Serializability Testing: Precedence Graph

Consider some schedule of a set of transactions T_1, T_2, ..., T_n

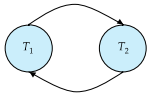
Precedence graph — a directed graph $G = (V, E)$ where the vertices (V) are the transactions.

We draw an arc from T_i to T_j, $T_i \rightarrow T_j$, if the two transactions are conflict, and T_i accessed the data item earlier.

- T_i executes write(Q) before T_j executes read(Q)
- T_i executes read(Q) before T_j executes write(Q)
- T_i executes write(Q) before T_j executes write(Q)

We may label the arc by the item that was accessed.

Example (of a precedence graph):

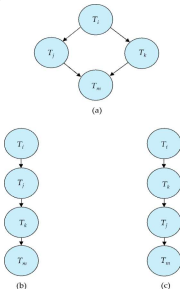


Conflict Serializability Testing

A schedule is conflict serializable if and only if its precedence graph is acyclic (cycle free).

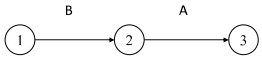
If the precedence graph is acyclic, the serializability order can be obtained by a *topological sorting* of the graph.

- This is a linear order consistent with the partial order of the graph.
- For (a), there are two linear orders (b) and (c).



Example

$r_2(A); r_1(B); w_2(A); r_3(A); w_1(B); w_3(A); r_2(B); w_2(B)$

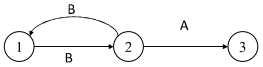


If there is **no** cycle in the precedence graph, this schedule is conflict-serializable

Note: Here we label the arc by the item that was accessed.

Example (2)

$r_2(A); r_1(B); w_2(A); r_2(B); r_3(A); w_1(B); w_3(A); w_2(B)$



If there is a cycle in the precedence graph, this schedule is NOT conflict-serializable

Note: Here we label the arc by the item that was accessed.

Example (3)

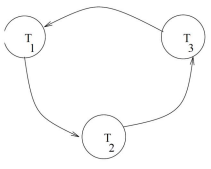
Example 1:

Schedule	T_1	T_2	T_3
read(A)	read(A)		
read(B)		read(B)	
$A \leftarrow f_1(A)$	$A \leftarrow f_1(A)$		
read(C)			read(C)
$B \leftarrow f_2(B)$		$B \leftarrow f_2(B)$	
write(B)		write(B)	
$C \leftarrow f_3(C)$			$C \leftarrow f_3(C)$
write(C)			write(C)
write(A)	write(A)		
read(B)			read(B)
read(A)		read(A)	
$A \leftarrow f_4(A)$		$A \leftarrow f_4(A)$	
read(C)	read(C)		
write(A)		write(A)	
$C \leftarrow f_5(C)$	$C \leftarrow f_5(C)$		
write(C)	write(C)		
$B \leftarrow f_6(B)$			$B \leftarrow f_6(B)$
write(B)			write(B)

Example (3) - Answer

Example 1:

Schedule	T_1	T_2	T_3
read(A)	read(A)		
read(B)		read(B)	
$A \leftarrow f_1(A)$	$A \leftarrow f_1(A)$		
read(C)			read(C)
$B \leftarrow f_2(B)$		$B \leftarrow f_2(B)$	
write(B)		write(B)	
$C \leftarrow f_3(C)$			$C \leftarrow f_3(C)$
write(C)			write(C)
write(A)	write(A)		
read(B)			read(B)
read(A)		read(A)	
$A \leftarrow f_4(A)$		$A \leftarrow f_4(A)$	
read(C)	read(C)		
write(A)		write(A)	
$C \leftarrow f_5(C)$	$C \leftarrow f_5(C)$		
write(C)	write(C)		
$B \leftarrow f_6(B)$			$B \leftarrow f_6(B)$
write(B)			write(B)



Example (4)

Example 2:

Schedule	T_1	T_2	T_3
read(A)	read(A)		
$A \leftarrow f_1(A)$	$A \leftarrow f_1(A)$		
read(C)	read(C)		
write(A)	write(A)		
$A \leftarrow f_2(C)$	$A \leftarrow f_2(C)$		
read(B)		read(B)	
write(C)	write(C)		
read(A)		read(A)	
read(C)			read(C)
$B \leftarrow f_3(B)$		$B \leftarrow f_3(B)$	
write(B)		write(B)	
$C \leftarrow f_4(C)$			$C \leftarrow f_4(C)$
read(B)			read(B)
write(C)			write(C)
$A \leftarrow f_5(A)$		$A \leftarrow f_5(A)$	
write(A)		write(A)	
$B \leftarrow f_6(B)$			$B \leftarrow f_6(B)$
write(B)			write(B)

Example (4) - Answer

Example 2:

Schedule	T_1	T_2	T_3
read(A)	read(A)		
$A \leftarrow f_1(A)$	$A \leftarrow f_1(A)$		
read(C)	read(C)		
write(A)	write(A)		
$A \leftarrow f_2(C)$	$A \leftarrow f_2(C)$		
read(B)		read(B)	
write(C)	write(C)		
read(A)		read(A)	
read(C)			read(C)
$B \leftarrow f_3(B)$		$B \leftarrow f_3(B)$	
write(B)		write(B)	
$C \leftarrow f_4(C)$			$C \leftarrow f_4(C)$
read(B)			read(B)
write(C)			write(C)
$A \leftarrow f_5(A)$		$A \leftarrow f_5(A)$	
write(A)		write(A)	
$B \leftarrow f_6(B)$			$B \leftarrow f_6(B)$
write(B)			write(B)