Note: Your TA probably will not cover all the problems. This is totally fine, the discussion worksheets are not designed to be finished in an hour. They are deliberately made long so they can serve as a resource you can use to practice, reinforce, and build upon concepts discussed in lecture, readings, and the homework.

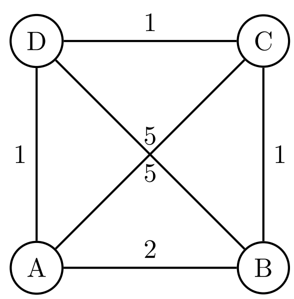
1 Independent Set Approximation

In the Max Independent Set problem, we are given a graph $G = (V, E)$ and asked to find the largest set $V' \subseteq V$ such that no two vertices in V' share an edge in E.

Given an undirected graph $G = (V, E)$ in which each node has degree $\leq d$, give an efficient algorithm that finds an independent set whose size is at least $1/(d+1)$ times that of the largest independent set. Only the main idea and the proof that the size is at least $1/(d+1)$ times the largest solution's size are needed.

2 Traveling Salesman Problem

In the lecture, we learned an approximation algorithm for the Traveling Salesman Problem based on computing an MST and a depth first traversal. Suppose we run this approximation algorithm on the following graph:



The algorithm will return different tours based on the choices it makes during its depth first traversal.

1. Which DFS traversal leads to the best possible output tour?

2. Which DFS traversal leads to the worst possible output tour?

3. What is the approximation ratio given by the algorithm in the worst case for the above instance? Why is it worse than 2? (*Hint*: Consider the triangle inequality on the graph).

3 Maximum Coverage

In the maximum coverage problem, we have m subsets of the set $\{1, 2, \ldots n\}$, denoted $S_1, S_2, \ldots S_m$. We are given an integer k, and we want to choose k sets whose union is as large as possible.

Give an efficient algorithm that finds k sets whose union has size at least $(1 - 1/e) \cdot OPT$, where OPT is the maximum number of elements in the union of any k sets. In other words, $OPT = \max_{i_1, i_2, \ldots i_k} |\cup_{j=1}^{k} S_{i_j}|$. Just the algorithm description and justification for the lower bound on the number of elements your solution contains is needed.

(Recall the set cover algorithm from lecture, and use that $(1 - 1/n)^n \leq 1/e$ for all integers n)

4 California Cycle

Prove that the following problem is NP-hard

Input: A directed graph $G = (V, E)$ with each vertex colored blue or gold, i.e., $V = V_{\text{blue}} \cup V_{\text{gold}}$

Goal: Find a *Californian cycle* which is a directed cycle through all vertices in G that alternates between blue and gold vertices (Hint : Directed Rudrata Cycle)