Supply Information

This document, as a supplement to the paper, consists of two sections. The first section gives the derivations of Section Ⅲ, part C 3); the remaining section give the simulation data and results in detail.

Section1: The Derivations of Section Ⅲ, part C 3)

Symbols

Symbol	meaning
$\mathrm{I_m}$	Set of nodes with operation data
$\mathrm{I_{um}}$	Set of nodes without operation data
I	$\mathrm{I} = \mathrm{I_m} \cup I_{\mathrm{um}}$ Set of all nodes
$\mathbf{a}_{\mathrm{r}}^{\mathrm{i},0/1}$	AGM parameter a identified under complete/fragmentary operation
$\mathbf{b}_{r}^{0/1}$	AGM parameter b identified under complete/fragmentary operation
$\mathbf{T}_{r,l}^{i}$	Return temperature of load node i
$\mathbf{h}_{l}^{i}$	Thermal load of node i
c	specific heat capacity
m_{i}	Equivalent mass flow rate of node i
T_{amb}	Ambient temperature
$\mathbf{T}$	Set of time index

$$\tilde{a}_r^k = \begin{bmatrix} \tilde{a}_r^{k,1,\Gamma} & \cdots & \tilde{a}_r^{k,1,\Gamma} \\ \vdots & \ddots & \vdots \\ \tilde{a}_r^{k,1,0} & \cdots & \tilde{a}_r^{k,N_l,0} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{\mathrm{r}}^{1,0/1} & \mathbf{a}_{\mathrm{r}}^{2,0/1} & \cdots & \mathbf{a}_{\mathrm{r}}^{\mathrm{i},0/1} \end{bmatrix}$$

Based on AGM, for single-source DHN,

$$T_{t,src}^{0} = \sum_{i \in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0\,\prime} \boldsymbol{T}_{r,l}^{i} + \sum_{i \in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\,\prime} \boldsymbol{T}_{r,l}^{j} + b_r^0 T_{amb} \tag{1}$$

Remark: for (1), in ideal DHN operation data, i.e. data generated by node method, accurate AGM parameters can be identified through estimator, let

$$T_{r,src}^{0} = T_{r,src} \tag{2}$$

Wherein, $T_{r,src}$ is the actual return temperature of source。

For fragmentary node operation data:

$$T_{t,src}^{1} = \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{i,1\prime} \boldsymbol{T}_{r,l}^{i} + b_r^1 T_{amb} \tag{3}$$

Under fragmentary measurement data, the parameter estimator will estimate a set of optimal parameters $T_{t,src}^{1}$ to make $T_{r,src}^{0}$ (i.e. the actual return water temperature) as close as possible.

The return water temperature of the load node has the following relationship with the thermal load:

$$\mathbf{h}_l = cm(\boldsymbol{T}_{s,l} - \boldsymbol{T}_{r,l}) \tag{4}$$

Here are the main derivations:

$$T_{r,src}^{0} - T_{r,src}^{1} = \sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0})' \boldsymbol{T}_{r,l}^{i} + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \boldsymbol{T}_{r,l}^{j} + (b_r^0 - b_r^0) T_{amb} \tag{5}$$

According to (4)

$$\begin{aligned} T_{r,src}^{0} - T_{r,src}^{1} &= \sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' (\boldsymbol{T}_{s,l}^{i} - \frac{\boldsymbol{h}_l^i}{cm^i}) + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} (\boldsymbol{T}_{s,l}^{j} - \frac{\boldsymbol{h}_l^j}{cm^j}) + (b_r^0 - b_r^1) T_{amb} \\ &= \sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' \boldsymbol{T}_{s,l}^{i} + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \boldsymbol{T}_{s,l}^{j} - \left[\sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' \frac{\boldsymbol{h}_l^i}{cm^i} + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} \right] + (b_r^0 - b_r^1) T_{amb} \end{aligned} \tag{6}$$

This derivation is based on a small- to medium-sized single-source network. It can be assumed that the water supply temperature of each load is approximately equal.

$$T_{r,src}^{0} - T_{r,src}^{1} = (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' \boldsymbol{T}_{s,l} - \left[\sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' \frac{\boldsymbol{h}_l^i}{cm^i} + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} \right] + (b_r^0 - b_r^1) T_{amb} \tag{7}$$

According to the AGM model of the heating network, there are normalization constraints between the coefficients. Similarly, when using fragmentary data for parameter estimation, it is assumed that the normalization constraints in the parameter estimator still exist. Therefore,

$$\begin{aligned} (b_r^0 - b_r^1) T_{amb} &= \left[(1 - \sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \mathbf{1}) - (1 - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1\prime} \mathbf{1}) \right] T_{amb} \\ &= \left(\sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1\prime} \mathbf{1} - \sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \mathbf{1} \right) T_{amb} \\ &= \left(\sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1\prime} - \sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \right) \boldsymbol{T}_{amb} \end{aligned} \tag{8}$$

By normalizing the constraints and writing the real number T_{amb} as a vector, the equivalent formula is rewritten as

$$\begin{aligned} T_{r,src}^{0} - T_{r,src}^{1} &= (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' (\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) - \left[\sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},0} - \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' \frac{\boldsymbol{h}_l^i}{cm^i} + \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} \right] \\ &= (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' (\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) - \left[\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \frac{\boldsymbol{h}_l^k}{cm^k} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{i,1\prime} \frac{\boldsymbol{h}_l^i}{cm^i} \right] \\ &= (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})' (\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) + \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{i,1\prime} \frac{\boldsymbol{h}_l^i}{cm^i} - \sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \frac{\boldsymbol{h}_l^k}{cm^k} \end{aligned} \tag{9}$$

The newly estimated parameters $\boldsymbol{a}_{\mathrm{r}}^{i,1}$ are expressed as

$$\boldsymbol{a}_{\mathrm{r}}^{i,1} = \boldsymbol{a}_{\mathrm{r}}^{i,0} + \Delta \boldsymbol{a}_{\mathrm{r}}^{i} \tag{10}$$

Where $\Delta \boldsymbol{a}_{\mathrm{r}}^{i}$ represents the coefficient increment caused by the loss of load nodes, the non-lost load nodes represent the coefficient increment caused by the lost node information, and $\boldsymbol{a}_{\mathrm{r}}^{i,0}$ represents the coefficient between the measured node i and the source node. (9) can be further written as follows

$$\begin{aligned} T_{r,src}^{0} - T_{r,src}^{1} &= (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})'(\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) + \sum_{i\in I_m} (\boldsymbol{a}_{\mathrm{r}}^{i,0} + \Delta \boldsymbol{a}_{r}^{i})' \frac{\boldsymbol{h}_l^i}{cm^i} - \sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0\prime} \frac{\boldsymbol{h}_l^k}{cm^k} \\ &= (\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})'(\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) + \sum_{i\in I_m} \Delta \boldsymbol{a}_{r}^{i\prime} \frac{\boldsymbol{h}_l^i}{cm^i} - \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} \end{aligned} \tag{11}$$

If the unmeasured node heat load can be expressed as follows, the unmeasured node heat load is a linear combination of the measured node heat load:

$$h_l^j = \sum_{i\in I_m} \alpha_{j,i} h_l^i, \quad j \in I_{um} \tag{12}$$

And there is a normalization constraint on the combination coefficients:

$$\sum_{i\in I_m} \alpha_{j,i} = 1, \quad j \in I_{um} \tag{13}$$

Let

$$\theta_{j,i} = \frac{\alpha_{j,i}}{m_j} \quad j \in I_{um}, i \in I_m \tag{14}$$

Assume that there exists $\Delta \boldsymbol{a}_{\mathrm{r}}^{i}$

$$\sum_{i\in I_m} \Delta \boldsymbol{a}_{r}^{i\prime} \frac{\boldsymbol{h}_l^i}{cm^i} = \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} \tag{15}$$

According to (14),

$$\sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} = \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \sum_{i\in I_m} \theta_{j,i} \boldsymbol{h}_l^i = \sum_{j\in I_{um}} \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} \boldsymbol{h}_l^i = \sum_{i\in I_m} \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} \boldsymbol{h}_l^i \tag{16}$$

According to (16), (15)

$$\begin{aligned} &\sum_{i\in I_m} \Delta \boldsymbol{a}_{r}^{i\prime} \frac{\boldsymbol{h}_l^i}{cm^i} = \sum_{i\in I_m} \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} \boldsymbol{h}_l^i \\ &\Delta \boldsymbol{a}_{r}^{i\prime} \frac{\boldsymbol{h}_l^i}{cm^i} = \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} \boldsymbol{h}_l^i \\ &\frac{\Delta \boldsymbol{a}_{r}^{i\prime}}{m^i} = \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} \\ &\Delta \boldsymbol{a}_{r}^{i\prime} = m^i \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \theta_{j,i} = \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{m^i}{m_j} \alpha_{j,i} \end{aligned} \tag{17}$$

So far, as long as (17) is satisfied, we can make(11)

$$\sum_{i\in I_m} \Delta \boldsymbol{a}_{r}^{i\prime} \frac{\boldsymbol{h}_l^i}{cm^i} - \sum_{j\in I_{um}} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0\prime} \frac{\boldsymbol{h}_l^j}{cm^j} = 0 \tag{18}$$

Now, let's analyze the other part of (11) when (17) is satisfied.

$$(\sum_{k\in I} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0} - \sum_{i\in I_m} \boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})'(\boldsymbol{T}_{s,l} - \boldsymbol{T}_{amb}) \tag{19}$$

$$\begin{aligned}\sum_{k\in I}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0}-\sum_{i\in I_m}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1} &= \sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}-\sum_{i\in I_m}\Delta\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i}} \\ &= \sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}-\sum_{i\in I_m}\sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}\frac{m^i}{m_j}\alpha_{j,i} \\ &= \sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}-\sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}\sum_{i\in I_m}\frac{m^i}{m_j}\alpha_{j,i} \\ &= \sum_{j\in I_{um}}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{j},0}(1-\sum_{i\in I_m}\frac{m^i}{m_j}\alpha_{j,i})\end{aligned} \tag{20}$$

If the flow rate of each load node is approximately equal, that is,

$$m^i = m^j \tag{21}$$

And the combination coefficients are normalized (13), then we have

$$\sum_{k\in I}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0}-\sum_{i\in I_m}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1}=0 \tag{22}$$

$$(\sum_{k\in I}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{k},0}-\sum_{i\in I_m}\boldsymbol{a}_{\mathrm{r}}^{\mathrm{i},1})'(\boldsymbol{T}_{s,l}-\boldsymbol{T}_{amb})=0 \tag{23}$$

According to (18)(23):

$$T_{r,src}^{0}-T_{r,src}^{1}=0 \tag{24}$$

Section2: Simulation Data and Results

■ 51-node DHN pipeline parameters

Pipeline NO.	from_node	to_node	length(m)	diameter(m)	roughness	conductivity	flowrate(t/h)
P1	0	1	900	0.4	0.0005	0.125	323
P2	1	25	600	0.1	0.0005	0.125	10
P3	1	2	600	0.4	0.0005	0.125	313
P4	2	26	600	0.1	0.0005	0.125	10
P5	2	3	1100	0.4	0.0005	0.125	303
P6	3	27	600	0.1	0.0005	0.125	10
P7	3	4	800	0.2	0.0005	0.125	84
P8	4	28	600	0.1	0.0005	0.125	12
P9	4	5	700	0.2	0.0005	0.125	72
P10	5	29	600	0.1	0.0005	0.125	12
P11	5	6	700	0.2	0.0005	0.125	60
P12	6	30	600	0.1	0.0005	0.125	12
P13	6	7	800	0.2	0.0005	0.125	48
P14	7	31	600	0.1	0.0005	0.125	12
P15	7	8	700	0.2	0.0005	0.125	36
P16	8	9	600	0.2	0.0005	0.125	24
P17	9	33	600	0.1	0.0005	0.125	12
P18	9	34	700	0.1	0.0005	0.125	12
P19	8	32	800	0.1	0.0005	0.125	12

P20	3	10	1000	0.3	0.0005	0.125	209
P21	10	35	400	0.1	0.0005	0.125	10
P22	10	11	600	0.3	0.0005	0.125	199
P23	11	36	400	0.1	0.0005	0.125	10
P24	11	12	600	0.3	0.0005	0.125	189
P25	12	37	400	0.1	0.0005	0.125	15
P26	12	13	1000	0.3	0.0005	0.125	174
P27	13	38	400	0.1	0.0005	0.125	15
P28	13	14	700	0.3	0.0005	0.125	159
P29	14	15	800	0.2	0.0005	0.125	114
P30	15	39	600	0.1	0.0005	0.125	15
P31	15	16	700	0.3	0.0005	0.125	99
P32	16	40	600	0.1	0.0005	0.125	15
P33	16	17	800	0.2	0.0005	0.125	84
P34	17	18	600	0.2	0.0005	0.125	36
P35	18	41	300	0.1	0.0005	0.125	12
P36	18	19	600	0.2	0.0005	0.125	24
P37	19	42	300	0.1	0.0005	0.125	12
P38	19	43	400	0.1	0.0005	0.125	12
P39	17	20	1400	0.2	0.0005	0.125	48
P40	20	21	700	0.2	0.0005	0.125	24
P41	21	44	300	0.1	0.0005	0.125	12
P42	21	45	600	0.1	0.0005	0.125	12
P43	20	22	700	0.2	0.0005	0.125	24
P44	22	46	400	0.1	0.0005	0.125	12
P45	22	47	700	0.1	0.0005	0.125	12
P46	14	23	600	0.2	0.0005	0.125	45
P47	23	48	400	0.1	0.0005	0.125	15
P48	23	24	700	0.2	0.0005	0.125	30
P49	24	49	400	0.1	0.0005	0.125	15
P50	24	50	800	0.1	0.0005	0.125	15

■ 51-node DHN node parameters

Node NO.	source flowrate(t/h)	load flowrate(t/h)	Tsmin	Tsmax	Trmin	Trmax
0	323	0	80	95	50	80
1	0	0	80	95	50	80
2	0	0	80	95	50	80
3	0	0	80	95	50	80
4	0	0	80	95	50	80
5	0	0	80	95	50	80
6	0	0	80	95	50	80

7	0	0	80	95	50	80
8	0	0	80	95	50	80
9	0	0	80	95	50	80
10	0	0	80	95	50	80
11	0	0	80	95	50	80
12	0	0	80	95	50	80
13	0	0	80	95	50	80
14	0	0	80	95	50	80
15	0	0	80	95	50	80
16	0	0	80	95	50	80
17	0	0	80	95	50	80
18	0	0	80	95	50	80
19	0	0	80	95	50	80
20	0	0	80	95	50	80
21	0	0	80	95	50	80
22	0	0	80	95	50	80
23	0	0	80	95	50	80
24	0	0	80	95	50	80
25	0	10	80	95	50	80
26	0	10	80	95	50	80
27	0	10	80	95	50	80
28	0	12	80	95	50	80
29	0	12	80	95	50	80
30	0	12	80	95	50	80
31	0	12	80	95	50	80
32	0	12	80	95	50	80
33	0	12	80	95	50	80
34	0	12	80	95	50	80
35	0	10	80	95	50	80
36	0	10	80	95	50	80
37	0	15	80	95	50	80
38	0	15	80	95	50	80
39	0	15	80	95	50	80
40	0	15	80	95	50	80
41	0	12	80	95	50	80
42	0	12	80	95	50	80
43	0	12	80	95	50	80
44	0	12	80	95	50	80
45	0	12	80	95	50	80
46	0	12	80	95	50	80
47	0	12	80	95	50	80
48	0	15	80	95	50	80

49	0	15	80	95	50	80
50	0	15	80	95	50	80

■ Simulation data

Performance of TSLE:

https://github.com/ZhengRongcheng/VL-AGM/blob/main/data_temperature-未平滑.xlsx

Verification of DHN Identification Specificity Under Affine Relation Thermal Load:

https://github.com/ZhengRongcheng/VL-AGM/blob/main/data_temperature-仿射关系.xlsx

Performance of VL-AGM Parameter Identification Method:

https://github.com/ZhengRongcheng/VL-AGM/blob/main/data_temperature-未平滑.xlsx