Title of Report

Author 1[1], Author 2[2], Author 3[3], Author 4[4], Author 5[5]

Faculty Mentors: Mentor 1[6], Mentor 2[7]

Abstract

- Summarize the results presented in the report, and the contributions of your research. This is a test.
- Readers should not have to look at the rest of the paper in order to understand the abstract.
- Keep it short and to the point.

1 Introduction

It should be written as much as possible in non-technical terms, so that a lay reader can understand the context and the contribution of the paper.

- Describe the problem you are trying to solve, the approach you took, and summarize your contribution and results.
- Review the history of this problem, and existing literature.
- Give an outline of the rest of the paper.

  Bathymetry is the measurement of the depth of water and can be used to understand the possible shifts of the ocean floor and its depth. Bathymetry is hard to understand and measure nearshore, due to the uncertainty of wave length, speed (celerity), and the depth from the wave to the ocean floor. Bathymetry can be used for marine transportation for the military and to predict the effects of storms on the ocean bottom. Several methods have been raised to measure bathymetry by the military such as the use of the Coastal Research Amphibious Buggy, it collects data which captures the bottom of the nearshore. The military has also used LIDAR, an airborne sensor that can get an accurate display of the ocean floor; however, it ony works in clear waters. GPS equipped jet skis with fathometers use echo sounding to determine the depth of the water and waves make it difficult for laborers to work through tough waves. All of the methods are useful but are very expensive and inefficient against strong infragravity (intermediate long waves) waves.

  Although there have been uncertainties in capturing the topography of the ocean nearshore, mathematical methods could prove to be possible solutions to this problem using the dispersion relationship between wavelength and the period.

$$sigma^2 = gktanh(kh)$$

  where sigma equals 2pi/T, where T is the period, g is the acceleration of gravity, k is 2pi/L, where L is the wavelength, and h is the depth of the wave from still water. Stockdon and Holman used vieo imagery, which compared true wave signal and remotely sensed video signal to create a linear representation between wave amplitudes and phases. Holman used a 2-dimenisional method with Kalman filtering to estimate the depth, h.

[1]Department, University
[2]Department, University
[3]Department, University
[4]Department, University
[5]Department, University
[6]Company
[7]University

Our research will compute the wave depth using wave length and wave number with a 1D model derived from using the energy flux method to create a correlation between the wave length and the wave depth from the surface. **briefly explain how we are exactly doing this with the model and true data**

**other possible methods **

2 The Forward Model

Although there have been uncertainties in capturing the topography of the ocean nearshore, mathematical methods could prove to be possible solutions to this problem using the dispersion relationship between wavelength and the period. Stockdon and Holman used video imagery, which compared true wave signal and remotely sensed video signal to create a linear representation between wave amplitudes and phases. Holman used a 2-dimenisional method with Kalman filtering to estimate the depth, h.

Our research will compute the wave depth using wave length and wave number with a 1D model derived from using the energy flux method to create a correlation between the wave length and the wave depth from the surface.

2.1 The Problem

We consider the following models for the forward problem.

$$\begin{cases} \frac{d}{dx}\left(EC_g\right) = \delta, \\ \sigma^2 = gk\tanh(kh), \end{cases} \tag{1}$$

where the speed at which the energy is transmitted, C_g, called linear theory group speed, which is given as

$$C_g = \frac{c}{2}\left(1 + \frac{2kh}{\sinh(2kh)}\right), \tag{2}$$

with $c = \frac{\sigma}{k}$ and from the linear theory, the wave energy, E is given as

$$E = \frac{1}{8}\rho g H^2, \tag{3}$$

ρ is the density of water, g is the gravitational acceleration, σ is the angular frequency, c is the local wave phase speed, k is the wave number, h is the water depth and H is the wave height. Here, we assume T, the wave period, is constant.

Observe that equation 1 is coupled using the fact that $\sigma = \frac{2\pi}{T}$. Hence, using equation 2 and 3 in 1, we obtain

$$\frac{d}{dx}\left(\frac{\lambda}{k}\left(1 + \frac{2kh}{\sinh(2kh)}\right)H^2\right) = -\delta, \tag{4}$$

and

$$f(k) = gk\tanh(kh) - \sigma^2, \tag{5}$$

where k is the zero of function f and $\lambda = \frac{\rho g \pi}{8T}$.
The wave breaking, δ, proposed by (Janssen and Battjes, 2007) given as

$$\delta = \frac{1}{4h}B\rho g f H_{rms}\left[(R^3 + \frac{3}{2}R)e^{-R^2} + \frac{3}{4}\sqrt{\pi}(1 - erf(R))\right], \tag{6}$$

where

$$R = \frac{H_b}{H_{rms}}, \quad H_b = 2\sqrt{2}H_{m0}, \quad H_b = 0.78h, \quad f = \frac{1}{T}, \quad B = 1.$$

2.2 Numerical Solution of the Forward Model

Finite difference scheme is applied to obtain numerical solution with appropriate initial and boundary conditions. In the scheme of finite differences, the derivatives are replaced using their finite-difference approximations. The goal is to provide wave height using the finite difference method. In the process, MATLAB fsolve function is applied to obtain the wave number. Furthermore, Newton-Raphson method is used to verify solution obtained for the wave number.

2.2.1 Discretization of the Model

Forward difference is applied depending on the nature of the model given in equation (4). The ordinary differential equation of the model applied in the method is as follows

$$\frac{d}{dx}\left(c^* H^2\right) = -\delta \tag{7}$$

where

$$c^* = \frac{c_g \lambda}{k}, \quad c_g = nc = \frac{2\pi n}{T},$$

with

$$\lambda = \frac{1}{8}\rho g, \quad n = \frac{1}{2}\left(1 + \frac{2hk}{sinh(2hk)}\right).$$

For implementation purposes, we let

$$\tilde{H} = H^2.$$

Thus, the required expression becomes

$$\frac{d}{dx}\left(c^* \tilde{H}\right) = -\delta$$

Applying the finite difference method the above expression becomes

$$\frac{c_i^* \tilde{H}_i - c_{i-1}^* \tilde{H}_{i-1}}{\triangle x} = \delta$$

$$\tilde{H}_i = \frac{c_{i-1}^* \tilde{H}_{i-1} + \triangle x \delta}{c_i^*}$$

After determining $\tilde{H}$ we take the square root on that to determine H at each index point in the discretization.

$$H = \sqrt{\tilde{H}}$$

fsolve

Wave numbers are generated at each index points applying MATLAB function fsolve. fsolve is applied in the dispersion relationship

$$\sigma^2 = gktanh(kh)$$

MATLAB function fsolve is used as non-linear solver. In this case $f(k) = gktanh(kh) - \sigma^2$ is provided as function handle in the input argument. Together to this, $\sigma/\sqrt{gh}$ is used as initial condition in fsolve.

Newton-Raphson Method

Newton-Raphson method is an widely used method for root finding. This method is used in the numerical experiment to verify the wave number extracted using fsolve. In Newton -Raphson method roots are calculated using the following expression

$$k_{i+1} = k_i - \frac{f(k_i)}{f'(k_i)}$$

This provides the wave number in each index.

2.2.2 Numerical Results

3 Data

Data for this project was collected by the US. Army Corps of Engineers Engineer Research and Development Center (ERDC) during October 2015 at the Field Research Facility (FRF) in Duck, NC in the Outerbanks.[8] The data comes from the Bathyduck project conducted by the Coastal and Hydraulic Laboratory (CHL). We make use of information about wave height, wave number, wave period, and bathymetry. These fields combine information collected by the Argus Beach Monitoring System, the Lighter Amphibious Resupply Cargo (LARC-5) vessel, the Coastal Research Amphibious Buggy (CRAB). The latter two platforms are shown in Figure 1.

Figure 1: The LARC (left) and CRAB (right) instruments are used to measure near coastal bathymetry. Image source: http://www.frf.usace.army.mil/aboutUS/equipment.shtml

The following sections discuss in more detail the observations and how they were used. Note the x coordinate of the observations was reversed to correspond to the modeling effort. In physical space, the boundary point used for the 1D problem is located 1150 m from the shoreline. The observations are transformed so that $x = 0$ m corresponds to the offshore boundary point and $x = 1150$ m is the shoreline.

3.1 Wave number

Wave number is a measure of the number of waves per unit distance. It is inversely proportional to wave speed.Hourly observations are collected for the month of October 2015 using an Argus video monitoring system mounted on shore. Photogrammetry is performed on the video to determine the dominant wave frequencies and wave numbers in the survey area (**?**). Data is available for a 2-dimensional area at the FRF survey site. A 1-dimensional profile is extracted from the 2-dimensional data along a transect corresponding to the position of the model boundary point. Figure 2 shows statistics for wave number, k, along the transect for October 2015. Wave number is shown to be more variable over time further from the coastline. Mean wave number decreases toward the shoreline, as expected.

[8]The data can be accessed online at http://chlthredds.erdc.dren.mil/thredds/catalog/frf/projects/bathyduck/catalog.html

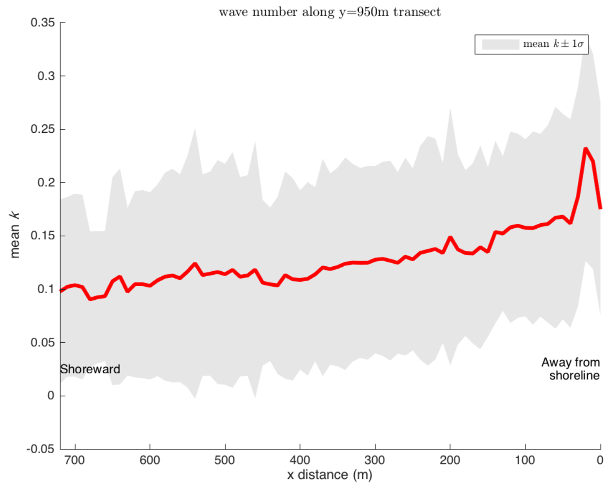


Figure 2: Wave number along the profile where the model boundary condition is located. Mean wave number, k, during October 2015 is shown in red. Gray envelopes show $\pm$ 1σ standard deviation in k. Wave number is observed to be relatively larger and more variable further from shore.

4 The Approach

4.1 The Additive Gaussian Noise Model

By assuming the measurements are corrupted by the additive Gaussian noise, the linear estimation model can be written as

$$\mathbf{d} \sim \text{Gaussian}(\mathbf{A}\mathbf{h}_t), \tag{8}$$

where

$$\begin{array}{lcl} \mathbf{d} & = & \text{a vector of measurements,} \\ \mathbf{A} & = & \text{a linear forward operator,} \\ \mathbf{h}_t & = & \text{the true bathymetry (depth).} \end{array}$$

Therefore, the Gaussian noise $\boldsymbol{\epsilon}$ corrupted measurements $\mathbf{d}$ with variance ν is given by

$$\mathbf{d} = \mathbf{A}\mathbf{h}_t + \boldsymbol{\epsilon}.$$

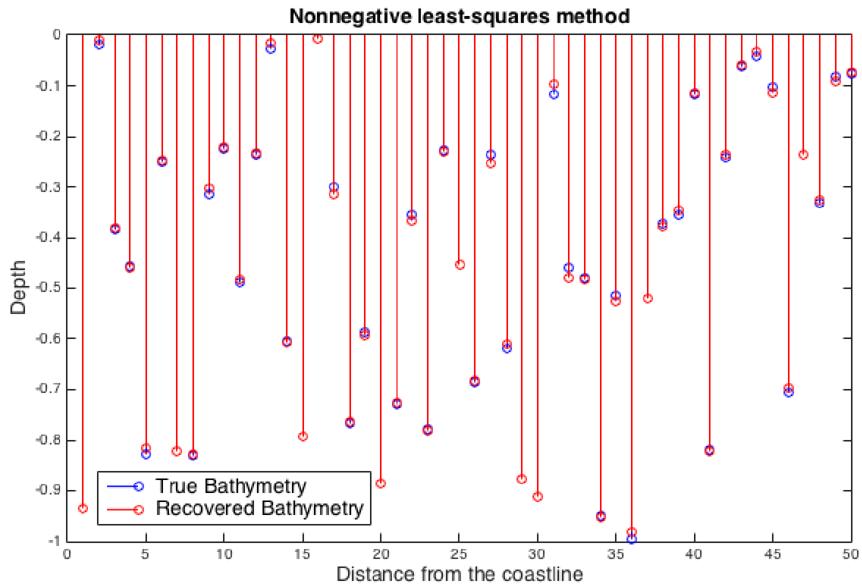
4.2 Bathymetric Inversion Method

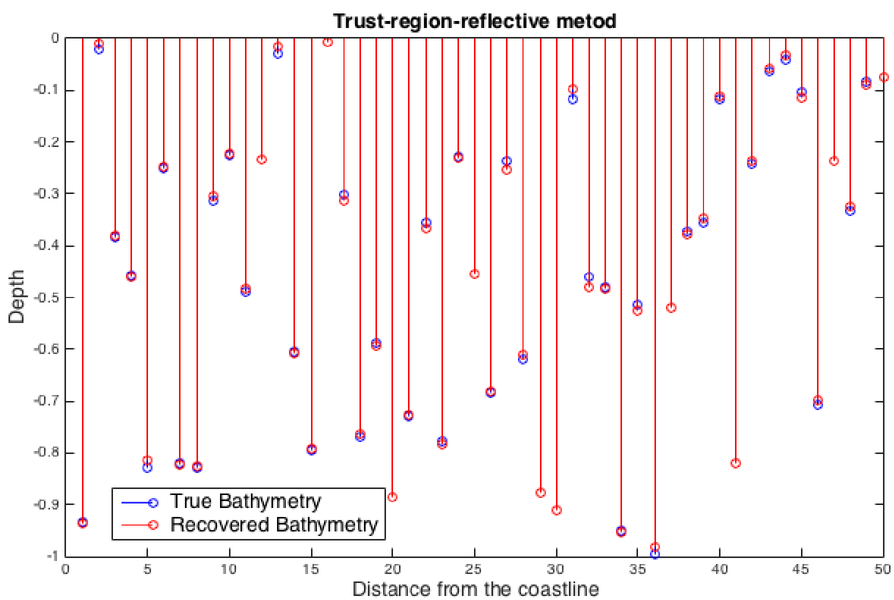
As a beginning to approximate the topographic heights of the sea-floor, we consider the following least-squares minimization problem,

$$\hat{\mathbf{h}} = \underset{\mathbf{h} \in \mathbb{R}^n}{\arg\min} \|\mathbf{A}\mathbf{h} - \mathbf{d}\|_2^2, \tag{9}$$

where we minimize the data misfit between the forward prediction and the measurements in least-squares sense. This least-squares minimization problem (9) can be solved using the following MATLAB functions:

(1) Nonnegative least-squares method: `lsqnonneg(A,b)`



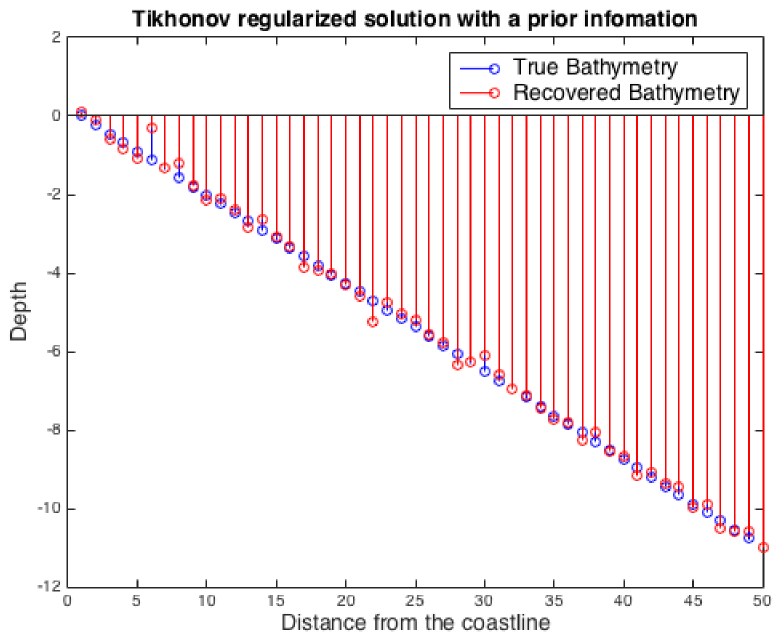


(2)

Challenges: What is the dimension of the $\mathbf{A}$? Is this a underdetermined or overdetermined system? If so, we have to consider the Tikhonov regularization,

$$\hat{\Psi} = \arg\min \|\mathbf{A}\mathbf{\Psi} - \mathbf{d}\|_2^2 + \alpha \|\mathbf{\Psi}\|_2^2,$$

where α is a regularization parameter (> 0).



Reference: I found a MATLAB package for analysis and solution of discrete ill-posed problems, which is available in http://www2.imm.dtu.dk/ pcha/Regutools/

4.3 Help

LSQR method: $\hat{\Psi}$ = lsqr(A,d,tol,maxit) it attempts to solve the least squares solution x that minimizes norm($\mathbf{d} - \mathbf{A\Psi}$) Note that $\mathbf{A}$ need not be square.

Conjugate gradients: $\hat{\Psi}$ = cgs(A,b) attempts to solve the system of linear equations $\mathbf{A\Psi} - \mathbf{d}$ for Ψ.

5 Computational Experiments

Give enough details so that readers can duplicate your experiments.

- Describe the precise purpose of the experiments, and what they are supposed to show.
- Describe and justify your test data, and any assumptions you made to simplify the problem.
- Describe the software you used, and the parameter values you selected.
- For every figure, describe the meaning and units of the coordinate axes, and what is being plotted.
- Describe the conclusions you can draw from your experiments

6 Summary and Future Work

- Briefly summarize your contributions, and their possible impact on the field (but don't just repeat the abstract or introduction).
- Identify the limitations of your approach.
- Suggest improvements for future work.
- Outline open problems.