

a) 分解规则  $\because AC \rightarrow BD \quad \therefore AC \rightarrow B$

传递律  $\because AC \rightarrow B \quad B \rightarrow E \quad \therefore AC \rightarrow E$

b)  $(A)^+ = \{A\}$

$(AC)^+ = \{A, B, C, D, E\}$

c)  $F = \{AC \rightarrow B, AC \rightarrow D, B \rightarrow C, C \rightarrow D, B \rightarrow E\}$

消除冗余后  $F_c = \{AC \rightarrow B, C \rightarrow D, B \rightarrow C, B \rightarrow E\}$

合并后  $F_c = \{AC \rightarrow B, C \rightarrow D, B \rightarrow CE\}$

d)  $X = \{A\} \quad Y = \{B, C, D\}$

$X^+ = \{A\}$ , 因此不是候选码

$(AB)^+ = U \quad (AC)^+ = U \quad (AD)^+ = \{A, D\}$

$\therefore$  候选码为  $(AB), (AC)$

对于  $C \rightarrow D$ ,  $D$  部分依赖于键  $AC$

因此,  $R \notin 2NF$

$\therefore R$  为  $1NF$

e)  $U_1 = \{A, B, C\}, F_1 = \{AC \rightarrow B\}$

$U_2 = \{C, D\}, F_2 = \{C \rightarrow D\}$

$U_3 = \{B, C, E\}, F_3 = \{B \rightarrow CE\}$

f)

|                | A        | B        | C     | D            | E            |                                 |
|----------------|----------|----------|-------|--------------|--------------|---------------------------------|
| $U_1(A, B, C)$ | $a_1$    | $a_2$    | $a_3$ | $a_4$        | $a_5$        | $\rightarrow$ 所以是 lossless-join |
| $U_2(C, D)$    | $b_{21}$ | $b_{22}$ | $a_3$ | $a_4$        | $b_{25} a_5$ |                                 |
| $U_3(B, C, E)$ | $b_{31}$ | $a_2$    | $a_3$ | $b_{34} a_4$ | $a_5$        |                                 |

$\therefore (F_1 \cup F_2 \cup F_3)^+ = F^+ \quad \therefore$  满足 dependency preservation

## 数据库系统原理第四次作业

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7.1 Suppose that we decompose the schema  $R = (A, B, C, D, E)$  into

$(A, B, C)$

$(A, D, E)$ .

Show that this decomposition is a lossless decomposition if the following set  $F$  of functional dependencies holds:

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

*Solution:*

*According to  $A \rightarrow BC$  in  $F$ :*

$(A, B, C) \cap (A, D, E) = A \rightarrow (A, B, C)$

*i.e.  $R_1 \cap R_2 \rightarrow R_1$*

*So, this decomposition is a lossless one.*

7.6 Compute the closure of the following set  $F$  of functional dependencies for relation schema  $R = (A, B, C, D, E)$ .

$A \rightarrow BC$

$CD \rightarrow E$

$B \rightarrow D$

$E \rightarrow A$

List the candidate keys for  $R$ .

*Solution:*

*Starting with  $A \rightarrow BC$ , we can get  $A \rightarrow B$  and  $A \rightarrow C$  (decomposition)*

*Since  $A \rightarrow B$  and  $B \rightarrow D$ , got  $A \rightarrow D$  (transitivity)*

Since  $A \rightarrow C, A \rightarrow D, CD \rightarrow E$ , got  $A \rightarrow E$  (union, transitivity)

So,  $A \rightarrow ABCDE$  (reflexivity, union)

Since  $E \rightarrow A$ , got  $E \rightarrow ABCDE$  (transitivity)

Since  $CD \rightarrow E$ , got  $CD \rightarrow ABCDE$  (transitivity)

Since  $B \rightarrow D$ , we got  $BC \rightarrow ABCDE$  (pseudotransitivity)

Besides, we have  $C \rightarrow C, B \rightarrow B, D \rightarrow D, BD \rightarrow BD$  (reflexivity)

Thus,  $BD \rightarrow B, BD \rightarrow D$  (decomposition)

Also,  $B \rightarrow BD$  (union)

In conclusion,  $F^+$  is  $B \rightarrow B, B \rightarrow D, C \rightarrow C, D \rightarrow D, B \rightarrow BD, BD \rightarrow B, BD \rightarrow D, BD \rightarrow BD$ , and all FDs whose LHS contains  $A, BC, CD$  or  $E$ , and whose RHS is any subset of  $\{A, B, C, D, E\}$ .

The candidate keys are  $A, BC, CD$  and  $E$ .

**7.30** Consider the following set  $F$  of functional dependencies on the relation schema  $(A, B, C, D, E, G)$ :

$$A \rightarrow BCD$$

$$BC \rightarrow DE$$

$$B \rightarrow D$$

$$D \rightarrow A$$

- Compute  $B^+$ .
- Prove (using Armstrong's axioms) that  $AG$  is a superkey.
- Compute a canonical cover for this set of functional dependencies  $F$ ; give each step of your derivation with an explanation.
- Give a 3NF decomposition of the given schema based on a canonical cover.

*Solution:*

a.  $X^{(0)} = \{B\}$

$X^{(1)} = \{B, D\}$ , according to  $B \rightarrow D$

$X^{(2)} = \{ A, B, D \}$ , according to  $D \rightarrow A$

$X^{(3)} = \{ A, B, C, D \}$ , according to  $A \rightarrow BCD$

$X^{(4)} = \{ A, B, C, D, E \}$ , according to  $BC \rightarrow DE$

So,  $B^+ = \{ A, B, C, D, E \}$ .

b. Since  $BC \rightarrow DE$ , we got  $BCD \rightarrow DE$  (augmentation)

Since  $A \rightarrow BCD$ ,  $BCD \rightarrow DE$ , we got  $A \rightarrow DE$  (transitivity)

Then we got  $A \rightarrow BCDE$  (union)

Thus,  $AG \rightarrow ABCDEG$  (augmentation)

So  $AG$  is a superkey.

c. (1)  $F = \{ A \rightarrow B, A \rightarrow C, A \rightarrow D, BC \rightarrow D, BC \rightarrow E, B \rightarrow D, D \rightarrow A \}$

(2)  $A \rightarrow D$  is implied by  $A \rightarrow B, B \rightarrow C$  and  $BC \rightarrow D$ , so  $A \rightarrow D$  is extraneous.

$F_c$  is now  $\{ A \rightarrow B, A \rightarrow C, BC \rightarrow D, BC \rightarrow E, B \rightarrow D, D \rightarrow A \}$ .

(3)  $BC \rightarrow D$  is implied by  $B \rightarrow D$ , so  $BC \rightarrow D$  is extraneous.

$F_c$  is now  $\{ A \rightarrow B, A \rightarrow C, BC \rightarrow E, B \rightarrow D, D \rightarrow A \}$ .

(4) combine  $A \rightarrow B$  with  $A \rightarrow C$  to get:

$$F_c = \{ A \rightarrow BC, BC \rightarrow E, B \rightarrow D, D \rightarrow A \}$$

d.

First, find all the candidate keys:

$$X = \{ G \}, Y = \{ A, B, C, D \}$$

Since  $X$  cannot be candidate keys, try:

$$(AG)^+ = R$$

$$(BG)^+ = R$$

$$(CG)^+ = \{ C, G \}$$

$$(DG)^+ = R$$

So the candidate keys are  $(AG), (BG), (DG)$ .

Then, give the 3NF decomposition:

$$U_1 = \{ A, B, C \}, F_1 = \{ A \rightarrow BC \}$$

$$U_2 = \{ B, C, E \}, F_2 = \{ BC \rightarrow E \}$$

$$U_3 = \{ B, D \}, F_3 = \{ B \rightarrow D \}$$

$$U_3 = \{ A, D \}, F_3 = \{ D \rightarrow A \}$$

*Since none of them contains candidate keys, add:*

$$U_4 = \{ A, G \}, F_4 = \emptyset$$

*In conclusion, the decomposition is  $(A, B, C), (B, C, E), (B, D), (A, D), (A, G)$ .*

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传递律  $\because AC \rightarrow B \quad B \rightarrow E \quad \therefore AC \rightarrow E$

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f)

|                | A        | B        | C     | D            | E            |
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$\rightarrow$  所以是 lossless-join

$\therefore (F_1 \cup F_2 \cup F_3)^+ = F^+ \quad \therefore$  满足 dependency preservation