X32B10T0 problem

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Slab of thickness L, initially at zero temperature and with temperature-independent properties, subject to heating by convection at x=0 with a constant environment temperature T_f with the boundary at $x = L$ thermally insulated, as illustrated in Fig. 1.

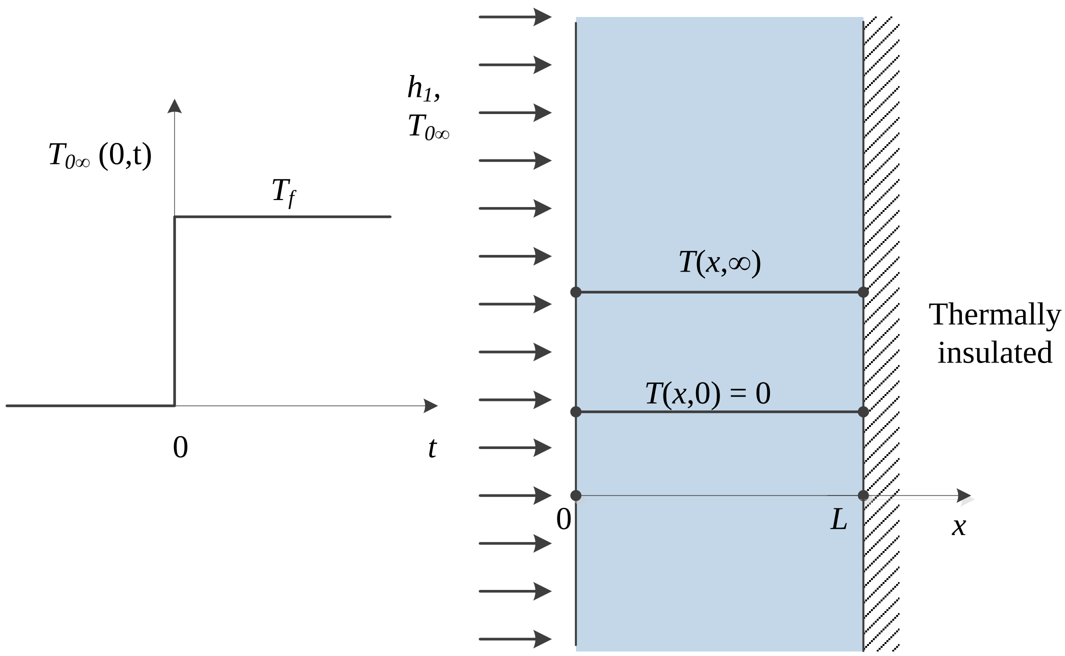

Fig. 1 – Schematic of the 1D transient X32B10T0 problem

Dimensional problem X32B10T0

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad 0 < x < L, \; t > 0 \tag{X32B1-1}$$

$$-k\frac{\partial T(0,t)}{\partial x} = h_1\left(T_f - T(0,t)\right) \tag{X32B1-2a}$$

$$\frac{\partial T(L,t)}{\partial x} = 0 \tag{X32B1-2b}$$

$$T(x,0) = 0 \tag{X32B1-2c}$$

Dimensionless problem X32B10T0

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} = \frac{\partial \tilde{T}}{\partial \tilde{t}}, \quad 0 < \tilde{x} < 1, \; \tilde{t} > 0 \tag{X32B1-3}$$

$$-\frac{\partial \tilde{T}(0,\tilde{t})}{\partial \tilde{x}} = B_1\left(1 - \tilde{T}(0,\tilde{t})\right) \tag{X32B1-4a}$$

$$\frac{\partial \tilde{T}(1,t)}{\partial \tilde{x}} = 0 \qquad \text{(X32B1-4b)}$$

$$\tilde{T}(\tilde{x},0) = 0 \qquad \text{(X32B1-4c)}$$

Dimensionless groups

$$\tilde{T}(\tilde{x},\tilde{t}) = \frac{T(x,t)}{T_f},\ \tilde{q} = \frac{q(x,t)}{h_1 T_f},\ \tilde{x} = \frac{x}{L},\ \tilde{t} = \frac{\alpha t}{L^2},\ B_1 = \frac{h_1 L}{k} \qquad \text{(X32B1-5)}$$

Solutions

For small times, values may be computed to accuracy of 10^{-A} until a time limited by

$$0 < \tilde{t} < \frac{0.12}{A}(2-\tilde{x})^2 \qquad \text{(X32B1-6)}$$

using this expression from the semi-infinite case

$$\tilde{T}_{X32B10T0}(\tilde{x},\tilde{t}) \approx \operatorname{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) - U(\tilde{x},\tilde{t},B_1) \qquad \text{(X32B1-7a)}$$

For intermediate time levels, values may be computed to an accuracy of 10^{-A} during the time interval limited by

$$\frac{0.12}{A}(2-\tilde{x})^2 < \tilde{t} < \frac{0.12}{A}(2+\tilde{x})^2 \qquad \text{(X32B1-8)}$$

using

$$\begin{aligned}\tilde{T}_{X32B10T0}(\tilde{x},\tilde{t}) \approx{}& \operatorname{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}}\right) - U(\tilde{x},\tilde{t},B_1) \\ &+ \operatorname{erfc}\left(\frac{2-\tilde{x}}{\sqrt{4\tilde{t}}}\right) - U(2-\tilde{x},\tilde{t},B_1)\end{aligned} \qquad \text{(X32B1-9)}$$

where

$$U(\tilde{x},\tilde{t},B_1) \equiv e^{B_1\tilde{x}+B_1^2\tilde{t}} \operatorname{erfc}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}} + B_1\sqrt{\tilde{t}}\right) = e^{-\frac{\tilde{x}^2}{4\tilde{t}}} \operatorname{erfcx}\left(\frac{\tilde{x}}{\sqrt{4\tilde{t}}} + B_1\sqrt{\tilde{t}}\right) \qquad \text{(X32B1-10)}$$

Values valid for all positive time can be computed to an accuracy of 10^{-A} from the infinite series solution below, but this expression is more efficient for

$$\tilde{t} > \frac{0.12}{A}(2+\tilde{x})^2 \qquad \text{(X32B1-11)}$$

Use

$$\tilde{T}_{X32B10T0}(\tilde{x},\tilde{t}) = 1 - 2B_1 \sum_{m=1}^{m_{\max}} \frac{(\beta_m^2 + B_1^2)\cos\left(\beta_m(1-\tilde{x})\right)\cos(\beta_m)}{\beta_m^2(\beta_m^2 + B_1^2 + B_1)} e^{-\beta_m^2\tilde{t}} \qquad \text{(X32B1-12a)}$$

$$\tilde{q}_{X32B10T0}(\tilde{x},\tilde{t}) = 2B_1 \sum_{m=1}^{m_{\max}} \frac{(\beta_m^2 + B_1^2)\sin\left(\beta_m(1-\tilde{x})\right)\cos(\beta_m)}{\beta_m(\beta_m^2 + B_1^2 + B_1)} e^{-\beta_m^2 \tilde{t}} \qquad \text{(X32B1-12b)}$$

Eigencondition

$$\tan(\beta_m) = \frac{B_1}{\beta_m},\ m = 1, 2, \cdots \qquad \text{(X32B1-13)}$$

Number of terms, $m_{\max}$, required for solution accuracy of 10^{-A} is

$$m_{\max} = \text{floor}\left(\frac{1}{\pi}\sqrt{\frac{A\ln(10)}{\tilde{t}}}\right) = \text{floor}\left(0.483\sqrt{\frac{A}{\tilde{t}}}\right) \qquad \text{(X32B1-14)}$$

Tables for computed values of dimensionless temperature at different dimensionless depths for given Bi_1

Bi= 0.01

time, t	T(x= 0.00)	T(x= 0.25)	T(x= 0.50)	T(x= 0.75)	T(x= 1.00)
0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0100	0.00112738	0.00004375	0.00000014	0.00000000	0.00000000
0.0200	0.00159377	0.00020218	0.00000801	0.00000008	0.00000000
0.0300	0.00195141	0.00039195	0.00003719	0.00000150	0.00000005
0.0400	0.00225276	0.00058433	0.00008745	0.00000702	0.00000057
0.0500	0.00251814	0.00077180	0.00015347	0.00001877	0.00000269
0.0600	0.00275796	0.00095242	0.00023043	0.00003760	0.00000785
0.0700	0.00297842	0.00112596	0.00031480	0.00006352	0.00001733
0.0800	0.00318356	0.00129274	0.00040419	0.00009616	0.00003203
0.0900	0.00337616	0.00145326	0.00049694	0.00013502	0.00005244
0.1000	0.00355829	0.00160806	0.00059195	0.00017957	0.00007874
0.1500	0.00435593	0.00231299	0.00108195	0.00047085	0.00029249
0.2000	0.00503170	0.00293661	0.00157874	0.00084266	0.00061316
0.2500	0.00563647	0.00351055	0.00207618	0.00126351	0.00100233
0.3000	0.00619832	0.00405410	0.00257344	0.00171417	0.00143364
0.3500	0.00673396	0.00457900	0.00307044	0.00218290	0.00189064
0.4000	0.00725352	0.00509245	0.00356716	0.00266258	0.00236320
0.4500	0.00776319	0.00559880	0.00406362	0.00314882	0.00284517
0.5000	0.00826673	0.00610074	0.00455982	0.00363897	0.00333278
0.5500	0.00876642	0.00659989	0.00505577	0.00413141	0.00382373
0.6000	0.00926369	0.00709724	0.00555147	0.00462515	0.00431662
0.6500	0.00975937	0.00759339	0.00604692	0.00511959	0.00481060
0.7000	0.01025399	0.00808872	0.00654213	0.00561435	0.00530514
0.7500	0.01074788	0.00858346	0.00703708	0.00610922	0.00579994
0.8000	0.01124121	0.00907773	0.00753179	0.00660405	0.00629479
0.8500	0.01173411	0.00957162	0.00802625	0.00709877	0.00678958
0.9000	0.01222665	0.01006519	0.00852047	0.00759333	0.00728423
0.9500	0.01271888	0.01055847	0.00901444	0.00808768	0.00777871
1.0000	0.01321082	0.01105147	0.00950816	0.00858182	0.00827298

Bi= 0.10

time, t	T(x= 0.00)	T(x= 0.25)	T(x= 0.50)	T(x= 0.75)	T(x= 1.00)
0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0100	0.01118454	0.00043549	0.00000143	0.00000000	0.00000000
0.0200	0.01575980	0.00200664	0.00007969	0.00000084	0.00000000
0.0300	0.01924796	0.00388108	0.00036914	0.00001493	0.00000048
0.0400	0.02217352	0.00577435	0.00086656	0.00006966	0.00000570
0.0500	0.02473961	0.00761335	0.00151829	0.00018604	0.00002672
0.0600	0.02705041	0.00937986	0.00227615	0.00037220	0.00007782

0.0700	0.02916780	0.01107235	0.00310529	0.00062803	0.00017163
0.0800	0.03113210	0.01269468	0.00398176	0.00094969	0.00031687
0.0900	0.03297133	0.01425228	0.00488944	0.00133199	0.00051828
0.1000	0.03470592	0.01575082	0.00581747	0.00176953	0.00077741
0.1500	0.04225131	0.02253235	0.01057806	0.00461815	0.00287450
0.2000	0.04858015	0.02847726	0.01536853	0.00823181	0.00600150
0.2500	0.05419811	0.03390783	0.02013498	0.01229877	0.00977472
0.3000	0.05938226	0.03901869	0.02487337	0.01663064	0.01393398
0.3500	0.06429617	0.04392749	0.02958521	0.02111357	0.01831813
0.4000	0.06903823	0.04870537	0.03427201	0.02567883	0.02282930
0.4500	0.07366787	0.05339538	0.03893478	0.03028489	0.02740825
0.5000	0.07822106	0.05802357	0.04357419	0.03490671	0.03201925
0.5500	0.08271956	0.06260575	0.04819070	0.03952917	0.03664073
0.6000	0.08717658	0.06715150	0.05278462	0.04414321	0.04125968
0.6500	0.09160014	0.07166666	0.05735618	0.04874337	0.04586830
0.7000	0.09599513	0.07615480	0.06190556	0.05332639	0.05046191
0.7500	0.10036454	0.08061812	0.06643292	0.05789037	0.05503771
0.8000	0.10471022	0.08505798	0.07093839	0.06243417	0.05959407
0.8500	0.10903334	0.08947527	0.07542209	0.06695716	0.06413004
0.9000	0.11333462	0.09387054	0.07988414	0.07145900	0.06864509
0.9500	0.11761457	0.09824417	0.08432464	0.07593952	0.07313893
1.0000	0.12187351	0.10259645	0.08874371	0.08039867	0.07761143

Bi= 1.00

time, t	T(x= 0.00)	T(x= 0.25)	T(x= 0.50)	T(x= 0.75)	T(x= 1.00)
0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0100	0.10354302	0.00416337	0.00001389	0.00000000	0.00000000
0.0200	0.14152038	0.01865811	0.00075624	0.00000805	0.00000004
0.0300	0.16894260	0.03530979	0.00343728	0.00014113	0.00000460
0.0400	0.19098048	0.05157829	0.00793626	0.00064910	0.00005379
0.0500	0.20962324	0.06691800	0.01369980	0.00171070	0.00024904
0.0600	0.22588454	0.08126088	0.02026166	0.00338083	0.00071736
0.0700	0.24036283	0.09466573	0.02729885	0.00564022	0.00156560
0.0800	0.25344573	0.10722154	0.03459908	0.00843897	0.00286177
0.0900	0.26540101	0.11901843	0.04202560	0.01171888	0.00463649
0.1000	0.27642276	0.13013913	0.04949155	0.01542288	0.00689175
0.1500	0.32167998	0.17788553	0.08611442	0.03864604	0.02447039
0.2000	0.35660922	0.21670227	0.12074519	0.06660471	0.04935822
0.2500	0.38567138	0.25013339	0.15351768	0.09664236	0.07799344
0.3000	0.41114951	0.28014243	0.18473652	0.12730868	0.10820450
0.3500	0.43429963	0.30783396	0.21461558	0.15781724	0.13877660
0.4000	0.45582922	0.33383695	0.24329431	0.18774922	0.16904964
0.4500	0.47614319	0.35851567	0.27086718	0.21688950	0.19867425
0.5000	0.49547807	0.38208668	0.29740274	0.24513556	0.22747362
0.5500	0.51397643	0.40468393	0.32295444	0.27244685	0.25536628

0.6000	0.53172816	0.42639489	0.34756679	0.29881674	0.28232319
0.6500	0.54879347	0.44728074	0.37127881	0.32425685	0.30834388
0.7000	0.56521571	0.46738757	0.39412592	0.34878840	0.33344321
0.7500	0.58102846	0.48675264	0.41614106	0.37243741	0.35764398
0.8000	0.59625957	0.50540789	0.43735528	0.39523207	0.38097290
0.8500	0.61093333	0.52338189	0.45779816	0.41720125	0.40345837
0.9000	0.62507176	0.54070091	0.47749797	0.43837375	0.42512919
0.9500	0.63869528	0.55738962	0.49648187	0.45877785	0.44601393
1.0000	0.65182315	0.57347140	0.51477594	0.47844108	0.46614060

Bi=10.00

time, t	T(x= 0.00)	T(x= 0.25)	T(x= 0.50)	T(x= 0.75)	T(x= 1.00)
0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0100	0.57241642	0.02866135	0.00010716	0.00000002	0.00000000
0.0200	0.66379600	0.10741527	0.00496733	0.00005761	0.00000031
0.0300	0.71265875	0.18013457	0.02006827	0.00090660	0.00003168
0.0400	0.74460432	0.24015517	0.04223413	0.00381684	0.00034100
0.0500	0.76767371	0.28956599	0.06755990	0.00933889	0.00147039
0.0600	0.78537366	0.33082033	0.09368394	0.01731763	0.00397937
0.0700	0.79952384	0.36581589	0.11938617	0.02733369	0.00821681
0.0800	0.81117900	0.39593935	0.14409611	0.03894825	0.01429104
0.0900	0.82100007	0.42220601	0.16759124	0.05178386	0.02213400
0.1000	0.82942619	0.44537196	0.18982991	0.06553621	0.03157579
0.1500	0.85889225	0.53093921	0.28479747	0.14117719	0.09524109
0.2000	0.87751776	0.58882469	0.36094311	0.21863826	0.17074527
0.2500	0.89134276	0.63360644	0.42564730	0.29214277	0.24633159
0.3000	0.90265372	0.67109312	0.48245039	0.35991829	0.31753540
0.3500	0.91240393	0.70377575	0.53309782	0.42167186	0.38299925
0.4000	0.92102473	0.73282526	0.57857550	0.47765227	0.44257325
0.4500	0.92873642	0.75887303	0.61953898	0.52828698	0.49654915
0.5000	0.93567104	0.78232072	0.65648726	0.57404222	0.54535944
0.5500	0.94192134	0.80346432	0.68983407	0.61537072	0.58946161
0.6000	0.94756057	0.82254470	0.71993847	0.65269386	0.62929521
0.6500	0.95265076	0.83976893	0.74711889	0.68639707	0.66526762
0.7000	0.95724627	0.85531984	0.77166059	0.71683037	0.69775083
0.7500	0.96139552	0.86936091	0.79382023	0.74431059	0.72708239
0.8000	0.96514200	0.88203907	0.81382921	0.76912409	0.75356772
0.8500	0.96852485	0.89348676	0.83189635	0.79152960	0.77748288
0.9000	0.97157939	0.90382345	0.84821012	0.81176077	0.79907721
0.9500	0.97433750	0.91315699	0.86294070	0.83002859	0.81857591
1.0000	0.97682794	0.92158475	0.87624174	0.84652361	0.83618236

Bi=100.00

time, t	T(x= 0.00)	T(x= 0.25)	T(x= 0.50)	T(x= 0.75)	T(x= 1.00)

0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0100	0.94385901	0.06662887	0.00032010	0.00000008	0.00000000
0.0200	0.96020477	0.19414686	0.01086432	0.00014719	0.00000091
0.0300	0.96748052	0.28888706	0.03748825	0.00193361	0.00007571
0.0400	0.97182565	0.35827172	0.07154091	0.00725248	0.00071719
0.0500	0.97479383	0.41120302	0.10696968	0.01636590	0.00282196
0.0600	0.97698620	0.45311078	0.14113138	0.02858100	0.00712604
0.0700	0.97869086	0.48729095	0.17307869	0.04306947	0.01393417
0.0800	0.98006545	0.51583609	0.20260975	0.05914724	0.02318985
0.0900	0.98120457	0.54013930	0.22983392	0.07630462	0.03462982
0.1000	0.98216900	0.56116845	0.25497648	0.09417129	0.04790638
0.1500	0.98547010	0.63634637	0.35770740	0.18678421	0.13045460
0.2000	0.98754159	0.68612265	0.43699424	0.27591201	0.22063836
0.2500	0.98910041	0.72468308	0.50319410	0.35721604	0.30642402
0.3000	0.99038900	0.75698961	0.56048255	0.43004033	0.38442260
0.3500	0.99149964	0.78498953	0.61078342	0.49483217	0.45420492
0.4000	0.99247326	0.80958896	0.65519892	0.55233203	0.51626337
0.4500	0.99333245	0.83131519	0.69450260	0.60331178	0.57132844
0.5000	0.99409257	0.85054261	0.72931146	0.64849436	0.62014644
0.5500	0.99476570	0.86757173	0.76014924	0.68853346	0.66341208
0.6000	0.99536202	0.88265832	0.78747221	0.72401276	0.70175212
0.6500	0.99589037	0.89602545	0.81168208	0.75545091	0.73572573
0.7000	0.99635851	0.90786959	0.83313393	0.78330800	0.76582966
0.7500	0.99677332	0.91836443	0.85214207	0.80799187	0.79250449
0.8000	0.99714088	0.92766374	0.86898492	0.82986395	0.81614077
0.8500	0.99746657	0.93590374	0.88390916	0.84924454	0.83708459
0.9000	0.99775516	0.94320510	0.89713334	0.86641744	0.85564266
0.9500	0.99801088	0.94967474	0.90885112	0.88163414	0.87208674
1.0000	0.99823746	0.95540740	0.91923410	0.89511747	0.88665764

Appendix X32B10T0 Matlab Codes

```
% fdX32B10T0(xd,td,B1,A) function Vector xd and td.
%  T_inf = 1 @ xd = 0, convection condition @ xd = 0
%   insulated boundary at x=1
% Revision History
% 4 2 12 written by James V. Beck
% 6 21 13 revised by Keith A. Woodbury from fdx31B10T0.m
% calling function: feigXIJ(Mmax,B1,B2) for the X32 eigenvalues
% Variables
%       Td = dimensionless temperature, could be a double subscripted array
%       xd = dimensionless location, measured from heating surface,
%            single value or an array of sizex
%       td = dimensionless time, single value or an array of sizet
%       B1 = convection coefficient at x=0
%        A = desired precision of results to 10^-A accuracy
%            used to compute the number of terms needed in the series soln
function Td=fdX32B10T0(xd,td,B1,A)
sizex= max(size(xd)); % start vector option
  sizet=max(size(td)); % end vector option
if( sizet > 1 )
    dt = td(2)-td(1);  % first entry might be zero
else
    dt = td(1);
end
xdv=xd; tdv=td;
Mmax=3*floor(1/2+1/pi*sqrt(A*log(10)/dt));
tp=0.48/A;
betar=feigXIJ(Mmax,B1,0); betav=betar(:,2);
for it=1:sizet
   td=tdv(it);
   for ix=1:sizex
       xd=xdv(ix);    Md(it,ix)=0; Td(it,ix)=0; %XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

    if td == 0
       Td(it,ix)=0;
    elseif  td < tp/4*(2-xd)^2    % short times soln
       Td(it,ix) = erfc(xd/sqrt(4*td)) - Ufunc(xd,td,B1);
    elseif  td < tp/4*(2+xd)^2    % intermediate times soln
       Td(it,ix)= erfc(xd/sqrt(4*td)) - Ufunc(xd,td,B1) + ...
                  erfc((2-xd)/sqrt(4*td)) - Ufunc(2-xd,td,B1);
    else
       M=floor(1/pi*sqrt(A*log(10)/td));
       Md(it,ix)=M;
       Td(it,ix)=1; % Start X32B10T0 case.
       for m=1:Mmax
          beta=betav(m);
          Td(it,ix)=Td(it,ix)-B1*2*exp(-beta^2*td)*(beta^2+B1^2)*cos(beta*(1-
xd))*cos(beta)/(beta^2+B1^2+B1)/beta^2;
       end % for m
    end % if
  end%ix
end%it
end % function

%
%  utility function to return U(x,t,B1)
function U=Ufunc(x,t,B1)
% U=exp(B1*x+B1^2*t)*erfc(x/sqrt(4*t)+B1*sqrt(t));
%  expression below better behaved esp. for large Bi
U = exp(-x^2/4/t)*erfcx( x/sqrt(4*t)+B1*sqrt(t) );
end
```

```
%feigXIJ(m,B1,B2).m 6/20/07 Matlab subroutine written by James V. Beck based on
%programming and method of Prof. A. Haji-Sheikh,
   %Haji-Sheikh, A. and Beck, J.V., "An Efficient Method of Computing Eigenvalues in
Heat Conduction",
   %Numerical Heat Transfer Part B: Fundamentals, Vol  38 No. 2, pp. 133-156, 2000.
%It gives the eigenvalues for the X11, X12, X13, ..., X33 cases. X33 is the general
case & includes the others.
%However, for X11 and X22, the eigenvalues are n *pi, for n = 1, 2, 3, ... .
%(Also n = 0 for X22.)  For X12 and X21, the eigenvalues are (2*n-1)*pi/2 for n = 1,
2, 3,
function XIJ=feigXIJ(m,B1,B2)
%Input quantities
%B1 is the one of the Biot numbers. For X31, it is for Bi_1.
%B2 is the other Biot number. For X13, it is for Bi_2. The eigenvalues for X13 and X31
are the same.
%For the boundary condition of the first kind (given temperature), Bi value goes to
infinity.
    %Use BI1=1.0e+15 or BI2 = 1.0e+15.
   %For the case of bc of the second kind, the Bi value goes to zero. Use BI1=0 or BI2
= 0
   %m is the number of eigenvalues
%Accuracy: Machine accuracy is expected, about 14 decimal place accuracy
BI1=B1; BI2=B2; AN=1:1:m;
%Immediately below generates the X33 eigenvalues using the Haji subroutine
CN =(AN-0.75)*pi; DN=(AN-0.5)*pi;  D=1.22*(AN-1)+0.76;  P23 = 1./AN;
      BX = BI1*BI2;       BS = BI1 + BI2;
      GAM = 1.0- 1.04*(sqrt((BS + BX+CN-pi/4.0)./(BS +BX+D))-(BS+BX+sqrt(D.*(CN-
pi/4.0)))./(BS+BX+D));
      X23 = (BI1+BI2-CN)./(BI1+BI2+CN);  X13 = BX./(BX+0.2+BS*(pi*pi*(AN-0.5)/2.0));
      G13=1+X13.*(1-X13).*(1-(0.85./AN)-(0.6-0.71./AN).*(X13+1).*(X13-0.6-0.25./AN));
      E23 = GAM.*(P23.*X23 + (1.0 - P23).*tanh(X23)/tanh(1)) ;
      E13 = 2.0*G13.*X13; Y23=CN+pi*E23/4.0;  Y13=DN+pi*E13/4.0;
      ZN =(Y13*BX./(BX+DN.^2)+Y23.*(1.0-BX./(BX+DN.^2)));
      for i=1:3
         A0=((6.0+3.0*BS+BX-ZN.*ZN).*cos(ZN)-(6.0+BS)*ZN.*sin(ZN))/6.0;
      B0=(ZN.^2-BS-BX).*cos(ZN)+(2.0+BS)*ZN.*sin(ZN);
      C0=(ZN.^2-BX).*sin(ZN)-BS*ZN.*cos(ZN); A0=(1.0+BS)*sin(ZN)+2.0*ZN.*cos(ZN)-
C0/2.0;
      E0=-C0./B0-(-A0.*C0.^3+A0.*C0.^2 .*B0)./(3.0*A0.*C0.*C0.*B0  -
2.0*A0.*C0.*B0.^2+B0.^4);
      ZN = ZN+E0;
      end

      if BI1 == 0 & BI2 == 0
         ZN(1)=0;
         end
         num=1:1:m;
         for ie=1:m
            XIJintrinsic(ie)=tan(ZN(ie))-ZN(ie)*(B1+B2)/(ZN(ie)^2-B1*B2);
            end%ie
            XIJ=[num' ZN' XIJintrinsic'];
            %fprintf('%7.1f  %15.15f %15.5e \n',XIJ')



%CX32B10T0.m, modified from CX31B10T0.m  by James Beck
% modified by Keith A. Woodbury 06/20/2013 to create plots

%CX30B1T0.m, modified from CX30B1T0.m  byJames Beck, 2 9 2013;
% Modified from CX30B0T1.m by Keith Woodbury 6/14/2013
clear all
 td=0.0:0.01:0.1; td=[  td 0.15:0.05:1]; sizet=length(td);
 xd=0:.25:1; sizex=length(xd);
```

```
%x = 0 results, vary B1
B1v=[0.01 0.1 1 10 100 1e-10 1e8];
sizeB=length(B1v);

for it=1:sizet
   %below for variable B1, various times
   for iB=1:sizeB
      B1=B1v(iB);
      Txt(:,:,iB) = fdX32B10T0(xd,td,B1,15);
   end%it
end%iB

 % write the tables for each Bi level
 for iB = 1:sizeB
     % write the header
     fprintf('\nBi=%5.2f',B1v(iB));
     fprintf('\n time, t',B1v(iB));
     for ix = 1:sizex
         fprintf('   T(x=%5.2f)       ',xd(ix));
     end %ix
     fprintf('\n');
     % now write the table
     for it = 1:sizet
         fprintf('%7.4f   ',td(it));
         fprintf('%12.8f      ',Txt(it,:,iB));
         fprintf('\n');
     end %it
     fprintf('\n');  % give an extra blank line
 end %iB

%sprintf(' time, t x=0    q(B=0.01)       q(B=0.1)         q(B=1)      q(B=10)
q(B=100)')
%fprintf('%7.4f   %12.8f    %12.8f     %12.8f    %12.8f    %12.8f\n',BBvq0')
%sprintf(' time, t x=1    q(B=0.01)       Tq(B=0.1)         q(B=1)      q(B=10)
q(B=100)')
%fprintf('%7.4f   %12.8f    %12.8f     %12.8f    %12.8f    %12.8f\n',BBvq1')
iBv_i = [ 2 3 4 ]; % these are the indicies of the Bi results to plot
for iB = 1:length(iBv_i)
    figure(iB)
    this_Bi = iBv_i(iB);
    plot(td,Txt(:,1,this_Bi),td,Txt(:,2,this_Bi),td,Txt(:,3,this_Bi), ...
        td,Txt(:,4,this_Bi),td,Txt(:,5,this_Bi))
    Bistr = sprintf('Bi = %5.2f',B1v(this_Bi));
    text(td(5),0.9*max(Txt(:,1,this_Bi)),Bistr);
    text(td(5),Txt(5,1,this_Bi)*1.03,'x = 0');
    text(td(7),Txt(7,2,this_Bi)*1.03,'x = 0.25');
    text(td(9),Txt(9,3,this_Bi)*1.03,'x = 0.50');
    text(td(9),Txt(9,4,this_Bi)*1.03,'x = 0.75');
    text(td(12),Txt(12,5,this_Bi)*1.03,'x = 1.00');
%text(td(5),Tx1(5,2)*1.03,'x = 0.5');
    xlabel('Dimensionless time')
    ylabel('Dimensionless temperature')
end %iB
```

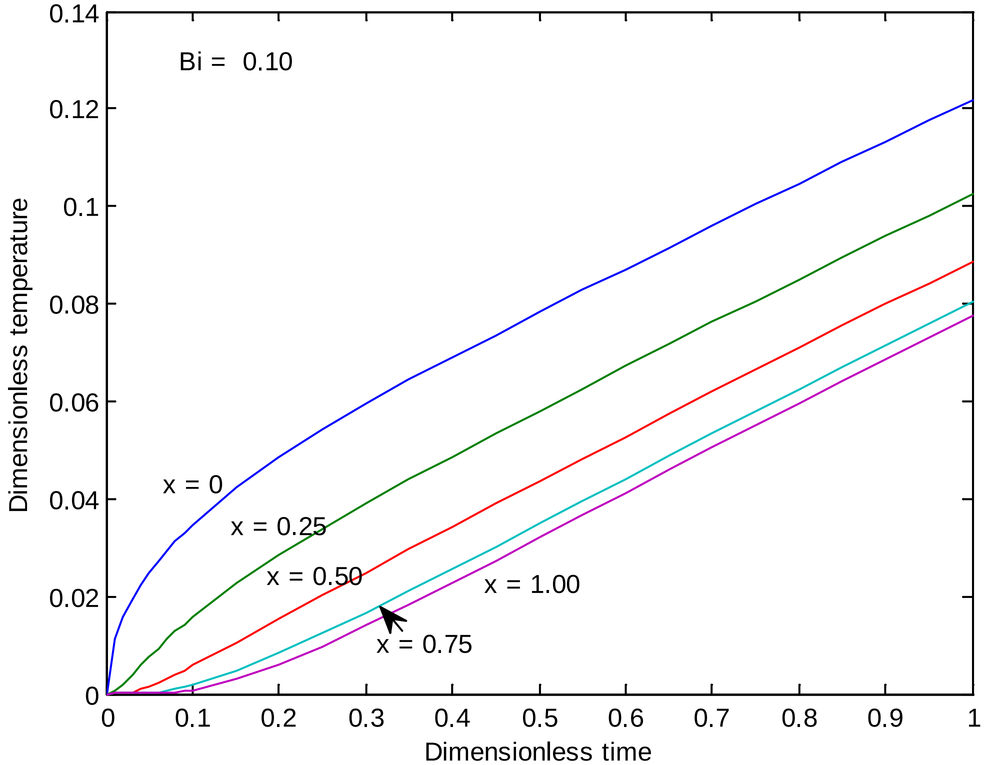

Fig. 2 – Temperature at depths for Bi = 0.10

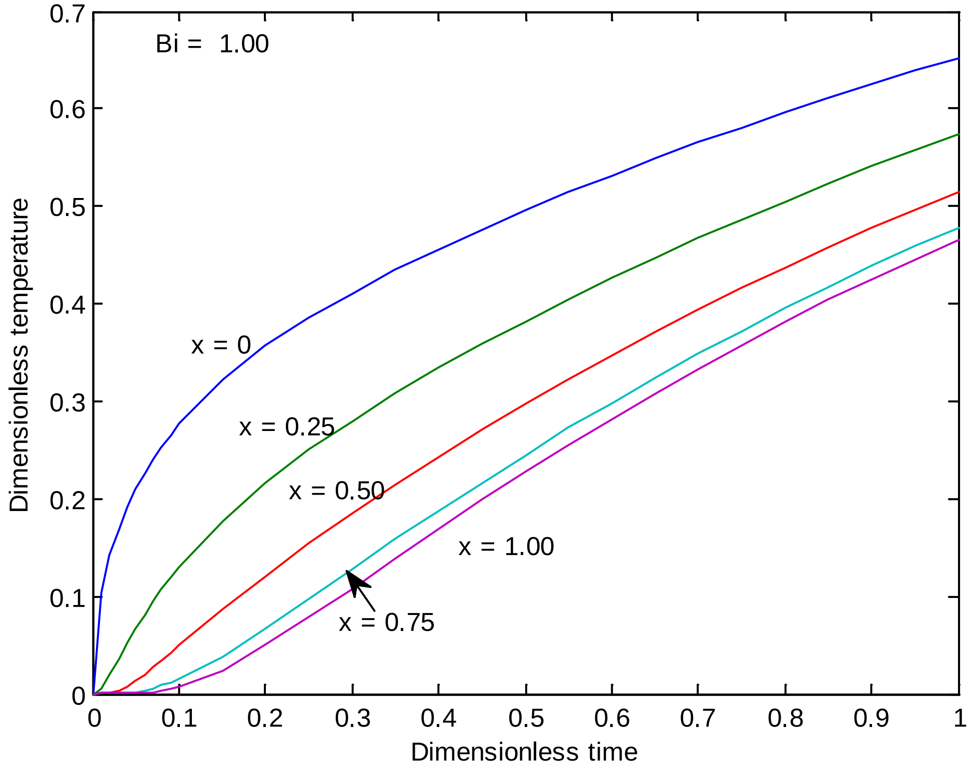

Fig. 3 – Temperature at depths for Bi = 1.0

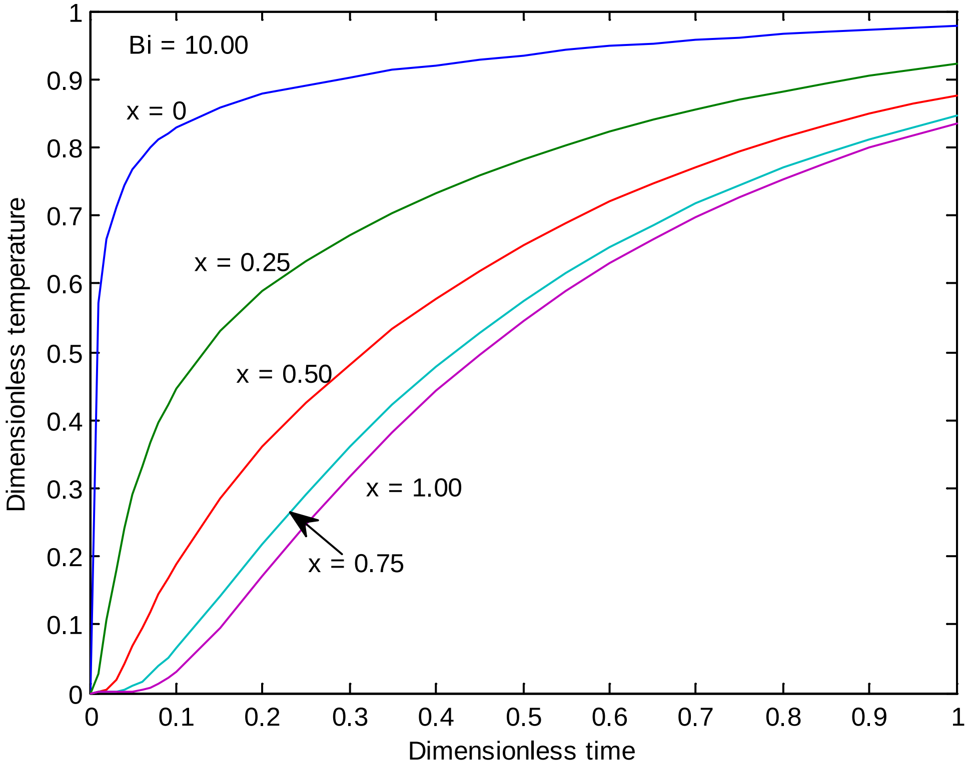

Fig. 4 – Temperature at depths for Bi = 10.0