 中的位置矢量，其模值分别为 r，r_{m} 和 r_{S}。太阳与月球的位置由星历给出。在地月空间外，采用日心二体动力学模型，在日心黄道惯性坐标系（HCI）中描述，其方程为

In the formula,, AndThey are the gravitational constants of the earth, the moon and the sun. AndAre the position vectors of the space station, the moon and the sun in the ECI respectively, and their modulus values are And。 The positions of the sun and moon are given by ephemeris. Outside the Earth-Moon space, the heliocentric two-body dynamic model is adopted, which is described in the heliocentric ecliptic inertial coordinate system (HCI), and its equation is

$$\ddot{\boldsymbol{r}} = -\frac{\mu_{\mathrm{S}}}{r^3}\boldsymbol{r} \tag{2}$$

式中，$\boldsymbol{r}$ 和 r 分别为空间站在 HCI 中的位置矢量及其模值。

In the formula,AndAre the position vector and its modulus of the space station in HCI, respectively.

在地月空间外，引力辅助采用简化模型设计，假设引力辅助瞬时完成，等效为一个瞬时脉冲

Outside the Earth-Moon space, the

gravitational assistance is designed with a simplified model, assuming that the gravitational assistance is completed instantaneously, which is equivalent to an instantaneous pulse.

$$\Delta \boldsymbol{v}_{\mathrm{GA}}=\boldsymbol{v}^{+}\left(t_{\mathrm{GA}}\right)-\boldsymbol{v}^{-}\left(t_{\mathrm{GA}}\right) \tag{3}$$

式中，t_{GA} 为引力辅助时刻，$\boldsymbol{v}^{-}\left(t_{\mathrm{GA}}\right)$ 与 $\boldsymbol{v}^{+}\left(t_{\mathrm{GA}}\right)$ 分别为引力辅助前后的空间站速度矢量。

In the formula,For gravitationally assisted moments,AndAre the space station velocity vectors before and after gravity assistance, respectively.

1.3. 初步分析

Preliminary analysis

赛题涉及行星际转移、地月空间限制性四体问题，单次运输任务包含不同动力学下的多段轨迹，整体任务约束复杂，具有挑战性。主要设计难点可以归纳为：(1) 行星际转移设计和地月空间内转移设计相互耦合约束；(2) 满足约束要求的行星际转移窗口稀疏；(3) 速度脉冲消耗限制较严格。(4) 地月空间限制性四体问题具有强非线性和非解析性，设计计算量大。

The competition involves interplanetary transfer and the restricted four-body problem in Earth-Moon space. A single transport mission contains multiple trajectories under different dynamics, and the overall mission constraints are complex and challenging. The main design difficulties can be summarized as follows: (1) the interplanetary transfer design and the transfer design in the Earth-Moon space are mutually coupled; (2) the interplanetary transfer window meeting the constraint requirements is sparse; (3) the velocity pulse consumption limit is relatively strict. (4) The restricted four-body problem in Earth-Moon space has strong nonlinearity and non-analyticity, and the design calculation is large.

单次运输飞行包含的二体模型下行星际转移和限制性四体模型下地月空间内转移互相牵连制约。行星际转移窗口选择不当易造成转移所需脉冲消耗过大，增加地月空间内轨迹设计的难度；地月空间内轨迹设计结果决定离开地月空间后的运动，对行星际转移中的约束特别是引力辅助约束的满足产生影响。

The downward interstellar transfer of the two-body model and the transfer in the Earth-Moon space of the restricted four-body model contained in a single transport flight are mutually related and restricted. Improper selection of the interplanetary transfer window can easily lead to excessive consumption of pulses required for transfer, which increases the difficulty of trajectory design in the Earth-Moon space. Trajectory design in the Earth-Moon space determines the motion after leaving the Earth-Moon space, and affects the constraints in the interplanetary transfer, especially the satisfaction of the gravitational auxiliary constraints.

题目约束中要求近地点对接飞船至近火点释放飞船的飞行时长不超过 300 天，并且到达火星时相对于火星的双曲线无穷远速度大小不超过 8 km/s；此外，较严苛的速度脉冲消耗约束要求出发时相对地球的无穷远速度也不能过大，这就导致满足上述要求的无脉冲地火转移窗口比较有限。在火星处，地火转移轨道和火地转移轨道还需满足引力辅助模型中的速度匹配约束和飞越高度约束，这进一步削减了可行的行星际转移窗口。因此，有必要进行深空机动设计以扩展行星际转移可行窗口。

The problem constraint requires that the flight time from the perigee docking spacecraft to the near fire point release spacecraft should not exceed 300 days, and the hyperbolic infinite velocity relative to Mars should not exceed 8 km/s when arriving at Mars. In addition, the more stringent velocity pulse consumption constraints require that the infinite velocity relative to the earth at the time of departure should not be too large, which leads to a relatively limited window of non-pulse fire transfer to meet the above requirements. At Mars, the Earth-Mars transfer orbit and the Fire-Earth transfer orbit also need to satisfy the velocity matching constraint and the flyby altitude constraint in the gravity-assisted model, which further reduces the feasible interplanetary transfer window. Therefore, it is necessary to design deep space maneuvers to expand the feasible window of interplanetary transfer.

对单次运输任务所需的速度脉冲消耗进行粗略估计。地火霍曼转移相对地球的双曲线无穷远速度大小约为 2.9 km/s，假定从火星返回仍按霍曼转移方式，则逃逸地球和被地球捕获共需要消耗约 5.8 km/s 的速度脉冲，该值大于题中所给速度脉冲消耗上限值 4.5 km/s。虽然上述估计方式存在不严谨之处，但一定程度上反映出了速度脉冲消耗限制较为严格。因此，充分利用限制性多体问题下的低能量轨道实现转移，以及设计合适的脉冲时机，是保证单次运输任务可行的关键。

Make a rough estimate of the speed pulse consumption required for a single transport mission. The hyperbolic infinite velocity of Hohmann transfer relative to the Earth is about 2.9 km/s. Assuming that the return from Mars is still in the way of Hohmann transfer, the total consumption of velocity pulse for escaping from the Earth and being captured by the Earth is about 5.8 km/s, which is greater than the upper limit value of velocity pulse consumption of 4.5 km/s given in the problem. Although the above estimation method is not rigorous, it reflects to some extent that the speed pulse consumption limit is more stringent. Therefore, the key to ensure the feasibility of a single transport mission is to make full use of the low energy orbit under the restricted multi-body problem to realize the transfer and to design a suitable pulse opportunity.

2. 设计方案

Design scheme

2.1. 整体设计思路

Overall design idea

由于地月空间的半径相比于从地球至火星的距离为小量，因此，在设计行星际转移轨道时，将地月空间整体视为与地球位置重合的质点，下面称为“地月空间点”。如此简化后，整个全局轨迹设计问题可分为两部分：

Since the radius of the cislunar space is small compared to the distance from the Earth to Mars, when designing an interplanetary transfer orbit, the whole of the cislunar space is considered to be a particle coincident with the position of the Earth, hereinafter referred to as a "cislunar point". After this simplification, the whole global trajectory design problem can be divided into two parts:

① 行星际转移部分：空间站从地月空间点至火星，经火星引力辅助后，再返回至地月空间点。

① Interplanetary transfer part: The space station travels from the Earth-Moon space point to Mars, and then returns to the Earth-Moon space point after being assisted by the gravity of Mars.

② 地月空间内部分：空间站从地月空间边界进入地月空间，进行 DRO 入轨（首次从 DRO 启航时没有该过程）；完成燃料补给后择机从 DRO 出发，前往地球与载人飞船对接，再飞向地月空间边界。

② The inner part of cislunar space: the space station enters cislunar space from the cislunar boundary and carries out DRO orbit insertion (there is no such process when it first sails from DRO); After completing the refueling, they will choose an opportunity to start from DRO, go to the Earth to dock with the manned spacecraft, and then fly to the Earth-Moon space boundary.

两部分的轨迹示意图如图 1 所示。在轨道设计时，将行星际转移部分与地月空间内部分分开进行，两部分在地月空间边界处拼接，需满足时刻、位置、速度相同的约束条件。

A schematic diagram of the trajectory of the two parts is shown in Figure 1. In the orbit design, the interplanetary transfer part and the inner part of the Earth-Moon space are separated, and the two parts are spliced at the boundary of the Earth-Moon space to meet the constraints of the same time, position and speed.

整体设计的过程为，首先进行行星际转移可行窗口搜索，为地月空间内转移设计提供边界约束条件；进行地月空间内逃逸阶段优化；筛选满足时间间隔约束且总载人数最大的运输

任务组合构成原始解集；对原始解集中的解优化地月空间捕获阶段轨迹以及设计相位调节机动；最后进行最少数量空间站的规划。

The overall design process is as follows: firstly, the feasible window search of interplanetary transfer is carried out to provide boundary constraints for the transfer design in the Earth-Moon space; Carry out that optimization of the escape phase in the Earth-Moon space; Lecting the transportation task combination which meets the time interval constraint and has the maximum total number of people to form an original solution set; Optimize the Earth-Moon space acquisition phase trajectory and design the phase adjustment maneuver for the solution in the original solution set; Finally, the planning of the minimum number of space stations is carried out.

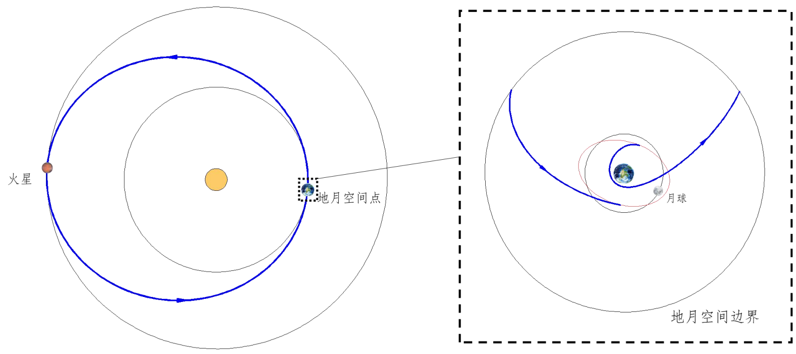


图 1 全局轨迹设计分析

Figure 1 Global Trajectory Design Analysis

Fig.1 Analysis of global trajectory design

2.2. 行星际转移设计

Interplanetary transfer design

行星际转移的动力学模型为二体模型，设计方法采用圆锥曲线拼接法，对火星或金星的引力辅助过程采用简化方法建模。本团队的设计方案中，未采用金星借力，探测器从地月空间点出发，进行地火转移飞往火星，经由火星的引力辅助（释放载人飞船）后，进行火地转移返回地月空间点。

The dynamic model of the interplanetary transfer is a two-body model, the design method is the conic splicing method, and the gravity assisted process of Mars or Venus is modeled by a simplified method. In the design scheme of this team, Venus is not used to borrow force. The probe starts from the Earth-Moon space point, carries out the transfer of fire to Mars, and returns to the Earth-Moon space point after the gravitational assistance of Mars (release of manned spacecraft).

在已有众多时间和能量约束限制的情况下，满足火星处引力辅助前后双曲线无穷远速度匹配约束以及飞越火星高度约束的行星际转移窗口极少。本团队的设计方案中，引入一种带深空机动的确定匹配引力辅助优化方法，可以有效拓展行星际转移窗口。下面依次介绍确定匹配的引力辅助后速度构造、带深空机动的引力辅助优化、窗口搜索。

In the case of many time and energy constraints, there are very few interplanetary transfer windows that satisfy the hyperbolic infinite velocity matching constraints before and after gravity assistance at Mars and the altitude constraints of Mars flyby. In the design scheme of our team, a determined matching gravity-assisted optimization method with deep space maneuvers is introduced, which can effectively expand the interplanetary transfer window. Velocity construction after determining the matching gravity assist, gravity assist optimization with deep space maneuvers, and window search are described in turn below.

2.2.1. 确定匹配的引力辅助后速度构造

Determining the matched gravitationally assisted post-velocity construction

首先阐述确定匹配的引力辅助后速度构造。定义描述借力后双曲线无穷远速度矢量的坐标系 $\boldsymbol{NWR}$，如图 2 所示，图中 $\boldsymbol{IJK}$ 坐标系为黄道 J2000 坐标系，$\boldsymbol{R}$ 轴指向引力辅助前双曲线无穷远速度矢量 $\boldsymbol{v}_\infty^-$ 方向，$\boldsymbol{N}$ 轴方向由 $\boldsymbol{N}=\boldsymbol{K}\times\boldsymbol{R}$ 确定，$\boldsymbol{W}$ 轴方向根据右手定则确定。在 $\boldsymbol{NWR}$ 坐标系中，满足引力辅助约束的借力后双曲线无穷远速度矢量 $\boldsymbol{v}_\infty^+$ 可以用图 3 中的转动角 δ 和方位角 α 描述，在 $\boldsymbol{NWR}$ 坐标系下描述为

The gravitationally assisted post-velocity construction that determines the match is first set forth. Define the coordinate system NWR describing the velocity vector at infinity of the post-gravity-assist hyperbola, as shown in Figure 2, whereThe coordinate system is the ecliptic J2000 coordinate system, and the r-axis points to the gravitationally assisted pre-hyperbolic velocity vector at infinity.Direction, the N axis direction is determined byOK, the W axis direction is determined according to the right-hand rule. In the NWR coordinate system, the velocity vector at infinity is hyperbolic after the gravitational auxiliary constraint is satisfied.The rotation angle in fig. 3 can be usedAnd azimuthDescription, described in the NWR coordinate system as

$$\boldsymbol{v}_\infty^+=\left[v_\infty^-\sin\delta\cos\alpha,\ v_\infty^-\sin\delta\sin\alpha,\ v_\infty^-\cos\delta\right]^{\mathrm{T}} \quad (4)$$

式中，v_∞^- 为 $\boldsymbol{v}_\infty^-$ 的模值，$\alpha\in[0,2\pi)$，$\delta\in[\delta_{\min},\delta_{\max}]$，转动角的最小值 $\delta_{\min}$ 和最大值 $\delta_{\max}$ 根据飞越高度的上下限确定

In the formula,ForThe modulus value of,，, the minimum value of the rotation angleAnd maximum valuesDetermined according to the upper and lower limits of the flyover altitude

$$\begin{aligned}\delta_{\min}&=2\arcsin\left(\frac{\mu_{\mathrm{M}}}{\mu_{\mathrm{M}}+r_{\mathrm{p,max}}{v_\infty}^2}\right)\\ \delta_{\max}&=2\arcsin\left(\frac{\mu_{\mathrm{M}}}{\mu_{\mathrm{M}}+r_{\mathrm{p,min}}{v_\infty}^2}\right)\end{aligned} \quad (5)$$

式中，μ_{M} 为火星的引力常数，$r_{\mathrm{p,min}}$ 和 $r_{\mathrm{p,max}}$ 分别为飞越火星的高度下限和上限，$v_\infty=v_\infty^-$。将 $\boldsymbol{v}_\infty^+$ 转换至黄道 J2000 系下描述，以方便参与运算，从 $\boldsymbol{NWR}$ 坐标系转换至 $\boldsymbol{IJK}$ 坐标系的坐标转换矩阵为

In the formula,Is the gravitational constant of Mars,AndThey are the lower and upper altitude limits of Mars flyby, respectively.1.willConvert to the description under the ecliptic J2000 coordinate system to facilitate participation in the calculation, and convert from the NWR coordinate system toThe coordinate transformation matrix of the coordinate system is

$$\boldsymbol{C}_{\mathrm{NI}}=\begin{bmatrix}1&0&0\\0&\cos\theta&\sin\theta\\0&-\sin\theta&\cos\theta\end{bmatrix}\begin{bmatrix}\cos\Omega&\sin\Omega&0\\-\sin\Omega&\cos\Omega&0\\0&0&1\end{bmatrix} \quad (6)$$

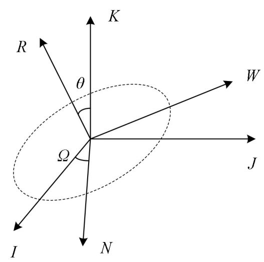


图 2 $\boldsymbol{NWR}$ 坐标系定义

Figure 2 Definition of NWR coordinate system

Fig.2 Definition of ***NWR*** coordinate

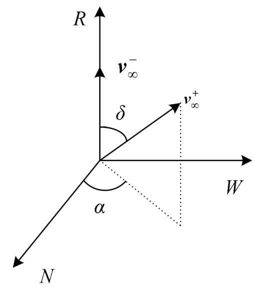


图 3 $\boldsymbol{v}_\infty^+$ 的几何描述

Figure 3 Geometric description of

Fig.3 Geometric description of $\boldsymbol{v}_\infty^+$

2.2.2. 带深空机动的引力辅助优化

Gravity Assisted Optimization with Deep Space Maneuver

在构造出确定匹配的引力辅助后速度基础上，进行带深空机动的引力辅助优化设计。深

空机动发生于火星引力辅助后，且与引力辅助间隔超过 30 天。优化变量为引力辅助中的转动角 δ 和方位角 α，以及引力辅助至深空机动的时间间隔 Δt，记为 $\chi = [\delta \quad \alpha \quad \Delta t]^{\mathrm{T}}$。当取定一组地月空间点出发时刻 t_0、到达火星时刻 t_1、返回地月空间点时刻 t_2 后，可由 Lambert 问题的求解确定到达火星时引力辅助前双曲线无穷远速度矢量 $\boldsymbol{v}_\infty^-$，进而可建立优化变量与引力辅助后日心轨道运动的映射。优化目标函数由两部分构成，分别为深空机动速度增量幅值和返回时近地点制动脉冲估计值

On the basis of constructing the determined matching velocity after gravity assistance, the optimization design of gravity assistance with deep space maneuver is carried out. The deep space maneuver occurred after the Mars gravity assist, and the interval between the deep space maneuver and the gravity assist was more than 30 days. The optimization variable is the rotation angle in the gravity assist.And azimuth, and the time interval between the gravity assist and the deep space maneuver, recorded as。 When the departure time of a group of Earth-Moon space points is determined,Time to reach Mars.Return to Earth-Moon space point timeThen, the vector at infinity of the gravitationally assisted front hyperbola can be determined by solving the Lambert problem.Then, the mapping between the optimization variables and the heliocentric orbital motion after gravity assistance can be established. The optimization objective function consists of two parts, namely, the amplitude of deep-space maneuvering velocity increment and the estimation of perigee braking impulse during return.

$$J_{\mathrm{GA}} = \Delta v_{\mathrm{DSM}} + \left(\sqrt{{v_{\infty 2}}^2 + 2\mu_{\mathrm{E}} / r_{\mathrm{p,E}}} - v_{\mathrm{p,E}} \right) \tag{7}$$

式中，Δv_{DSM} 为深空机动速度增量幅值，$v_{\infty 2}$ 为返回时相对地月空间点的双曲线无穷远速度大小，μ_{E} 为地球引力常数，$r_{\mathrm{p.E}}$ 为施加制动脉冲时的近地点高度，近似取为常值 6778 km，$v_{\mathrm{p.E}}$ 为制动后的近地点速度，根据大量计算取经验值 10.6 km/s。带深空机动的引力辅助优化问题表述为

In the formula, Is the increment amplitude of the deep space maneuvering speed,Is the hyperbolic infinite velocity relative to the Earth-Moon space point at the time of return,Is the Earth's gravitational constant,Is the altitude of perigee when the braking pulse is applied, which is approximately taken as a constant value of 6778 km.Is the perigee velocity after braking, an empirical value of 10.6 km/s based on extensive calculations. The gravity-assisted optimization problem with deep space maneuvers is formulated as

$$\begin{aligned} &\min \quad J_{\mathrm{GA}} = \Delta v_{\mathrm{DSM}} + \left(\sqrt{{v_{\infty 2}}^2 + 2\mu_{\mathrm{E}} / r_{\mathrm{p,E}}} - v_{\mathrm{p,E}} \right) \\ &s.t. \\ &\delta_{\min} \le \delta \le \delta_{\max} \\ &0 \le \alpha < 2\pi \\ &30 \text{ day} \le \Delta t \le t_2 - t_1 \end{aligned} \tag{8}$$

求解上述模型，可以获得指标最优的引力辅助变量。

Solving the above model, we can obtain the optimal gravitational auxiliary variables.

2.2.3. 窗口搜索

Window search

进一步，利用网格搜索确定满足能量约束和时间约束的转移窗口，搜索变量为前面已定义过的 t_0，t_1，t_2。给定 t_0，t_1，t_2 后，可以计算出发时相对地月空间点的双曲线无穷远速度大小 $v_{\infty 0}$，到达火星时相对火星的双曲线无穷远速度大小 $v_{\infty 1}$，以及经过优化的指标 J_{GA}。所考虑的约束包括：

Furthermore, the transition window satisfying the energy constraint and time constraint is determined by grid search, and the search variable is defined as 。 Given Then, the hyperbolic infinite velocity relative to the Earth-Moon space point at the time of departure can be calculated.The magnitude of the hyperbolic infinite velocity relative to Mars at the time of arrival at Mars., and optimized metrics。 The constraints considered are:

① $0 \le t_0 < t_1 < t_2 \le$ MJD 7304；

② $t_1 - t_0 \le 300$ day；

③ $t_2 - t_0 \le 5$ year；

④ $v_{\infty 1} \le 8$ km/s；

⑤ $v_{\infty 0} \le 6$ km/s；

⑥ $J_{GA} \le 1.8$ km/s 。

为了减小计算复杂度，首先只进行 t_0 与 t_1 的搜索，将满足约束①、②、④、⑤的 t_0，t_1 取值记录下来；然后对于每一组满足要求的 t_0，t_1，进一步搜索满足剩余约束的 t_2。搜索的步长设置为 2 天。搜索结果见图 4。可以看出，可行窗口的分布具有明显的区间特性，根据出发时刻的先后分布于 8 个区间内。

In order to reduce the computational complexity, first onlyAndSearch for, the constraint will be satisfied ①、②、④、⑤ Yes The value is recorded; Then for each set of , further search for satisfying the remaining constraint。 The step size of the search is set to 2 days. The search results are shown in Figure 4. It can be seen that the distribution of feasible windows has obvious interval characteristics, which are distributed in eight intervals according to the departure time.

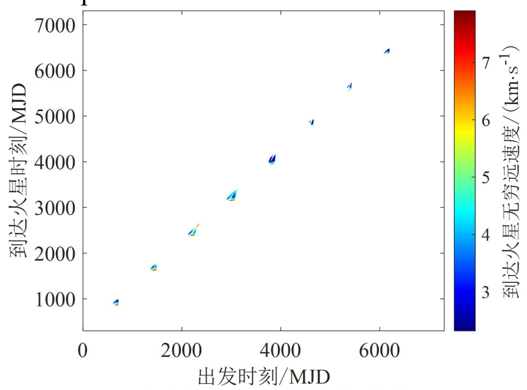


图 4 行星际转移窗口搜索结果

Figure 4 Interplanetary transfer window search results

Fig.4 The search result of interplanetary transfer windows

2.3. 地月空间内逃逸与捕获设计

Design of escape and capture in Earth-Moon space

地月空间内空间站的运动遵循日—地—月—空间站限制性四体动力学。在行星际转移与地月空间内转移分离设计的整体思路下，地月空间内轨迹必须满足在地月空间边界处的位置、速度及时间拼接约束。地月空间内轨迹设计的目标为，在满足燃料消耗约束和时间约束的前提下，设计逃逸轨迹和捕获轨迹，使得空间站在 t_{out} 时刻，位于地月空间边界上给定位置 $\boldsymbol{r}_{out}$ 且速度为 $\boldsymbol{v}_{out}$（逃逸）；在 t_{in} 时刻，位于地月空间边界上给定位置 $\boldsymbol{r}_{in}$ 且速度为 $\boldsymbol{v}_{in}$（捕获）。在设计中，构建并求解了时间—相位解耦的非线性规划模型，并通过 DRO 相位调节机动满足离轨与入轨的状态约束。由于地月空间捕获段是地月空间逃逸段简化约束后的逆过程，设计思路与逃逸段类似，故下文以地月空间逃逸段为例介绍设计方法。

The motion of the space station in the Earth-Moon space follows the Sun-Earth-Moon-Space Station restricted four-body dynamics. Under the overall idea of separating the interplanetary transfer and the transfer in the Earth-Moon space, the trajectory in the Earth-Moon space must meet the constraints of position, velocity and time splicing at the boundary of the Earth-moon space. The goal of the trajectory design in the Earth-Moon space is to design the escape trajectory and the capture trajectory on the premise of meeting the fuel consumption constraint and the time constraint, so that the space stationTime, at a given location on the boundary of Earth-Moon spaceAnd that speed is(Escape); AtTime, at a given location on the boundary of Earth-Moon spaceAnd that speed is(Capture). In the design, a time-phase decoupled nonlinear programming model is constructed and solved, and the state constraints of deorbit and orbit insertion are satisfied by DRO phase adjustment maneuver. Since the Earth-Moon space capture segment is the inverse process of the Earth-Moon space escape segment after simplified constraints, the design idea is similar to that of the escape segment, so the design method is introduced below by taking the Earth-Lunar space escape segment as an example.

2.3.1. 时间—相位解耦优化

Time-phase decoupling optimization

空间站在离轨时刻与 DRO 服务站的位置速度重合，现将 DRO 离轨时刻与离轨相位进行解耦。定义“伪离轨”事件，空间站于伪离轨时刻 t_{dep} 在 DRO 上相位 ϕ 处离轨，而此时服务站不必位于相位 ϕ 处。用时刻 t_{pha} 描述相位

ϕ， $t_{pha} \in [0,\ T_{DRO}]$， T_{DRO} 为 DRO 的周期。

The position and velocity of the space station at the time of deorbit coincide with the position and velocity of the DRO service station, and the time of DRO deorbit is decoupled from the phase of deorbit. A "pseudo-deorbit" event is defined, and the space station is in the pseudo-deorbit moment.Phase on DRODeorbit at, while the service station does not have to be in phaseDepartment. Use the momentDescribe the phase,， Is the period of the DRO.

在离开 DRO 后，空间站需近距离飞越地球对接载人飞船，将满足对接条件的近地点称为第一次近地点（忽略此前不满足对接条件的近地点）。根据题目约束，在第一次近地点时刻前后 1 天内不得施加机动。考虑到脉冲施加于速度较快的近地点处，效率较高，因此，采用在第二次近地点施加逃逸脉冲的方法。故伪离轨后至地月空间边界的逃逸轨迹可分为三段：伪离轨至第一次近地点（对接载人飞船）、第一次近地点至第二次近地点、第二次近地点至地月空间边界。在该过程中共施加三次脉冲，包括伪离轨脉冲、第二次近地点附近的逃逸脉冲、地月空间边界处的匹配脉冲。

After leaving DRO, the space station needs to fly close to the Earth to dock with the manned spacecraft, and the perigee that meets the docking conditions is called the first perigee (ignoring the perigee that does not meet the docking conditions before). Subject to the constraints of the title, no manoeuvre shall be imposed within one day before or after the first time of perigee. Considering that the pulse is applied to the perigee with faster speed, the method of applying the escape pulse at the second perigee is adopted. Therefore, the escape trajectory from the pseudo-deorbit to the Earth-Moon space boundary can be divided into three sections: the pseudo-deorbit to the first perigee (docking manned spacecraft), the first perigee to the second perigee, and the second perigee to the Earth-Moon space boundary. In this process, three pulses are applied, including the pseudo-deorbit pulse, the escape pulse near the second perigee, and the matching pulse at the Earth-Moon space boundary.

基于上述思想构建优化伪离轨后逃逸轨迹的非线性规划模型，优化变量为 5 维列向量 $\boldsymbol{\xi} = \begin{bmatrix} t_{pha} & t_{dep} & \lambda & \tau & \Delta t_D \end{bmatrix}^T$，其中 λ 为伪离轨脉冲施加后切向速度与原速度之比，τ 为伪离轨脉冲施加后法向速度与原速度之比，Δt_D 为第二次近地点至逃逸脉冲施加时刻的时间间隔，引入该变量的目的在于提高算法局部最优性，取值限制为很小的正数。

Based on the above idea, a nonlinear programming model is constructed to optimize the escape trajectory after pseudo-deorbit, and the optimization variables are 5-dimensional column vectors., of whichIs the ratio of the tangential velocity to the original velocity after the pseudo-deorbit pulse is applied,Is the ratio of the normal velocity to the original velocity after the pseudo-deorbit pulse is applied,Is the time interval between the second perigee and the application time of the escape pulse. The purpose of introducing this variable is to improve the local optimality of the algorithm, and its value is limited to a very small positive number.

首先根据 DRO 服务站的星历确定 t_{pha} 时刻对应的位置 $\boldsymbol{r}_{DRO}$ 和速度 $\boldsymbol{v}_{DRO}$，施加离轨脉冲后的位置速度为

First, it is determined according to the ephemeris of DRO service stationPosition corresponding to timeAnd speed, the position velocity after the application of the deorbit pulse is

$$\begin{aligned} \boldsymbol{r}_{DRO}^{+} &= \boldsymbol{r}_{DRO} \\ \boldsymbol{v}_{DRO}^{+} &= \lambda \boldsymbol{v}_{DRO} + \tau \left\| \boldsymbol{v}_{DRO} \right\| \boldsymbol{h}_{DRO} \end{aligned} \tag{9}$$

式中 $\boldsymbol{h}_{DRO}$ 为 DRO 的角动量方向单位矢量。以 $\boldsymbol{r}_{DRO}^{+}$ 和 $\boldsymbol{v}_{DRO}^{+}$ 为初始状态，以 t_{dep} 为初始时刻，在限制性四体动力学下积分，记录首次满足对接载人飞船约束的近地点时刻 t_{rdz}、位置 $\boldsymbol{r}_{rdz}$ 与速度 $\boldsymbol{v}_{rdz}$，所要求的约束为

In the formulaIs the unit vector of the angular momentum direction of the DRO. WithAndIs the initial state, withIs the initial time, and is integrated under the restricted four-body dynamics to record the perigee time satisfying the constraints of the docking manned spacecraft for the first time.LocationAnd speed, the required constraint is

$$\begin{aligned} &\boldsymbol{r}_{\mathrm{rdz}} \cdot \boldsymbol{v}_{\mathrm{rdz}} = 0 \\ &\boldsymbol{v}_{\mathrm{rdz}} \cdot \boldsymbol{v}_{\mathrm{rdz}} + \boldsymbol{r}_{\mathrm{rdz}} \cdot \boldsymbol{a}_{\mathrm{rdz}} > 0 \\ &400\ \mathrm{km} \le \|\boldsymbol{r}_{\mathrm{rdz}}\| - r_{\mathrm{E}} \le 800\ \mathrm{km} \\ &\|\boldsymbol{v}_{\mathrm{rdz}}\| \le 12\ \ \mathrm{km/s} \end{aligned} \tag{10}$$

式中，$\boldsymbol{a}_{\mathrm{rdz}}$为近地点时刻空间站的加速度，$r_{\mathrm{E}}$为地球半径。以第一次近地点的状态为初值，继续积分至第二次近地点，若近地点高度满足约束，则记录下第二次近地点时刻t_{secp}、位置$\boldsymbol{r}_{\mathrm{secp}}$和速度$\boldsymbol{v}_{\mathrm{secp}}$。为保证近地点具有较大的速度，除题目给定的最小高度限制外，为第二次近地点设定了最大高度的约束。继续积分至$t_{\mathrm{secp}} + \Delta t_{\mathrm{D}}$时刻，在该时刻施加逃逸脉冲$\Delta\boldsymbol{v}_{\mathrm{esc}}$，$t_{\mathrm{secp}} + \Delta t_{\mathrm{D}}$与$t_{\mathrm{out}}$时刻的位置作为已知，则逃逸脉冲的大小与方向由固定时间单段打靶法求解，为保证在地月空间边界处的速度满足匹配约束，t_{out}时刻需执行一个匹配脉冲$\Delta\boldsymbol{v}_{\mathrm{mat}}$。所构建的非线性规划模型为

In the formula,Is the acceleration of the space station at perigee,Is the radius of the earth. Take the state of the first perigee as the initial value, continue to integrate to the second perigee, if the perigee height meets the constraint, record the time of the second perigeonLocationAnd speed。In order to ensure that the perigee has a large velocity, in addition to the minimum altitude limit given by the problem, the maximum altitude constraint is set for the second perigee. Continue to integrate toTime at which the escape pulse is applied， AndThen the magnitude and direction of the escape pulse are solved by the fixed time single segment shooting method. In order to ensure that the velocity at the Earth-Moon space boundary meets the matching constraint,A matching pulse is to be executed at all times。The nonlinear programming model constructed is

$$\begin{aligned} &\min\ J_{\mathrm{esc}} = \left\|\boldsymbol{v}_{\mathrm{DRO}}^{+} - \boldsymbol{v}_{\mathrm{DRO}}\right\| + 1.33\left(\|\Delta\boldsymbol{v}_{\mathrm{esc}}\| + \|\Delta\boldsymbol{v}_{\mathrm{mat}}\|\right) \\ &s.t. \\ &0 \le t_{\mathrm{pha}} \le T_{\mathrm{DRO}} \\ &t_{\mathrm{out}} - dT_1 \le t_{\mathrm{dep}} \le t_{\mathrm{out}} - dT_2 \\ &\lambda_{\min} \le \lambda \le \lambda_{\max} \\ &\tau_{\min} \le \tau \le \tau_{\max} \\ &0 \le \Delta t_{\mathrm{D}} \le 0.1\ \mathrm{day} \end{aligned} \tag{11}$$

其中，对于直接离轨方式，$\lambda_{\min} = 0.08$，$\lambda_{\max} = 0.3$；$\tau_{\min} = -0.3$，$\tau_{\max} = 0.3$；$dT_1 \approx 45$，$dT_2 \approx 18$；而对于大范围长时间转移方式，$\lambda_{\min} = 0.8$，$\lambda_{\max} = 1.2$；$\tau_{\min} = -0.3$，$\tau_{\max} = 0.3$；$dT_1 \approx 200$，$dT_2 \approx 70$。优化设计中主要采取直接离轨方式。此外，在目标函数构造中用地心 Lambert 问题的求解代替微分修正近似求解$\Delta\boldsymbol{v}_{\mathrm{esc}}$和$\Delta\boldsymbol{v}_{\mathrm{mat}}$，以提高优化效率，数值结果表明，近似值与精确值相差通常不超过 0.1 km/s。

Wherein, for the direct deorbit mode, , ; , ; ; , ; And for a wide range of long time transfer mode, , ; ; , ; , 。 In the optimization design, the direct off-orbit mode is mainly adopted. In addition, the solution of the geocentric Lambert problem is used to replace the approximate solution of the differential correction in the construction of the objective function.And, to improve the efficiency of the optimization, and the numerical results show that the approximate values do not differ from the exact values by more than 0.1 km/s in general.

2.3.2. DRO 相位调节机动

DRO Phase Adjustment Maneuver

为保证在 DRO 离轨时刻空间站的状态与 DRO 服务站的状态相同，在伪离轨前需进行相位调节机动。脱离 DRO 服务站的时刻为真实离轨时刻t_{sep}，相位调节机动的目标是在t_{sep}时刻从 DRO 服务站出发，在t_{dep}时刻达到伪离轨状态，即服务站在t_{pha}时刻的位置速度。设计了一种基于微分修正的 4 脉冲优化调相方法，在t_{sep}时刻施加离轨脉冲$\Delta\boldsymbol{v}_{\mathrm{d0}}$，此后在时刻$t_{\mathrm{d1}}, t_{\mathrm{d2}}, t_{\mathrm{dep}}$分别施加三次脉冲$\Delta\boldsymbol{v}_{\mathrm{d1}}, \Delta\boldsymbol{v}_{\mathrm{d2}}, \Delta\boldsymbol{v}_{\mathrm{d3}}$。

In order to ensure that the state of the space station is the same as that of the DRO service station at the time of DRO deorbit, the phase adjustment maneuver should be carried out before the pseudo-deorbit. The time of leaving the DRO service station is the actual time of leaving the orbit., the goal of the phase adjustment maneuver is toDepart from DRO service station at any time.The pseudo-deorbit state is reached at the moment, that is, the service station is in thePosition and velocity at time. A 4-

pulse optimal phase modulation method based on differential correction is designed.Apply off-track pulse at all times, thereafter at the moment,,Three pulses were applied separately,,。

优化变量为

The optimization variable is

$$\begin{bmatrix} t_{\text{sep}} & t_{\text{d1}} & t_{\text{d2}} & \lambda_0 & \tau_0 & \lambda_1 & \tau_1 \end{bmatrix}^{\text{T}}$$

其中 λ_0 和 τ_0 分别为 t_{sep} 时刻脉冲后切向速度幅值和法向速度幅值与脉冲前速度幅值之比；λ_1 和 τ_1 分别为 t_{d1} 时刻脉冲后切向速度幅值和法向速度幅值与脉冲前速度幅值之比。由服务站星历计算出在真实离轨时刻 t_{sep} 服务站的位置 $\boldsymbol{r}_{\text{st0}}$ 和速度 $\boldsymbol{v}_{\text{st0}}$，执行离轨脉冲后空间站的位置和速度为

AmongAndRespectivelyThe ratio of the tangential velocity amplitude and the normal velocity amplitude after the pulse to the velocity amplitude before the pulse;AndRespectivelyThe ratio of the tangential velocity amplitude and the normal velocity amplitude after the pulse to the velocity amplitude before the pulse. Calculate that actual deorbit time by the ephemeris of the service stationLocation of the service stationAnd speed, the position and velocity of the space station after the deorbiting pulse are

$$\begin{aligned} \boldsymbol{r}_{\text{st0}}^{+} &= \boldsymbol{r}_{\text{st0}} \\ \boldsymbol{v}_{\text{st0}}^{+} &= \lambda_0 \boldsymbol{v}_{\text{st0}} + \tau_0 \left\| \boldsymbol{v}_{\text{st0}} \right\| \boldsymbol{h}_{\text{DRO}} \end{aligned} \tag{12}$$

限制性四体模型下外推至 t_{d1} 时刻，得到空间站位置 $\boldsymbol{r}_{\text{st1}}$ 和速度 $\boldsymbol{v}_{\text{st1}}$，脉冲后位置和速度为

Extrapolation under the restricted four-body model toTime, get the space station location.And speed, the position and velocity after the pulse are

$$\begin{aligned} \boldsymbol{r}_{\text{st1}}^{+} &= \boldsymbol{r}_{\text{st1}} \\ \boldsymbol{v}_{\text{st1}}^{+} &= \lambda_1 \left[\boldsymbol{v}_{\text{st1}} - \left(\boldsymbol{v}_{\text{st1}} \cdot \boldsymbol{h}_{\text{DRO}} \right) \boldsymbol{h}_{\text{DRO}} \right] \\ &\quad + \tau_1 \left\| \boldsymbol{v}_{\text{st1}} - \left(\boldsymbol{v}_{\text{st1}} \cdot \boldsymbol{h}_{\text{DRO}} \right) \boldsymbol{h}_{\text{DRO}} \right\| \boldsymbol{h}_{\text{DRO}} \end{aligned} \tag{13}$$

继续外推至 t_{d2} 时刻得空间站位置 $\boldsymbol{r}_{\text{st2}}$ 和速度 $\boldsymbol{v}_{\text{st2}}$，$t_{\text{d2}}$ 时刻与 t_{dep} 时刻的速度脉冲 $\Delta\boldsymbol{v}_{\text{d2}}$ 与 $\Delta\boldsymbol{v}_{\text{d3}}$ 由固定时间单段打靶法求解。优化目标函数为四次速度增量幅值之和，表述为

Continue to extrapolate toStation position at time of dayAnd speed， Moment andSpeed pulse at the momentAndIt is solved by the fixed time single shooting method. The optimization objective function is the sum of the amplitudes of the four velocity increments, expressed as

$$J_{\text{pha}} = \left\| \boldsymbol{v}_{\text{st0}}^{+} - \boldsymbol{v}_{\text{st0}} \right\| + \left\| \boldsymbol{v}_{\text{st1}}^{+} - \boldsymbol{v}_{\text{st1}} \right\| + \left\| \Delta\boldsymbol{v}_{\text{d2}} \right\| + \left\| \Delta\boldsymbol{v}_{\text{d3}} \right\| \tag{14}$$

约束为

The constraint is

$$\begin{aligned} & t_{\text{dep}} - dT_{\text{pha}} \le t_{\text{sep}} \le t_{\text{d1}} \le t_{\text{d2}} \le t_{\text{dep}} \\ & 0.9 \le \lambda_0 \le 1.1 \\ & -0.1 \le \tau_0 \le 0.1 \\ & 0.9 \le \lambda_1 \le 1.1 \\ & -0.1 \le \tau_1 \le 0.1 \end{aligned} \tag{15}$$

dT_{pha} 的值可灵活设置，以使 DRO 服务站启航时间间隔约束得到满足。

The value of can be flexibly set so that the DRO service station departure time interval constraint is satisfied.

2.3.3. 地月空间捕获设计

Design of Earth-Moon Space Capture

地月空间捕获及 DRO 入轨是地月空间逃逸阶段的逆过程，设计时通过反向积分和反向微分修正技术代替逃逸阶段的正向积分和微分修正技术即可。实际上捕获阶段的设计更为简单，因为进入地月空间后在第一次近地点便可施加制动捕获脉冲，并且设计相位调节机动时，不存在 DRO 入轨的时间间隔约束问题。

Earth-Moon space capture and DRO injection are the inverse process of the Earth-Moon space escape phase, and the forward integration and differential correction technology in the escape phase can be replaced by the reverse integration and reverse differential correction technology in the design. In fact, the design of the capture phase is simpler, because the braking capture pulse can be applied at the first perigee after entering the Earth-Moon space, and there is no time interval constraint for DRO orbit injection when designing the phase adjustment maneuver.

2.4. 地月空间边界拼接

Boundary Splicing of Earth-Moon Space

本节介绍行星际转移与地月空间内转移在地月空间边界上的拼接方法。以地月空间逃逸为例，行星际转移窗口搜索完成后，可得到

HCI坐标系中空间站在t_0时刻相对地月空间点的逃逸速度$\boldsymbol{v}_{\infty 0}$。$t_0$时刻根据行星星历确定地球位置$\boldsymbol{r}_{\mathrm{E}}$和速度$\boldsymbol{v}_{\mathrm{E}}$，以$\boldsymbol{r}_{\mathrm{E}}$和$\boldsymbol{v}_{\mathrm{E}}+\boldsymbol{v}_{\infty 0}$为HCI系中空间站的状态初值，用二体模型对空间站进行轨道外推，地球的位置用行星星历外推，当空间站与地球距离为500万km时，记录下对应时刻t_{bd}及HCI坐标系下空间站的位置$\boldsymbol{r}_{\mathrm{bd}}^{\mathrm{HCI}}$和速度$\boldsymbol{v}_{\mathrm{bd}}^{\mathrm{HCI}}$，进一步转换到ECI坐标系中描述，记为$\boldsymbol{r}_{\mathrm{bd}}^{\mathrm{ECI}}$与$\boldsymbol{v}_{\mathrm{bd}}^{\mathrm{ECI}}$。由此便获得了在地月空间边界上的期望到达时刻及期望位置速度，作为地月空间内轨迹设计的边界条件。

This section describes the splicing method of interplanetary transfer and intra-cislunar transfer on the cislunar boundary. Taking the Earth-Moon space escape as an example, after the interplanetary transfer window search is completed, the space station in the HCI coordinate system can be obtained.Escape velocity relative to the Earth-Moon space point at any time。Determining the position of the earth according to the planetary ephemeris at any timeAnd speedIn order toAndIs the initial value of the state of the space station in the HCI system. The two-body model is used to extrapolate the orbit of the space station. The position of the earth is extrapolated by the planetary calendar. When the distance between the space station and the earth is 5 million km, the corresponding time is recorded.And the position of the space station in the HCI coordinate systemAnd speed, further converted to the description in the ECI coordinate system, denoted asAnd。 Therefore, the expected arrival time and the expected position and velocity on the boundary of the Earth-Moon space are obtained, which are used as the boundary conditions for trajectory design in the Earth-Moon space.

2.5. 批量任务规划

Batch task planning

本节介绍从众多满足燃料消耗要求的运输任务中筛选载人数最多任务集合的方法。行星际转移可行窗口分布于8个相对独立的区间中，不同区间内的运输任务不存在DRO离轨、近地点对接载人飞船、近火点释放载人飞船间隔10天的冲突，而同一区间内的运输任务有可能存在冲突。因此，在同一区间内，需要规划出满足时间间隔约束且总载人数最大的组合。考虑到返回地月空间时DRO入轨并无时间间隔约束，故先对单个窗口内搜索出的所有行星际转移可行解，进行地月空间逃逸时间—相位解耦优化；选出整体燃耗估计值满足约束的运输任务，然后进一步筛选出满足时间间隔约束且载人数最大化的组合；最后进行地月空间捕获优化与DRO相位调节机动设计。

This section describes the method for selecting the most manned task set from a large number of transport tasks that meet the fuel consumption requirements. The feasible windows of interplanetary transfer are distributed in eight relatively independent intervals, and there is no conflict between the transportation missions in different intervals, such as DRO deorbit, perigee docking manned spacecraft, and the interval of 10 days between the release of manned spacecraft near the fire point, while there may be conflict between the transportation missions in the same interval. Therefore, in the same interval, it is necessary to plan a combination that meets the time interval constraints and has the largest total number of passengers. Considering that there is no time interval constraint for DRO to enter orbit when returning to the Earth-Moon space, all the feasible solutions of interplanetary transfer searched in a single window are first optimized by decoupling the escape time-phase in the Earth-Moon space. The transportation tasks whose overall burnup estimates meet the constraints are selected, and then the combinations which meet the time interval constraints and maximize the number of passengers are further screened out. Finally, the Earth-Moon space acquisition optimization and the design of DRO phase adjustment maneuver are carried out.

整体燃耗估计值满足约束的判断依据为，指标J_{esc}与J_{GA}之和小于Δv_{u}，其中Δv_{u}取值在3.5 km/s~4.5 km/s之间，与具体所在区间有关。然后对各区间内燃耗估计值满足约束的运输任务进一步筛选出满足10天间隔约束的组合，筛选算法的流程如图5所示。

The judgment basis for the overall burnup estimation value to satisfy the constraint is that the indexAndThe sum is less than, of whichThe value is between 3.5 km/s and 4.5 km/s, which is related to the specific interval. Then, the transportation tasks whose internal combustion consumption estimates in each interval meet the constraints are further screened out to meet the 10-day interval constraints. The flow of the screening algorithm is shown in Figure 5.

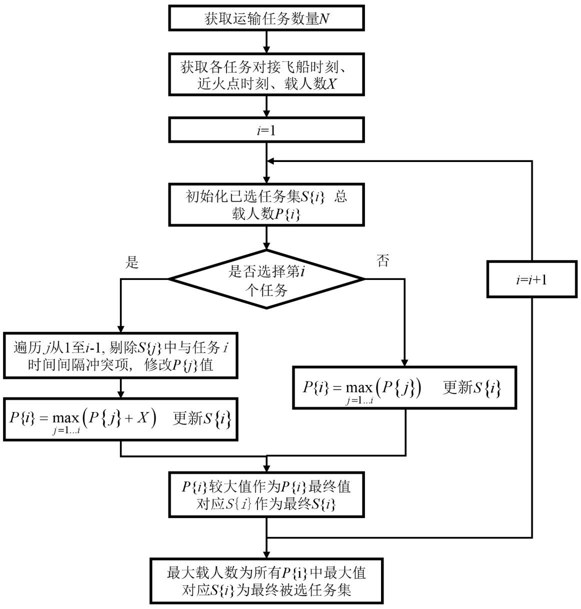


图 5 批量任务规划流程图

Fig. 5 Flow chart of batch task planning

Fig.5 Flowchart of massive missions planning

2.6. 空间站最小数量规划

Minimum number planning of space station

本节介绍在已经获得各区间内运输任务的基础上，根据各任务的出发时刻和返回时刻，规划最少所需空间站数量的方法。约束条件为所有任务都被执行，且每一空间站执行完上一任务返回地球的时刻，与其执行下一任务的出发时刻，间隔大于等于 10 天，并且一个飞船一次只能执行一个任务。利用动态规划思想设计规划算法如下：

This section describes the method of planning the minimum number of space stations required according to the departure time and return time of each mission on the basis of the transportation tasks in each interval. The constraint condition is that all tasks are executed, and the interval between the time when each space station returns to the earth after completing the previous task and the time when it starts to perform the next task is greater than or equal to 10 days, and a spacecraft can only perform one task at a time. The planning algorithm is designed by using the dynamic programming idea as follows:

a) 获取 M 个运输任务离开 DRO 服务站时刻 $t_{\mathrm{d}}(i)$，返回 DRO 服务站时刻 $t_{\mathrm{a}}(i)$；
Acquire that time when M transportation tasks leave the DRO service station? Return to DRO service station time;

b) 定义执行第 i 个任务的空间站编号为 $s(i)$，执行 1~i 号任务所需最小空间站数量为 $n(i)$，第 k 个空间站返回时刻 $t_{\mathrm{s}}(k)$；
Define the execution sectionThe space station number of the mission is, execute 1 ~The minimum number of space stations required for the mission is, dTime of space station return;

c) 初始化 $n(1)=1$， $s(1)=1$， $t_{\mathrm{s}}(1)=t_{\mathrm{a}}(1)$；
Initialize, , ;

d) 对第 i 个任务，遍历 $j=1\ldots i-1$，若存在 $t_{\mathrm{s}}\left[s(j)\right]+10<t_{\mathrm{d}}(i)$，则令 $s(i)=s(j)$，$n(i)=n(j)$，$t_{\mathrm{s}}\left[s(i)\right]=t_{\mathrm{a}}(i)$；若不存在 j 使得 $t_{\mathrm{s}}\left[s(j)\right]+10<t_{\mathrm{d}}(i)$，则令 $n(i)=n(i-1)+1$，$s(i)=n(i)$，$t_{\mathrm{s}}\left[s(i)\right]=t_{\mathrm{a}}(i)$。
Yes, firstTasks, traversing, if present, then make , , ; If does not existMake, then make ,。

e) 对 i 从 1 至 M，重复步骤 d)。
YesRepeat step d) from 1 to M.

通过上述算法，可得到需要空间站的最小数量 $n(M)$，以及每个空间站执行的任务集。

Through the above algorithm, the minimum number of space stations can be obtained., and the set of tasks performed by each station.

3. 设计结果

Design results

按照第 2 节中的设计方法，最终获得的设

计指标结果为：20 年时间内，使用 24 艘空间站共执行 61 次运输任务，移民总人数为 4320 人，总速度脉冲消耗 239.57 km/s。表 1 给出了所有运输任务的汇总。表中的时刻都为距离 MJD 60676 （2025 年 1 月 1 日）的时长，脉冲消耗考虑了从对接载人飞船到释放载人飞船过程的燃料消耗放大系数 1.33，最终移民总人数的构成为实际运送的 4290 人加上提交结果时间奖励 30 人。

According to the design method in Section 2, the final design index results are as follows: within 20 years, 24 space stations are used to carry out 61 transportation missions, the total number of immigrants is 4320, and the total speed pulse consumption is 239.57 km/s. Table 1 gives a summary of all transportation tasks. The time in the table is the time from MJD 60676 (January 1, 2025). The impulse consumption takes into account the fuel consumption amplification factor of 1.33 from the docking of manned spacecraft to the release of manned spacecraft. The final total number of immigrants is composed of 4290 people actually transported plus 30 people awarded for the time of submitting results.

以某次运输任务为例，行星际转移轨迹如图 6 所示，地月空间内逃逸阶段轨迹如图 7 所示，捕获阶段轨迹如图 8 所示。可以看出，该次任务地月空间逃逸阶段借助了月球借力以及 WSB 型的大范围转移轨道来降低转移燃料消耗。

Taking a transport mission as an example, the interplanetary transfer trajectory is shown in Figure 6, the escape phase trajectory in the Earth-Moon space is shown in Figure 7, and the capture phase trajectory is shown in Fig. 8. It can be seen that during the Earth-Moon space escape phase of the mission, the transfer fuel consumption was reduced by means of the lunar force and the WSB type of large-scale transfer orbit.

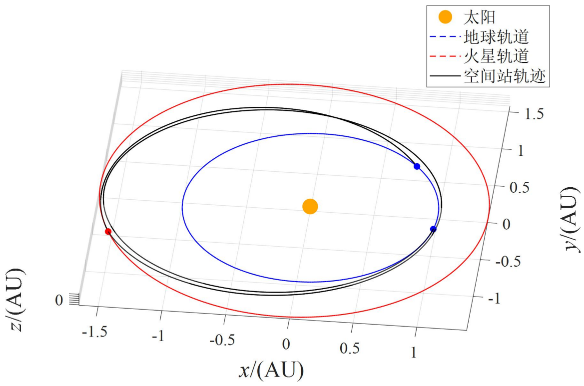
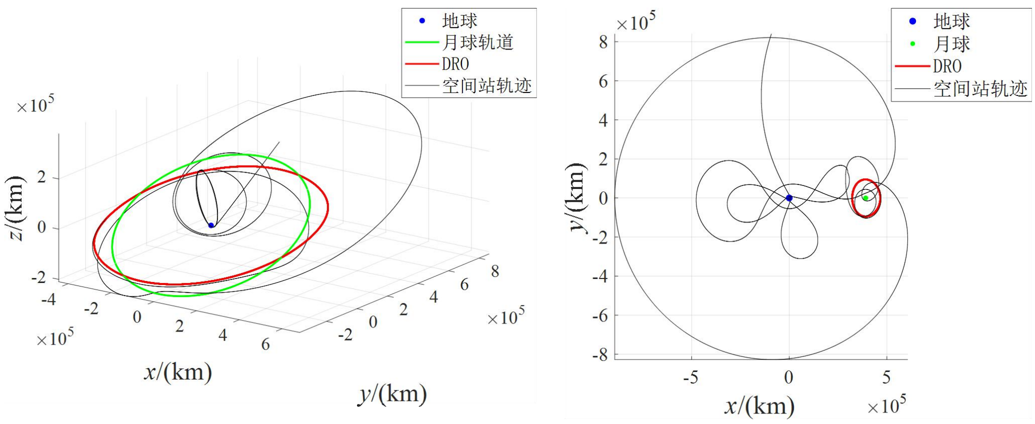


图 6 行星际转移轨迹

Figure 6 Interplanetary Transfer Trajectory

Fig.6 Trajectories of interplanetary transfer

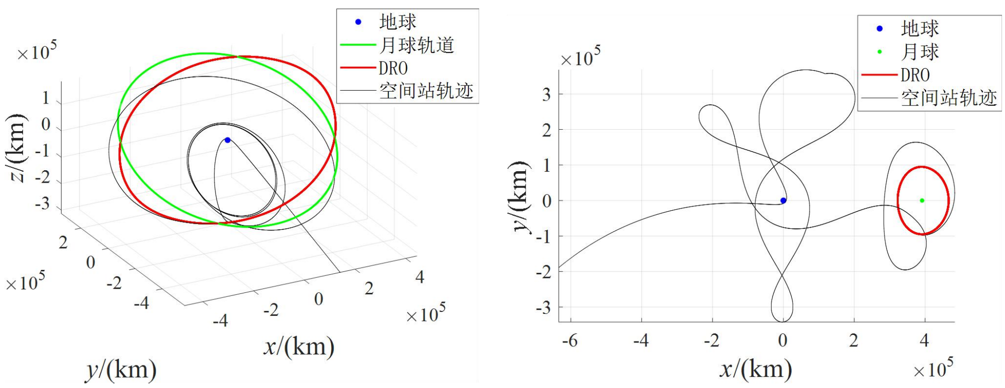


a) 地心赤道惯性系下　b) 地月旋转系下

图 7 地月空间内逃逸阶段轨迹

Fig. 7 Trajectory of the escape phase in the Earth-Moon space

Fig.7 Trajectories of escaping phase in Earth-Moon system



a) 地心赤道惯性系下　　b) 地月旋转系下

图 8 地月空间内捕获阶段轨迹

Fig. 8 Trajectory of capture phase in Earth-Moon space

Fig.8 Trajectories of capturing phase in Earth-Moon system

表 1 运输任务汇总

Tab.1 Summarizing of transport missions

运输任务编号	空间站编号 (No. of Station)	DRO 启航时刻 (DRO departure) (MJD)	对接飞船时刻 (ship dock) (MJD)	释放飞船时刻 (shiprelease) (MJD)	返回 DRO 时刻 (DRO return) (MJD)	脉冲消耗 (dv) (km/s)	载人数 (人)
1	1	529.05	634.84	908	1875.17	3.93	60
2	2	540.88	622.18	920	1845.65	3.83	70
3	4	553.78	668.49	946	1881.43	4.37	70
4	7	584.07	702.53	986	1805.14	4.48	70
5	3	603.51	656.97	934	1874.37	3.86	70
6	5	615.77	679.81	956	1800.65	3.79	80
7	6	630.48	690.01	968	1843.60	3.88	80
8	8	1249.21	1392.46	1668	2978.03	4.16	60
9	24	1292.49	1375.85	1658	2970.69	4.50	60
10	12	1306.23	1416.07	1716	2959.87	3.82	80
11	9	1320.72	1403.04	1680	3003.26	3.43	70
12	10	1337.45	1444.11	1692	2970.32	3.91	60
13	11	1347.61	1454.13	1704	2971.63	3.80	70
14	13	1358.71	1430.96	1730	2964.22	4.13	80
15	3	2016.22	2168.32	2452	3725.45	4.13	70
16	2	2061.38	2153.61	2440	3678.73	3.81	70
17	7	2076.20	2134.49	2430	3778.97	4.46	70
18	5	2091.91	2186.01	2486	3831.28	3.91	70
19	15	2113.68	2113.68	2498	3751.88	4.46	60
20	4	2138.23	2237.93	2464	3726.57	3.75	70
21	6	2154.76	2208.44	2410	3734.04	4.24	40
22	1	2174.86	2224.00	2420	3762.06	3.93	50
23	14	2195.90	2253.15	2474	3750.90	3.65	60
24	21	2825.50	2933.52	3206	4596.88	3.55	60
25	17	2887.80	2991.25	3286	4597.75	2.99	60
26	18	2902.68	2980.09	3276	4552.98	3.87	60
27	22	2912.94	3015.66	3306	4569.00	3.16	60
28	16	2923.44	3003.71	3296	4597.54	2.84	60
29	20	2935.38	3028.75	3246	4617.82	2.97	70
30	19	2956.30	3051.15	3236	4567.78	3.09	70
31	10	2986.03	3062.44	3222	4561.42	3.62	70
32	8	2999.38	3038.86	3196	4558.38	3.56	60
33	11	3010.87	3091.06	3346	4597.75	4.39	50
34	9	3032.95	3075.13	3318	4610.95	4.29	60

35	13	3674.40	3774.12	4044	5016.89	3.82	80
36	12	3685.50	3784.70	4008	5361.42	3.67	80
37	2	3705.35	3794.95	3992	5333.53	3.68	70
38	3	3740.72	3805.14	4034	5046.98	3.69	80
39	6	3753.97	3859.00	4070	5064.98	4.02	80
40	4	3766.00	3826.92	4060	5055.92	3.48	80
41	1	3790.53	3847.13	4020	5373.35	3.78	80
42	7	3817.49	3869.09	4086	4992.05	4.45	80
43	23	4501.59	4604.26	4820	5779.66	4.48	70
44	14	4526.43	4621.02	4852	5813.48	3.92	80
45	15	4547.97	4641.68	4880	5788.80	4.44	90
46	5	4582.53	4631.25	4840	5801.59	4.27	80
47	7	5252.46	5337.27	5632	6560.22	4.44	80
48	4	5274.98	5355.87	5654	6531.71	4.37	80
49	3	5285.02	5397.59	5642	6557.33	4.46	80
50	6	5311.58	5366.11	5666	6542.58	4.06	70
51	8	5330.17	5408.42	5686	6554.55	4.03	80
52	2	5361.07	5418.47	5700	6562.72	4.28	90
53	1	5976.39	6102.15	6402	7290.08	4.34	70
54	14	5987.21	6087.65	6380	7301.14	4.46	60
55	9	6000.41	6126.26	6416	7301.25	3.35	70
56	5	6017.90	6114.73	6390	7301.43	4.04	60
57	10	6030.28	6138.63	6428	7302.49	3.84	70
58	11	6045.86	6149.92	6440	7301.51	3.43	80
59	12	6074.78	6161.59	6452	7272.90	4.20	80
60	13	6093.55	6174.03	6462	7282.35	3.96	80
61	15	6119.22	6185.26	6482	7277.00	4.05	70

4. 结论

Conclusion

第十二届全国（中国）空间轨道设计竞赛甲组赛题以可重复使用地火运输空间站火星移民为背景，综合地月空间限制性四体问题与行星际转移二体问题，为参赛队提出了一个复杂深空轨道动力学环境下非线性和多约束的任务规划与轨道优化问题，极具趣味性与挑战性。

Group A of the 12th National (China) Space Orbit Design Competition, based on the background of Mars migration of reusable earth-fire transport space station, integrates the restricted four-body problem of Earth-Moon space and the two-body problem of interplanetary transfer, and proposes a nonlinear and multi-constrained mission planning and orbit optimization problem for the participating teams in the complex deep space orbit dynamics environment, which is very interesting and challenging.

本文介绍了北京航空航天大学和航天科工二院联队的设计方法。将行星际转移与地月空间内转移解耦，提出一种确定匹配的引力辅助优化方法，拓展了行星际转移窗口；并通过构建与求解时间—相位解耦的非线性规划模型，以及设计相位调节机动实现地月空间低能量逃逸与捕获；设计了一套有效筛选满足约束且最大化运输人数任务集合的算法，并规划了需使用空间站的最小数量。最终设计结果可实现在 20 年内，使用 24 艘空间站将 4320 人移民火星，获得了甲组赛道季军。

This paper introduces the design method of Beijing University of Aeronautics and Astronautics and the Second Academy of Aerospace Science and Engineering. The interplanetary transfer is decoupled from the intra-Earth-Moon transfer, and a gravity-assisted optimization method is proposed to determine the matching, which expands the interplanetary transfer window. By constructing and solving a time-phase decoupled nonlinear programming model, and designing a phase adjustment maneuver, the Earth-Moon space low energy escape and capture is realized. A set of algorithm is designed to effectively filter the task set that meets the constraints and maximizes the number of people transported, and the minimum number of space stations to be used is planned. The final design result can be achieved in 20 years, using 24 space stations to immigrate 4320 people to Mars, and won the third place in Group A track.

本团队的设计方案仍有不足之处。在行星际转移部分未考虑地火转移阶段施加深空机动及金星借力以进一步拓展可行窗口；地月空间内部分，月球借力、WSB 型转移等低能转移设计依赖于数值法，未从理论上给出辅助设计的定性或定量建议；此外，将行星际部分与地月空间内部分结合起来进行一体化设计，在算力允许的情况下，有可能进一步降低速度脉冲消耗，提高设计指标，甚至获得全局最优；在搜索和筛选的参数取值方面，主观因素较大，因此遗漏了较多满足约束的运输任务。

There are still some shortcomings in the design of the team. In the phase of interplanetary transfer without considering the earth-fire transfer, deep space maneuver and Venus force are applied to further expand the feasible window. In the inner part of the Earth-Moon space, the design of low-energy transfer such as lunar gravity-assist and WSB type transfer depends on numerical methods, and no qualitative or quantitative suggestions for aided design are given in theory. In addition, by combining the interplanetary part with the inner part of the Earth-Moon space for integrated design, it is possible to further reduce the speed pulse consumption, improve the design index, and even obtain the global optimum if the computing power permits. In terms of the parameter values of search and screening, the subjective factors are large, so more transportation tasks meeting the constraints are omitted.