

CS 170 Homework 4

Due **2021-09-27, at 10:00 pm**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write none.

In addition, we would like to share correct student solutions that are well-written with the class after each homework. Are you okay with your correct solutions being used for this purpose? Answer “Yes”, “Yes but anonymously”, or “No”

2 Running Errands

You need to run a set of k errands in Berkeley. Berkeley is represented as a directed weighted graph G , where each vertex v is a location in Berkeley, and there is an edge (u, v) with weight w_{uv} if it takes w_{uv} minutes to go from u to v . The errands must be completed in order, we'll assume the i th errand can be completed immediately upon visiting any vertex in the set S_i (for example, if you need to buy snacks, you could do it at any grocery store). Your home in Berkeley is the vertex h .

Given G, h , and all S_i as input, given an efficient algorithm that computes the time needed to complete all the errands starting at h . That is, find the shortest path in G that starts at h and passes through a vertex in S_1 , then a vertex in S_2 , then in S_3 , etc.

Give a 3-part solution.

3 MST Variant

Give an undirected graph $G = (V, E \cup S)$ with edge weight $c(e)$. Note that S is disjoint with E . Design an algorithm to find a minimum one among all spanning trees having at most one edge from S and others from E .

Input: A graph $G = (V, E \cup S)$, and a cost function $c(e)$ defined for every $e \in E \cup S$.

Output: A tree $T = (V, E')$ such that T is connected (there is a path in T between any two vertices in V), $E' \subseteq E \cup S$, $\sum_{e \in E'} c(e)$ is minimized, and $|E' \cap S| \leq 1$.

Give a 3-part solution.

4 Blackout in the Igloos

It's the year 3077, exactly 1000 years after Cyberpunk happened. PNPenguins live in igloos that are equipped with one-directional portals, allowing them to instantly travel from one igloo to another. It is polar night now, so lights are necessary for PNPenguins to navigate inside their igloos – without light, igloos are completely dark inside. There are n igloos in total, and in each igloo, there are k one-directional portals numbered $\{1 \dots k\}$. Each portal can teleport PNPenguins to a certain igloo (including the one that this portal is located).

Recently, PG&E (Penguin Gaming and Electric Company - yes, gaming is considered a utility in 3077) issued a notice indicating power could be shut off at any time. PNPenguins want to buy an emergency power generator for one igloo, and in the case of a power shut off, all PNPenguins need to rendezvous there. PNPenguins are terrible at memorizing which igloo they are in, but they all have a very good sense of direction. When the power is shut off at an igloo, they still can choose any portal and travel through it to another igloo.

In the event of a power shut off, PNPenguins need a way to know how to get to the igloo with the power generator. Thus, there has to be a fixed sequence of numbers p_1, \dots, p_l such that starting from any igloo, if a PNPenguin goes into portals p_1, \dots, p_l , it ends up at the igloo with the power generator.

PNPenguins are wondering, if there exists any fixed sequence of portal numbers p_1, \dots, p_l , such that regardless of the igloo they are currently in, after going through these portals, they end up in the same igloo? They only need to know if such a sequence of portals exist without knowing the sequence itself.

More formally, there are k functions f_1, \dots, f_k and each function maps each igloo to an igloo (possible the same). Your goal is to verify if there exists a sequence of numbers p_1, \dots, p_l such that $f_{p_1} \circ f_{p_2} \circ \dots \circ f_{p_{l-1}} \circ f_{p_l}$ outputs the same igloo regardless of the igloo a PNPenguin starts from, i.e.,

$$\forall x, y \in \{1 \dots n\} f_{p_1} \circ f_{p_2} \circ \dots \circ f_{p_{l-1}} \circ f_{p_l}(x) = f_{p_1} \circ f_{p_2} \circ \dots \circ f_{p_{l-1}} \circ f_{p_l}(y)$$

\circ is function composition:

$$f_{p_1} \circ f_{p_2} \circ \dots \circ f_{p_{l-1}} \circ f_{p_l}(x) = f_{p_1}(f_{p_2}(\dots(f_{p_{l-1}}(f_{p_l}(x)))))$$

- (a) Provide a 3-part solution for the described problem.
- (b) Go to the online contest system at <https://hellfire.ocf.berkeley.edu/>, access the HW4 contest and provide a program to solve this problem.

Hint First, think about just two igloos. Can you find an algorithm that will find if there is a sequence of doors that will map these two igloos to the same igloo? Then, try to generalize this to all igloos by "merging" together pairs of igloos. Based on this, try to find an "if and only if" condition for the existence of a sequence of doors that will map all igloos to the same igloo. Can you develop an algorithm to verify if this "if and only if" condition holds?

Note that there are several ways to solve this problem with different time complexities, ranging from a very slow to a very fast solution. This is a hard problem and we do not expect everyone to get to the best possible solution. Because of this, we will be awarding partial credit for slower solution, even for a solution that is exponential in the input size. Try to do your best. We will be providing Office Hours support for this question, but note that we will emphasize conceptual help and we do not guarantee that a staff member will be able to debug your code.