Circuit Theory I Solved Problems

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1 Chapter 1 - Circuit Variables

1.1 Electrical Engineering - An Overview

This section is primarily a vocabulary only based section so there are no example problems.

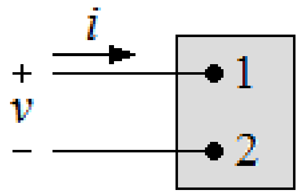
1.2 Units

Assume a telephone travels through a cable at two-thirds the speed of light. How long does it take the signal to get from New York City to Miami if the distance is approximately 1100 miles?
We know

- The speed of light is 3×10^8 m/s
- There are 1.6 km in a mile
- There are 1,000 meters in a kilometer

$$
\begin{aligned}
&= \left(\frac{second}{\frac{2}{3} \times 3 \times 10^8 meters}\right) \times \left(\frac{1.6km}{mile}\right) \times \left(\frac{1000m}{km}\right) \times 1100miles \\
&= \left(\frac{second}{\frac{2}{3} \times 3 \times 10^8 \cancel{meters}}\right) \times \left(\frac{1.6\cancel{km}}{\cancel{mile}}\right) \times \left(\frac{1000\cancel{meters}}{\cancel{km}}\right) \times 1100\cancel{miles} \\
&= \frac{1.6 \times 1000 \times 1100 \times seconds}{\frac{2}{3} \times 3 \times 10^8} \\
&= \frac{1760000}{2e8} \\
&= 0.0088seconds \\
&= 8.8ms
\end{aligned}
$$

1.3 Voltage, Current, Power, and Energy



The current at the terminals of the element in the above diagram is:

- $i = 0, t < 0$
- $i = 20e^{-5000t}, t \geq 0$

Calculate the total charge (in microcoulombs) entering the element at its upper terminal.
We know the equation for current is:

$i = \frac{dq}{dt}$ Therefore the equation for total charge is:

$q(t) = \int_0^t i(x)dx$

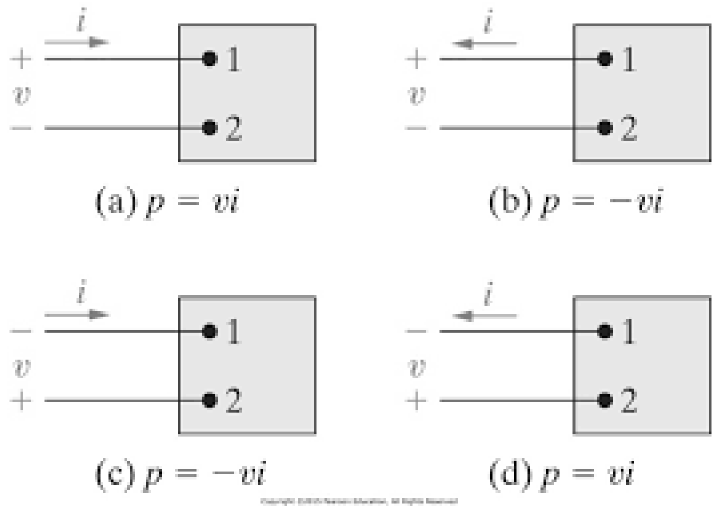
To find the total charge we need to find the current from 0, to ∞. Substituing our equation for i and we get:

$q_{total} = \int_0^\infty 20e^{(-5000t)}dx$

The solving we get:

$$q_{total} = \frac{20}{-5000} e^{-5000x}\Big|_0^\infty$$
$$q_{total} = \frac{20}{-5000} \left(e^{-\infty} - e^0\right)$$
$$q_{total} = \frac{20}{-5000} (0 - 1)$$
$$q_{total} = \frac{20}{-5000}$$
$$q_{total} = 0.004C$$
$$q_{total} = 4000\mu C$$

1.4 Voltage, Current, Power, and Energy



Assume that a 20 V voltage drop occurs across an element from terminal 2 to terminal 1 and that a current of 4 A enters terminal 2.

1. Specify the values of v and i for the polarity references shown in the figure above.

2. State whether the circuit inside the box is absorbing or delivering power.

3. How much power is the circuit absorbing?

1) We are told the positive terminal is connected to terminal 2, and the negative terminal is connected to terminal 1. We are also told that a current of 4 A enters terminal 2. a) The positive terminal is connected to terminal 2 which means the voltage is negatively referenced. The current is referenced entering terminal 1, but our current is entering terminal 2. Therefore the current is negatively referenced. $v = -20V$, $i = -4A$
b) The positive terminal is connected to terminal 2 which means the voltage is negatively referenced. The current is referenced leaving terminal 1, and our current is referenced entering terminal 2, which means the current is positively referenced. $v = -20V$, $i = 4A$
c) The positive terminal is connected to terminal 2, which is also the positive reference. Therefore voltage will be positive. The current is referenced as entering terminal 1, but our current is entering terminal 2. Therefore the current is negatively referenced. $v = 20V$, $i = -4A$
d) The positive terminal is connected to the positive reference point. The current is flowing in the appropriate direction. Therefore both the voltage and the current are positively referenced. $v = 20V$, $i = 4A$
2) We will use the reference shown in Figure (a). Using the passive sign convention we can see that the voltage and the current will be negative. Therefore:

$$p = v \times i$$
$$p = (-20) \times (-4)$$
$$p = 80W$$

Because the power is greater than 0, the box is absorbing power.
3) We can see from the calculation above that the box is absorbing 80 W

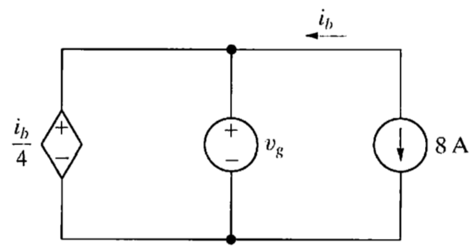
1.5 Voltage, Current, Power, and Energy

Find the current drawn from a 115-V line by a dc electric motor that delivers 1 hp. Assume 100% efficency.

$$
\begin{aligned}
P &= VI \\
I &= \frac{P}{V} \\
1hp &= 745.7W \\
I &= \left(\frac{1hp}{115V}\right) \times \left(\frac{745.7W}{1hp}\right) \\
I &= \left(\frac{1\cancel{hp}}{115V}\right) \times \left(\frac{745.7W}{1\cancel{hp}}\right) \\
I &= 6.48\frac{W}{V} \\
I &= 6.48A
\end{aligned}
$$

2 Circuit Elements

2.1 Voltage and Current Sources



For the above circuit:

1. What value of v_g is required in order for the interconnection to be valid?
2. For this value of v_g find the power associated with the 8 A source.

1) Notice that the current i_b is in the same circuit branch as the 8 A source, but is defined in the opposite direction. Therefore we can say that.
$i_b = -8$A
Next, we can see that the dependent and independent voltage source are in parallel with the same polarity. Therefore, we know that they must be equal. So if the dependent source is:

$$V_{dependent} = \frac{i_b}{4}$$
$$V_{dependent} = V_g$$
$$V_g = \frac{i_b}{4}$$
$$V_g = \frac{-8}{4}$$
$$V_g = -2V$$

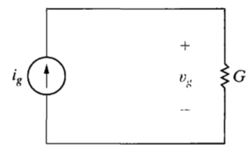
Therefore the answer to part 1 is -2 V.

2) To find the power that corresponds to the 8 A source, we find the voltage drop across the source which is V_i. This will also need to be equal to V_g. Therefore, V_i will also equal -2 V. Using passive sign convention we get:

$$p_s = (8A)(V_i)$$
$$p_s = (8A)(-2V)$$
$$p_s = -16W$$

Therefore the current source generates 16 W of power.

2.2 Ohm's Law



For the above circuit

1. If $i_g = 0.5$ A and $G = 50$ mS, find v_g and the power delivered by the current source
2. If $v_g = 15$ V and the power delivered to the conductor is 9 W, find the conductance G and the source current i_g.
3. If $G = 200\mu S$ and the power delivered to the conductance is 8 W, find i_g and v_g

1) You can see in the circuit that since G and i_g are in the same branch, they must have the same current. The voltage drop across the current source and G must also be equal, because they are in parallel. Therefore, using Ohm's law we can solve for the voltage:

$$
\begin{aligned}
V &= iR \\
V &= \frac{i}{conductance} \\
v_g &= \frac{i_g}{G} \\
v_g &= \frac{0.5A}{50mS} \\
v_g &= 10V
\end{aligned}
$$

Now we know the voltage we can solve for the power.

$$
\begin{aligned}
P &= Vi \\
p_{source} &= -v_i i_g \\
p_{source} &= -(10V)(0.5A) \\
p_{source} &= -5W
\end{aligned}
$$

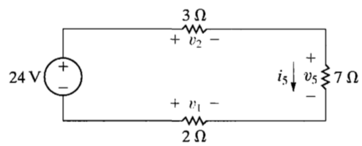
Thus the current source delivers 5 W to the circuit.
2) First, we find the value of the conductance using the power and voltage.

$$
\begin{aligned}
P &= VI \\
V &= IR \\
I &= \frac{V}{R} \\
I &= V \times G \\
P &= V^2 G \\
p_g &= G v_g^2 \\
G &= \frac{p_g}{v_g^2} \\
G &= \frac{9}{15^2} \\
G &= 0.04S \\
G &= 40mS
\end{aligned}
$$

Then we can solve for the current:

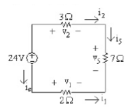
$$
\begin{aligned}
i_g &= G v_g \\
i_g &= (40mS)(15V) \\
i_g &= 0.6A
\end{aligned}
$$

2.3 Kirchhoff's Law



For the problem above, calculate:

1. i_5
2. v_1
3. v_2
4. v_5
5. The power delivered by the 24 V source



1) First redraw the circuit as shown above, adding in the new labels. Then, write a KVL equation clockwise around the circuit.
$-24V + v_2 + v5 - v1 = 0$
Next, we can use Ohm's law to calculate the three unknown voltages.

$$v2 = 3i_2$$
$$v5 = 7i_5$$
$$v1 = 2i_1$$

Using KCL in the upper right corner we can see that $i_2 = i_5$. In the bottom right corner we see that $i_5 = -i_1$. In the upper left corner we can see that $i_s = -i_2$. We can then substitute these into the previous three equations to get.

$$v2 = 3i_2 = 3i_5$$
$$v5 = 7i_5$$
$$v1 = 2i_1 = -2i_5$$

Then we can substitute these equations into the first equation to get:

$$24 = v_2 + v_5 - v_1$$
$$24 = 3i_5 + 7i_5 - (-2i_5)$$
$$24 = 12i_5$$
$$2A = i_5$$

We can then use this information to solve for the remaining variables. 2)

$$v_1 = -2i_5$$
$$v_1 = -2(2)$$
$$v_1 = -4V$$

3)

$$v_2 = 3i_5$$
$$v_2 = 3(2)$$
$$v_2 = 6V$$

4)

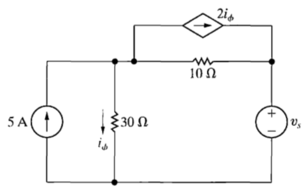
$$v_5 = 7i_5$$
$$v_5 = 7(2)$$
$$v_5 = 14V$$

5) A KCL equation at the lower left node gives $i_s = i_1$ and since we know that $i_1 = -i_5$, then $i_s = -2A$. Therefore:

$$p_24 = (24)i_s$$
$$p_24 = (24)(-2)$$
$$p_24 = -48W$$

So we can see the source is delivering 48 W.

2.4 Analysis of Circuit Containing Dependent Sources



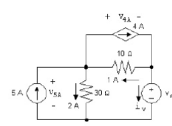
For the above circuit. The current i_ϕ is 2 A. Calculate:

1. v_s
2. The power absorbed by the independent voltage source,
3. The power delivered by the independent current source,
4. the power delivered by the controlled current source,
5. the total power dissipated in the two resistors

The problem tells us that i_ϕ is 2A, so we know that the current in the dependent source is $2i_\phi = 2(2) = 4A$ We can then write a KCL equation in the left node to find the current in the 10Ω resistor.

$$
\begin{aligned}
0 &= -5A + 2A + 4A + i_{10\Omega} \\
i_{10\Omega} &= 5A - 2A - 4A \\
i_{10\Omega} &= -1A
\end{aligned}
$$

We can then redraw the circuit as shown below.



1) To find v_s, we can write a KVL equation that sums the voltages counter-clockwise around the lower right loop.

$$
\begin{aligned}
0 &= -v_s + (1A)(10\Omega) + (2A)(30\Omega) \\
v_s &= 10V + 60V \\
v_s &= 70V
\end{aligned}
$$

2) The current in the voltage source can be found by writing a KCL equation at the right-hand node.

$$
\begin{aligned}
0 &= -4A + 1A + i_v \\
i_v &= 4A - 1A \\
i_v &= 3A
\end{aligned}
$$

Therefore power is:

$$
\begin{aligned}
p &= vi \\
p &= (70V)(3A) \\
p &= 210W
\end{aligned}
$$

Thus the voltage source absorbs 210 W.

3) The voltage drop across the independent current source can be found through a KVL equation around the left loop.

$$
\begin{aligned}
0 &= -v_{5a} + (2A)(30\Omega) \\
v_{5a} &= 60V \\
p &= -v_{5a} i \\
p &= -(60V)(5A) \\
p &= -300W
\end{aligned}
$$

Therefore the source delivers 300 W to the circuit.

4) The voltage across the controlled current source can be found by writing a KVL equation around the upper right loop in a clockwise direction:

$$
\begin{aligned}
0 &= v_{4A} + (10\Omega)(1A) \\
v_{4A} &= -10V \\
p &= v_{4A} i \\
p &= (-10V)(4A) \\
p &= -40W
\end{aligned}
$$

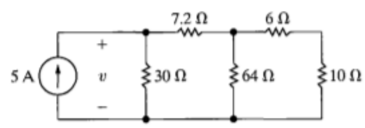
Thus the source delivers 40 W of power to the circuit.

5) The total power dissipated by the resistors is:

$$
\begin{aligned}
&= (i_{30\Omega})^2(30\Omega) + (i_{10\Omega})^2(10\Omega) \\
&= (2)^2(30\Omega) + (1)^2(10\Omega) \\
&= 120 + 10 \\
&= 130W
\end{aligned}
$$

3 Resistive Circuits

3.1 Resistors in Series and Parallel

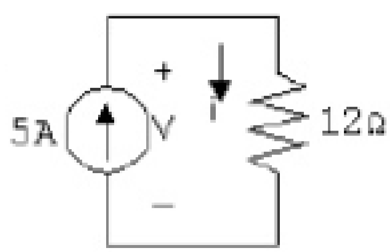


For the circuit shown above, find:

1. The voltage v
2. The power delivered to the circuit by the current source
3. The power dissipated in the 10Ω resistor

We begin by starting from the farthest right side of the circuit. Then, by alternating combing series and parallel resistor we can reduce down to one resistor.

1. First combine the 6Ω and 10Ω in series to get 16Ω
2. Now combine that in parallel with the 64_{Ω} $\frac{(16)(64)}{16+63} = 12.8\Omega$
3. Now comine that in series with the 7.2Ω resistor to get 20Ω
4. Then combine that in parallel with the 30Ω $\frac{(20)(30)}{20+30} = 12\Omega$



This gives us the below circuit:

With the simplified circuit the problem becomes easier. 1) Using Ohm's Law we can see that:
$V = IR = (5A)(12\Omega) = 60V$
2) Now that we know the value of the voltage drop accross the current source we can find the power:
$p = -(V)(I) = -(60V)(5A) = -300W$
Therefore 300 W is delivered to the circuit. 3) Now we know that the voltage drop across the current source is 60 V. This is also the voltage drop across the 30Ω resistor. Therefore the current through this resistor is:
$i_A = \frac{60V}{30\Omega} = 2A$

Using KCL in the upper left node, we can solve for i_B

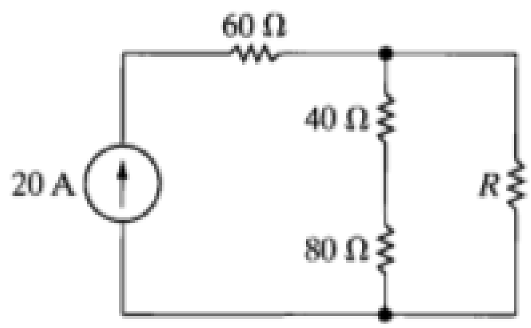
$$\begin{aligned} 0 &= -5A + i_A + i_B \\ i_B &= 5A - i_A \\ i_B &= 5A - 2A \\ i_B &= 3A \end{aligned}$$

Next, we can create a KVL equation of the outer loop of the circuit. and solve for i_C

$$\begin{aligned} 0 &= -v + 7.2i_B + 6i_C \\ 16i_C &= v - 7.2i_B \\ 16i_C &= 60V - (7.2)(3) \\ i_c &= \frac{38.4}{16} \\ i_c &= 2.4A \end{aligned}$$

Now that we know the current through the 10Ω resistor, we can use that to find the power:
$p_{10\Omega} = (10)(2.4)^2 = 57.6W$

3.2 The Voltage-Divider and Current-Divider Circuits



In the above Circuit:

1. Find the value of R that will cause 4A of current to flow through the 80Ω resistor in the crcuit shown.

2. How much power will the resistor R from part 1 need to dissipate?

3. How much power will the current source generate for the value of R from part 1?

1) The current will be divided between the branch with R and the branch with the 40Ω and 80Ω resistors. Therefore:

$$\begin{aligned}
i_{80\Omega} &= \frac{R}{R + 40\Omega + 80\Omega} \times (20A) \\
i_{80\Omega} &= 4A \\
4A &= \frac{R}{R + 40\Omega + 80\Omega} \times (20A) \\
20R &= 4(R + 120) \\
16R &= 480 \\
R &= \frac{480}{16} \\
R &= 30\Omega
\end{aligned}$$

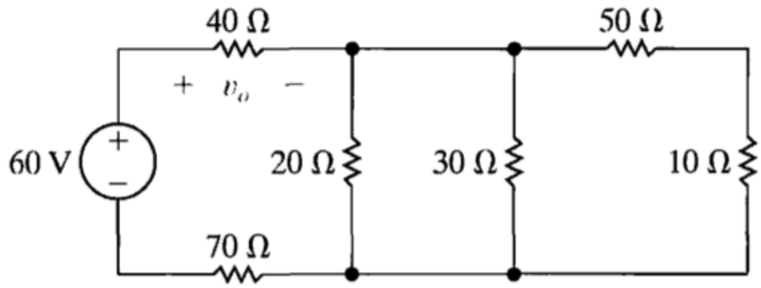
2) With $R = 30\Omega$ we can then calculate the current through R using current division, then solve for the power.

$$\begin{aligned}
i_R &= \frac{40 + 80}{40 + 80 + 30}(20A) \\
i_R &= 16A \\
p_R &= (30)(16)^2 \\
p_R &= 7680W
\end{aligned}$$

3) Finally, we can make a KVL equation around the outer loop to solve for the voltage v, then solve for power:

$$\begin{aligned}
0 &= -v + (60\Omega)(20A) + (30\Omega)(16A) \\
v &= 1200 + 480 \\
v &= 1680V \\
p_{source} &= -vi \\
p_{source} &= -(1680V)(20A) \\
p_{source} &= -33,600W
\end{aligned}$$

3.3 Voltage and Current Divisions



Using the above circuit, answer the following questions:

1. Use voltage division to determine the voltage v_o across the 40Ω resistor.
2. Use v_o from part 1 to determine the current through the 40Ω resistor. Use this current and current division to calculate the current in the 30Ω resistor.
3. How much power is abosrbed by the 50Ω resistor?

1) First we need to find the equivalent resistance of all the resistors to the right of the 40Ω and 70Ω Resistors.
$R_{eq} = 20\Omega \parallel 30\Omega \parallel (50\Omega + 10\Omega)$

$\frac{1}{R_{eq}} = \frac{1}{20}\frac{1}{30} + \frac{1}{60} = \frac{1}{10}$
Therefore $R_{eq} = 10\Omega$ and the voltage is:
$v_o = \frac{40}{40+10+70}(60V) = 20V$

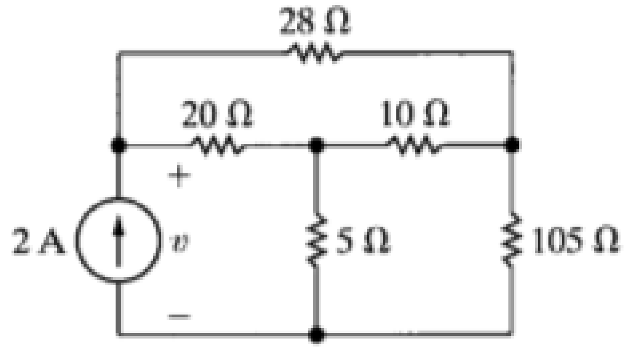
2) First we can find the current through the 40Ω resistor:
$i_{40\Omega} = \frac{v_o}{40} = \frac{20V}{40\Omega} = 0.5A$

Then we will need to find the equivalent circuit of the two parallel branches in order to use current division.
$20\Omega \parallel (50\Omega + 10\Omega) = \frac{(20)(60)}{(20+60)} = 15\Omega$

$i_{30\Omega} = \frac{15}{15+30} \times 0.5A = 0.16667A = 166.67mA$

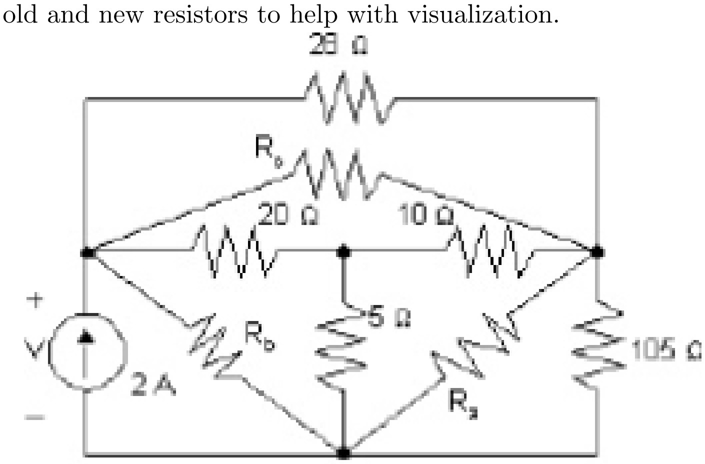
3) We can find the power dissipated by the 50Ω resistor by fnding the current through it. We use current division to find the current in the 40Ω resistor, but first the equivalent resistance of the 20Ω and 30Ω branches must be calculated:
$20\Omega \parallel 30\Omega = \frac{(20(30)}{20+30} = 12\Omega$
Then current division gives:
$i_{50\Omega} = \frac{12}{12+50+10} \times 0.5A = 0.08333A$
Thus:
$p_{50\Omega} = (50)(0.083333)^2 = 0.034722W = 347.22mW$

3.4 Delta-Wye Equivalent Circuits



Use a Y to Δ transformation to find the voltage v in the circuit shown.

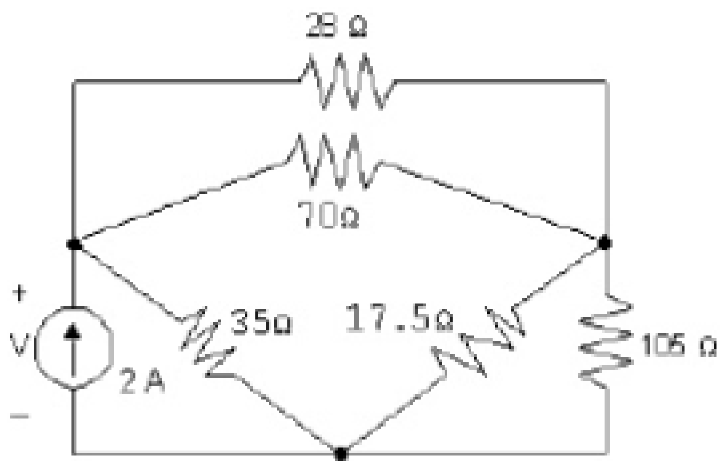
First we can convert the three Y-connected resistors (20 10, and 5 Ω) to three Δ-connected resistors as shown below. Note that this diagram shows the old and new resistors to help with visualization.



Then we can solve for the new resistors:

$$R_a = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\Omega$$
$$R_b = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\Omega$$
$$R_c = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\Omega$$

This gives us the following circuit:



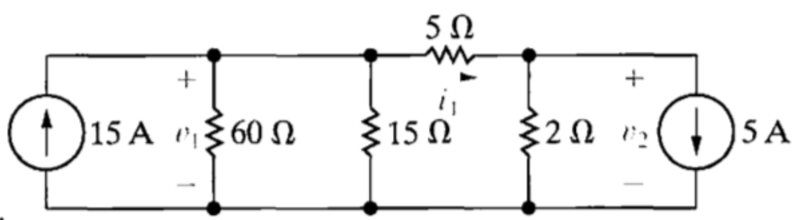
From this circuit we can see the 70Ω resistor is in parallel with the 28Ω Resistor and that the 17.5Ω resistor is in parallel with the 105Ω resistor.
$70\Omega \parallel 28\Omega = \frac{(70)(28)}{70+28} = 20\Omega$

$17.5\Omega \parallel 105\Omega = \frac{(17.5)(105)}{17.5+105} = 15\Omega$

Now we can see that the equivalent 20Ω resistor is in series with the equivalent 15Ω reistor, giving a 35Ω equivalent resistance. Putting this resistor in parallel with the other 35Ω resistor means:
$35\Omega \parallel 35\Omega = \frac{(35)(35)}{35+35} = 17.5\Omega$
This means the resistance seen by the 2 A source is 17.5Ω. Therefore:
$v = ir = (17.5\Omega)(2A) = 35V$

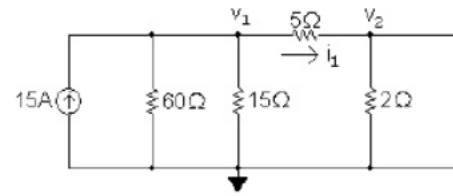
4 Techniques of Circuit Analysis

4.1 The Node-Voltage Method



1. For the circuit shown, use the node-voltage method to find $v_1, v_2, andi_1$
2. How much power is delivered to the circuit by the 15 A Source?

3. Repeat 2 for the 5 A source.



First we can redraw the circuit labeling the node-voltages as follows.

The two node voltage equations are:

$$
\begin{aligned}
-15 + \frac{v_1}{60} + \frac{v_1}{15} \qquad & + \frac{v_1 - v_2}{5} = 0 \\
5 + \frac{v_2}{2} \qquad & + \frac{v_2 - v_1}{5} = 0
\end{aligned}
$$

We can then put them into standard form and solve the simultaneous equations:

$$
\begin{aligned}
v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2(\frac{-1}{5}) \qquad & = 15 \\
v_1\left(\frac{-1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) \qquad & = -5
\end{aligned}
$$

Solving gives us $v_1 = 60V$ and $v_2 = 10V$ Which means that:

$$
\begin{aligned}
i_1 &= \frac{v_1 - v_2}{5} \\
i_1 &= 10A
\end{aligned}
$$

Next we can solve for the power of the 15 A source as follows:

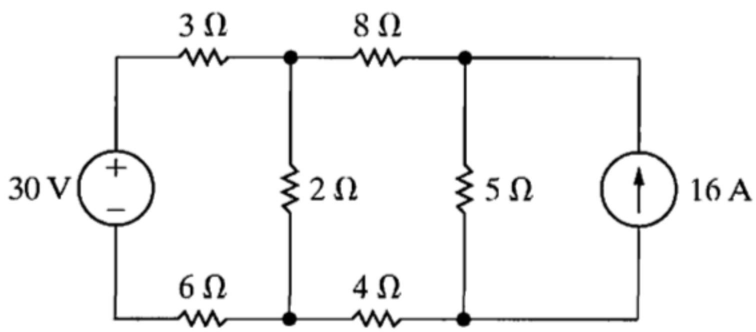
$$
\begin{aligned}
p_{15A} &= -(15A)v_1 \\
p_{15A} &= -(15A)(60V) \\
p_{15A} &= -900W
\end{aligned}
$$

Which means the source delivers 900 W.
Finally we can solve for the 5 A source as follows:

$$p_{5A} = (5A)v_2$$
$$p_{5A} = (5A)(10V)$$
$$p_{5A} = 50W$$

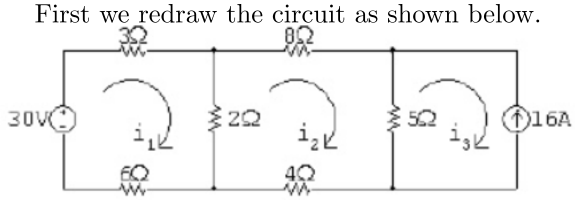
Which means -50 W is delivered.

4.2 The Mesh-Current Method



For the circuit shown, use the mesh-current method to find the power dissipated in the 2Ω resistor.

First we redraw the circuit as shown below.



Since there is a current source on the far right, we know that i_3 is equal to -16A. The remaining two mesh equations are:

$$-30 + 3i_1 + 2(i_1 - i_2) + 6i_1 = 0$$
$$8i_2 + 5(i_2 + 16) + 4(i_2) + 2(i_2 - i_1) = 0$$

Simplifying gives us:

$$11i_1 - 2i_2 = 30$$
$$-2i_1 + 19i_2 = -80$$

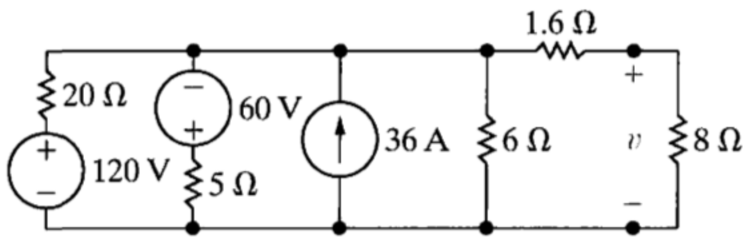
Solviing gives us:

$$i_1 = 2A$$
$$i_2 = -4A$$
$$i_3 = -16A$$

The current in the 2Ω resistor is therefore:
$i_1 - i_2 = 6A$
$p_{2\Omega} = (6)^2(2) = 72W$

Thus the 2Ω resistor dissipates 72 W.

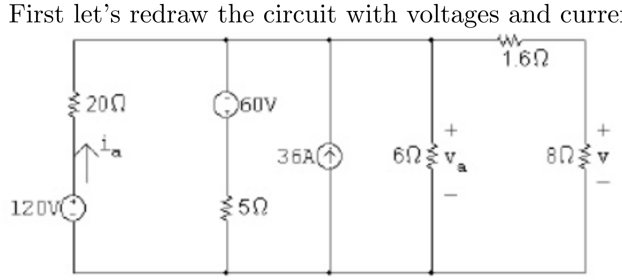
4.3 Source Transformations



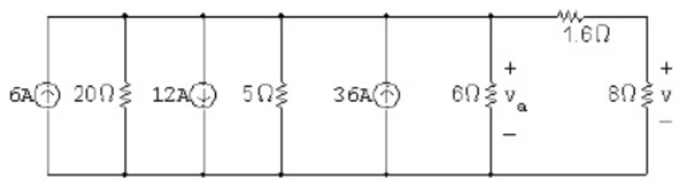
Using the above circuit:

1. Use a series of source transformations to find the voltage v in the circuit shown.
2. How much power does the 120 V source deliver to the circuit?

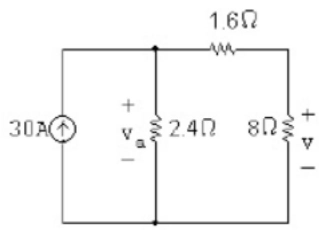
First let's redraw the circuit with voltages and currents labeled:



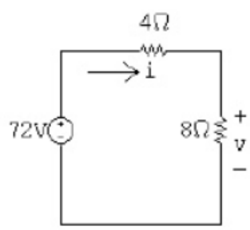
Now we can transform the 120 V source in series with the 20Ω resistor into a 6 A source in parallel with the 20Ω resistor. Then, transform the -60 V source in series with the 5Ω resistor into a -12 A source in parallel with the 5Ω resistor:



Because they are in parallel, we can use KCL to combine the 20Ω, 5Ω, and 6Ω resistors. We can similarly combine the 3 current sources. This is shown below:



Next, we can combine the resulting current source with the 2.4Ω resistor into a 72 V source in series with a 2.4Ω resistor, which can then be added to the 1.6Ω resistor. This is shown below:

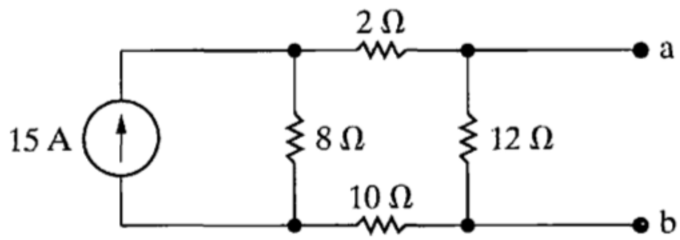


Now we can use voltage division to calcuate v:
$v = \frac{8}{12} \times 72 = 48V$

We can use the last circuit to calculate i:
$i = \frac{v}{8} = \frac{48}{8} = 6A$

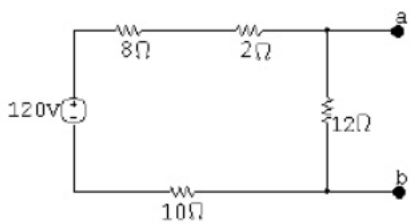
Now we can use i to calculate v_a in the circuit on the left:
$v_a = 6(1.6 + 8) = 57.6V$
We can see in the original equation that v_a is also the voltage drop across the series combination of 120V and 20Ω

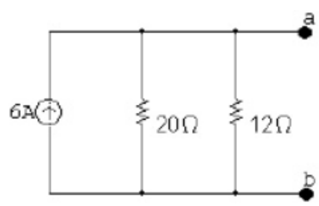
4.4 Thevenin's and Norton's Equivalents



Find the Norton Equivalent circuit with respect to terminals a and b. To solve the problem follow the below steps:

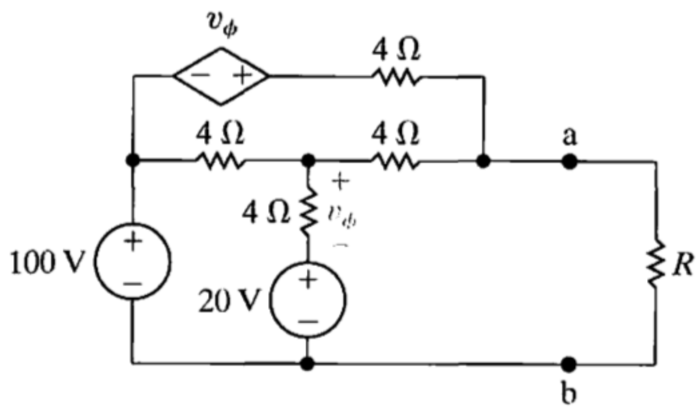
1. Perform a source transformation turning 15 A source and 8Ω resistor into a series combination of 120 V and 8Ω resistor. See the first circuit below.
2. Combine the 2Ω, 8Ω, and 10Ω resistors in series to get 20Ω
3. Combine the 20Ω resistor and 120 V source into a parallel combination of 6 A source and 20Ω resistor. This is shown in the second circuit below.
4. Combine the 20Ω and 12Ω parallel resistors to give 7.5Ω.





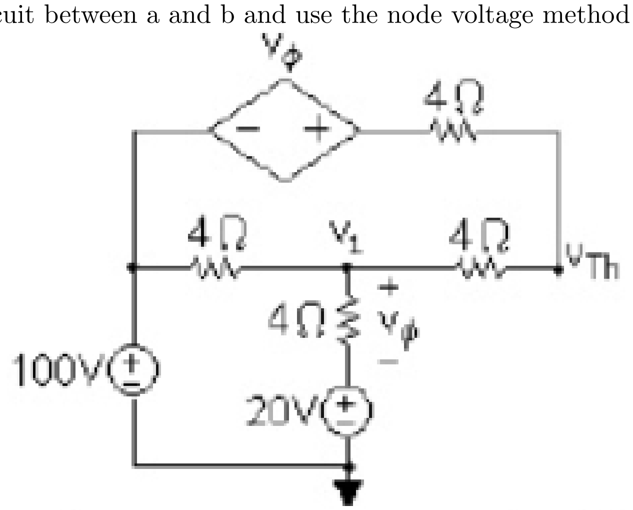
Thus the Norton equivalent is a 6A source and a 7.5Ω Resistor.

4.5 Maximum Power Transfer



1. Find the value of R that enables the circuit shown to deliver maximum power to the terminals a and b.
2. Find the maximum power delivered to R.

First we find the Thevenin equivalent circuit. To find v_{TH} we create an open circuit between a and b and use the node voltage method to solve:



$$0 = \frac{v_{th} - (100 + v_\phi)}{4} + v_{th} - v_1 4$$

$$0 = \frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{th}}{4}$$

The dependent source is:
$v_\phi = v_1 - 20$

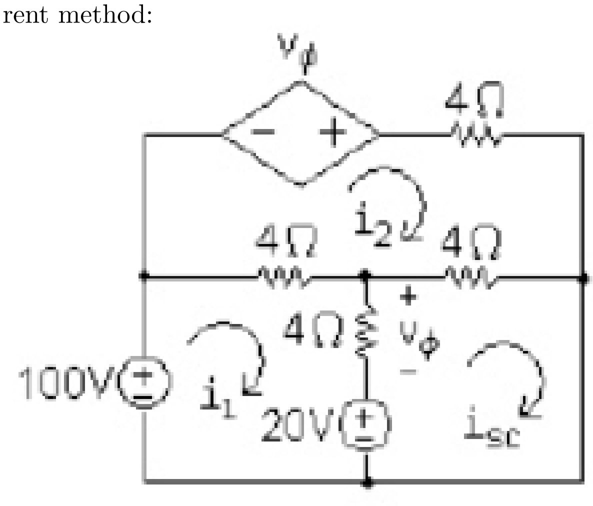
Putting the three equations together we get:

$$\begin{aligned}
v_{th}\left(\frac{1}{4}+\frac{1}{4}\right)+v_1\left(\frac{-1}{4}\right) & \qquad +v_{phi}\left(\frac{-1}{4}\right)=25\\
v_{th}\left(\frac{-1}{4}\right)+v_1\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) & \qquad +v_\phi(0)=30\\
v_{th}(0)+v_1(1) & \qquad +v_\phi(-1)=20
\end{aligned}$$

Solving for all three equations gives us:

$$\begin{aligned}
v_{th} &= 120V\\
v_1 &= 80V\\
v_\phi &= 60V
\end{aligned}$$

Now we create a short circuit between nodes a and b and use the mesh current method:



The mesh current equations are:

$$\begin{aligned}
0 &= -100 + 4(i_1 - i_2) + v_\phi + 20\\
0 &= -v_\phi + 4i_2 + 4(i_2 - i_{sc}) + 4(i_2 - i_1)\\
0 &= -20 - v_\phi + 4(i_{sc} - i_2)
\end{aligned}$$

The dependent source equation is:
$v_\phi = 4(i_1 - i_{sc})$
Putting the four equation together gives:

$$
\begin{aligned}
4i_1 - 4i_2 + 0i_{sc} + v_\phi &= 80 \\
-4i_1 + 12i_2 - 4i_{sc} + v_\phi &= 0 \\
0i_1 - 4i_2 + 4i_{sc} - v_\phi &= 20 \\
4i_1 + 0i_2 - 4i_{sc} - v_\phi &= 0
\end{aligned}
$$

Solving gives:

$$
\begin{aligned}
i_1 &= 45A \\
i_2 &= 30A \\
i_{sc} &= 40A \\
v_\phi &= 20V
\end{aligned}
$$

Then:
$R_{TH} = \frac{v_{TH}}{i_{sc}} = \frac{120}{40} = 3\Omega$

For maximum power transfer, $R = R_{TH} = 3\Omega$

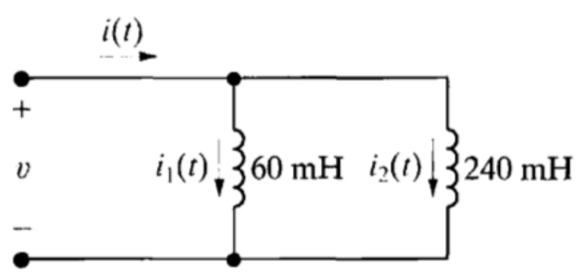
The Thevenin voltage voltage, $v_{TH} = 120V$, splits equally between the Thevenin resistance and the load resistance so:
$v_{load} = \frac{120}{2} = 60V$

$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200W$

4.6 Superposition

5 Inductors and Capacitors

5.1 Inductors in Series and PArallel



The initial value of i_1 and i_2 in the circuit shown are +3 A and -5 A respectively. The voltage at the terminals of the parallel inductors for $t \geq 0$ is $-30e^{-5t}$ mV.

1. If the parallel inductors are replaced by a single inductor, what is the inductance?
2. What i the initial current and its reference direction in the equivalent inductor?
3. Use the equivalent inductor to find $i(t)$.
4. Find $i_1(t)$ and $i_2(t)$. Verify the solutions for $i_1(t)$, $i_2(t)$, and $i(t)$ satisfy Kirchhoff's current law.

1) First we find the equivalent inductance of the two parallel inductors:

$$L_{eq} = \frac{L_1 * L_2}{L_1 + L_2}$$
$$L_{eq} = \frac{60 * 240}{60 + 240}$$
$$L_{eq} = 48mH$$

2) Next we can find the inital current of the combined inductor by adding the values of the current through the two individual inductors.

$$\begin{aligned} i(0^+) &= i_{L1} + i_{L2} \\ &= 3 + (-5) \\ &= -2A \end{aligned}$$

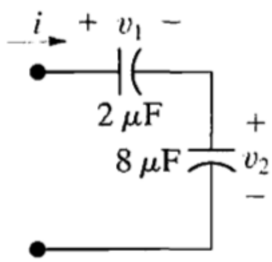
3) Now we can use the equivalent inductor to find $i(t)$. Remember that the voltage when t is greater than 0 is given in the problem statement. Also notice we just found the current when t = 0.

$$L_{eq} = \frac{1}{0.06} + \frac{1}{0.240}$$
$$L_{eq} = \frac{125}{6}$$
$$i = \frac{1}{L_{eq}} \int_{t_0}^{t} vdt + i(t_0)$$
$$i = \left(\frac{125}{6}\right) \int_{0^+} t(-0.03e^{-5x})dx - 2$$
$$i = 0.125e^{-5t} - 2.125A$$

4) Now we can solve for $i_1(t)$ and $i_2(t)$ as follows:

$$i_1 = L_{eq} * \left(\frac{L_1}{L1 + L2}\right) \int_{t_0}^{t} vdt + i(t_0)$$
$$i_1 = \frac{125}{6} * \left(\frac{0.06}{0.06 + 0.24}\right) \int_{0^+}^{t} (-0.03e^{-5x})dx + 3$$
$$i_1 = 0.1e^{-5t} + 2.9A$$
$$i_2 = \frac{25}{6} \int_{0^+}^{t} (-0.03e^{-5x})dx - 5$$
$$i_2 = 0.025e^{-5t} - 5.025A$$
$$i_1 + i_2 = i$$

5.2 Capacitors in series and parallel



The current at the terminals of the two capacitors shown is $240e^{-10t}\mu A$ for $t \geq 0$. The initial values of v_1 and v_2 are -10V and -5V respectively. Calculate the total energy trapped in the capacitors as t approaches ∞.

Using the hint we can see that we should not try to combine the two capacitors and instead should solve for each voltage individually.

$$v = \frac{1}{C} \int_{t_0}^{t} itx + v(0)$$
$$v_1 = \frac{1}{2\mu F} \int_{0^+}^{t} 240 \times 10^{-6} e^{-10x} dx - 10$$
$$v_1 = 0.5x10^6 \int_{0^+}^{t} 240 \times 10^{-6} e^{-10x} dx - 10$$
$$v_1 = -12e^{-10t} + 2V$$
$$v_2 = 0.125 \times 10^6 \int_{0^+}^{t} 240 \times 10^{-6} e^{-10x} dx - 5$$
$$v_2 = -3e^{-10t} - 2V$$

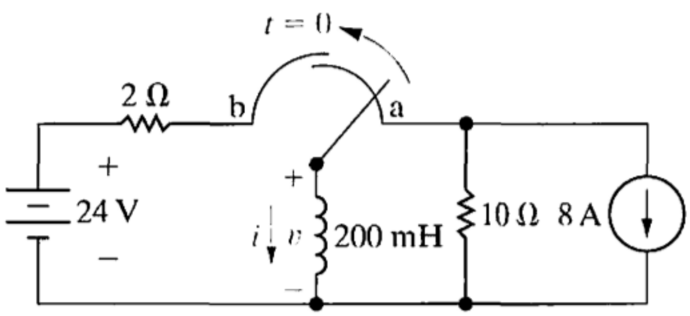
We can see that as time approaches infinity we have:

$$v_1(\infty) = 2V$$
$$v_2(\infty) = -2V$$

The total energy trapped is therefore:
$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\mu J$

6 Response of First Order Circuits

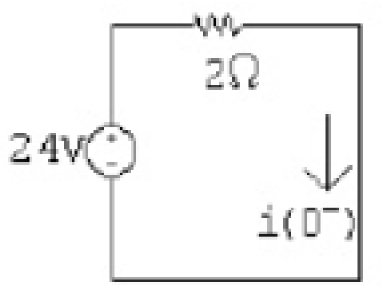
6.1 The Step Response of RC and RL Circuits



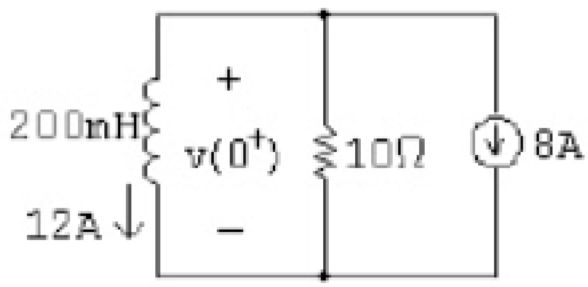
Assume that the switch in the circuit shown in the figure above has been in position b for a long time, and at $t = 0$ it moves to position a. Find:

1. $i(0^+$
2. $v(0^+)$
3. $\tau, t > 0$
4. $i(t), t \geq 0$
5. $v(t), t \geq 0^+$

We begin my solving for $i(0^+)$:
Using the circuit shown below at t ¡ 0, we can calculate the initial current in the inductor.

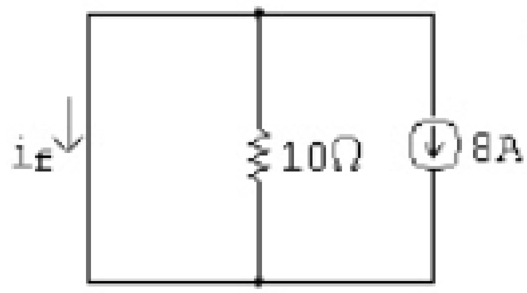


We can solve for the current at time $i(0^-)$ because the current in an inductor is continuous.
$i(0^- = \frac{24}{2} = 12A = i(0^+)$
Next, using the current at $t = 0^+$, as shown below, we can calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the 10Ω resistor, which has current from two sources, 8A from the current source and 12 A from the initial current through the inductor.



$v(0^+) = -10(8 + 12) = -200V$
Now, to calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the 10Ω resistor is connected to the inductor for $t > 0$. Therefore:
$\tau = \frac{L}{R} = \frac{200^{-3}}{10} = 20ms$

Then, to find $i(t)$ we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:



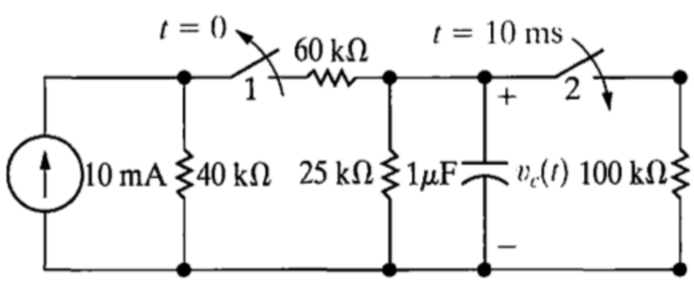
Its important to notice that the inductor behaves like a short circuit and all of the current from the 8 A source flows through the short circuit. Thus:
$i_f = -8A$
Therefore:

$$i(t) = i_f + [i(0^+) - i_f]e^{\frac{-t}{\tau}}$$
$$i(t) = -8 + [12 - (-8)]e^{\frac{-t}{0.02}}$$
$$i(t) = -8 + 20e^{-50t}A, t \geq 0$$

Finally, to find $v(t)$, we can use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt}$$
$$v(t) = (200 \times 10^{-3})(-50)(20e^{-50t})$$
$$v(t) = -200e^{-50t}V, t \geq 0^+$$

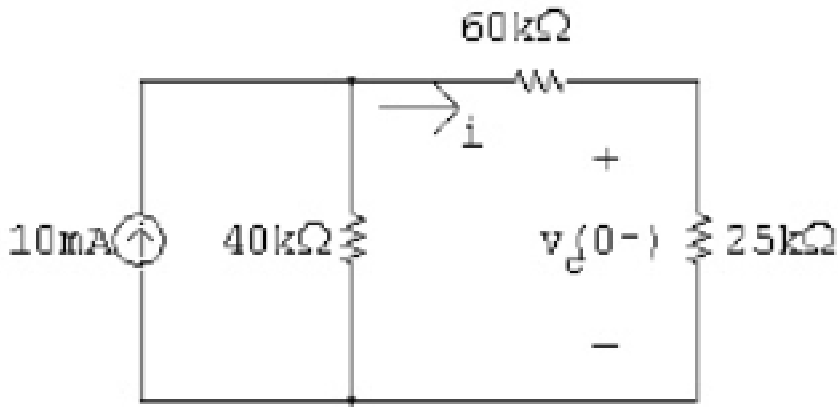
6.2 Sequential Switching



In the circuit shown, switch 1 has been closed and switch 2 has been open for a long time. At t=0, switch 1 is opened. Then 10 ms later, switch 2 is closed. Find:

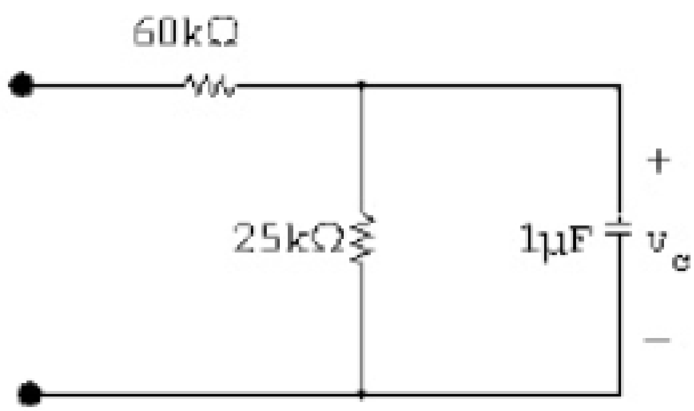
1. $v_c(t) for 0 \leq t \leq 0.01$ s
2. $v_c(t) for t \geq 0.01$ s
3. the total energy dissipated in the $25k\Omega$ resistor
4. the total energy dissipated in the $100k\Omega$ resistor

1) Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$i = \left(\frac{40\times10^3}{125\times10^3}\right)(10 \times 10^{-3} = 3.2mA$

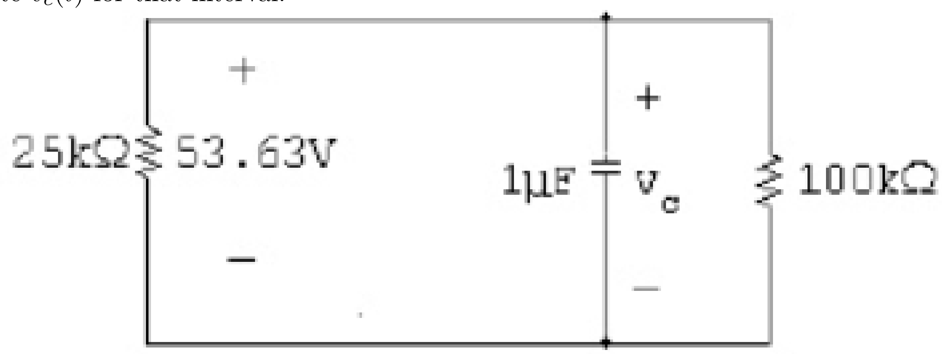
$v_c(0^- = (3.2 \times 10^{-3})(25 \times 10^3) = 80V = v_c(0^+)$
Now we ca use the following circuit, valid for $0 \leq t \leq 10ms$ to calculate $v_c(t)$ for that interval:



For $0 \leq t \leq 100ms$:

$$\begin{aligned}
\tau &= RC \\
\tau &= (25 \times 10^3)(1 \times 10^{-6}) \\
\tau &= 25ms \\
v_c(t) &= v_c(0^-)e^{\frac{t}{\tau}} \\
&= 80e^{-40t}V, 0 \leq t \leq 10ms
\end{aligned}$$

2) Now we can calculate the starting capacitor voltage in the interval $t \geq 10ms$, to calculate $v_c(t)$ for that interval:



For $t \geq 10ms$:
$R_{eq} = 25k\Omega \parallel 100k\Omega = 20k\Omega$
$\tau = R_{eq}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02s$
Therefore:

$$\begin{aligned}
v_c(t) &= v_c(0.01^+)e^{-(t-0.02)/\tau} \\
&= 53.63e^{-50(t-0.01)}V, t \geq 0.01s
\end{aligned}$$

3) To calculate the energy disiated in the $25k\Omega$ resistor, we can integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed.

$$\begin{aligned}
w_{25k} &= \int 0^{0.01} \frac{[80e^{40t}]^2}{25000} dt + \int 0.01^{\infty} \frac{[53.53e^-50(t-0.01)]^2}{25000} dt \\
w_{25k} &== 2.91mJ
\end{aligned}$$

4) We can repeat the process from (3), but keeping in mind that the voltage

across this resistor is non-zero only for the second interval:

$$w_{100k\Omega} = \int 0.01\infty \frac{[53.63e^{-50(t-0.01)}]^2}{100000} dt$$

$$w_{100k\Omega} = 0.29mJ$$