

Multiplayer Allocations in the Presence of Diminishing Marginal Contributions: Cooperative Game Analysis and Applications in Management Science

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罗春林教授论文被Management Science接受

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5月20日, 信息管理学院教授罗春林与香港岭南大学冷明明教授、梁丽萍教授合作的合作博弈研究论文《*Multi-Player Allocations in the Presence of Diminishing Marginal Contributions: Cooperative Game Analysis and Applications in Management Science*》正式被Management Science (MS) 接受。

该研究初稿最早完成于2014年, 历经五年半的投稿、修改。研究主要基于经济学中的边际收益递减原理, 提出了一类边际贡献几乎递减的合作博弈。针对该类博弈, 研究了核(core)非空的充分必要条件, 以保证合作博弈存在稳定的分配方案, 进而分析了基于联盟满意度的分配方法—核仁, 以及基于联盟贡献度的分配方法—夏普利值。当核为空时, 探讨了为保证大联盟稳定可能需要的惩罚力度。最后, 研究了该类博弈在航空领域(航班共享)、采购领域(团购)和制造领域(调度)中的应用。

Management Science是管理学领域中的标杆性期刊, 被誉为“管理学界的Science”, 属于UT-Dallas24、FinancialTimes(FT)45、ABS四星期刊。(文/信息管理学院 编辑/姜莹 审核/阮艳平)

Content

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- 3 Insight (What insights we have gained)

1 Authors

- Professor Leng is a Chair Professor of Computing and Decision Sciences and the Dean of the Faculty of Business.
- Research Interests: Game theory (with a focus on cooperative game theory); operations and supply chain management; interface between operations and other disciplines
- 5 papers in UTD journals until 2022
- A member in the Editorial Review Board of *Production and Operations Management* (May2015 -- present)



Leng Mingming (冷明明)

1 Authors

- Born in 1971
- BEng (SIT)
- MEng (WUT) in 1996
- Industrial experiences for 5 years
- PhD in (McMaster) in 2005
- joined in 2005, tenured in 2010

现任岭南大学电脑及决策科学讲座教授的冷明明教授，在少年时代曾经经历过一次数学考试带来的挫折，从而成为他的人生转折点。那一年他12岁，在中学一次数学考试里考了零分。但这个零分成绩，反而让少年冷明明对数学产生浓厚兴趣，钻研不倦，终成大器，对博弈论、运营和供应链管理，以及运营和其他学科之间的相互关系开展深入研究，成为今日学界翘楚。

“我听了父亲的建议，上课前先阅读课本。出乎意外的是，我觉得这些课本上内容明白易懂。”冷教授说，“于是，我很快就对数学问题和逻辑思维产生了极为浓厚的兴趣，从那时起就完全沉迷其中。即使是现在，对我来说最好的周末放松方法，就是做一两道数学奥林匹克竞赛旧卷子上的题目。”

1980年代末，冷明明参加中国高考。他带着一腔对数学的热情，不用一小时就完成了时限为两小时的数学考试，还拿到了满分佳绩。带着热情和才华，他在数学研究领域一路深造，在加拿大麦克马斯特大学取得管理科学博士学位。从2001年开始，他的学术研究重点在于动态程序设计、随机过程，特别是博弈论及其应用。

1 Authors

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- 1996年9月 -2000年7月, 南昌大学数学系, 理学学士
- 2000年9月 -2003年7月, 南昌大学数学系, 理学硕士
- 2007年9月 -2010年7月, 江西财经大学信息管理学院, 管理学博士
- 2015年12月-2019年4月, 中国科学院大学经济与管理学院, 博士后
- 2011年8月-2012年8月, 香港岭南大学商学院, Research Assistant (RA)
- 2013年7月-2013年8月, 香港岭南大学商学院, Research Assistant (RA)
- 2014年8月-2014年12月, 香港岭南大学商学院, Research Assistant (RA)
- 2019年5月-2020年6月, 美国Rutgers大学商学院, 访问学者
- 主要从事物流与供应链管理、博弈论及其应用等研究

1. Management Science: 1 篇论文
2. Omega: 1 篇论文
3. Computers & Industrial Engineering: 1 篇论文
4. Transportation Research Part E: 1 篇论文 (已被接受)
5. European Journal of Operational Research: 3 篇论文
6. Operations Research Letters: 1 篇论文
7. INFOR: Information Systems and Operational Research: 1 篇论文
8. Asia-Pacific Journal of Operational Research: 1 篇论文
9. International Journal of Production Economics: 1 篇论文
10. Journal of System Sciences and Information: 1 篇论文
11. Journal of Systems Science and Complexity: 1 篇论文
12. Porcedia Computer Science: 1 篇论文
13. Managerial and Decision Economics: 1 篇论文

近日获悉, 由我校信息管理学院罗春林老师、黄健副教授与香港岭南大学冷明明教授、梁丽萍博士合作的论文《Supply chain analysis under a price-discount incentive scheme for electric vehicles》, 在国际A类期刊《European Journal of Operational Research》(欧洲运筹研究, 简称EJOR) 2014年第235卷第1期上发表。该文基于合作博弈理论研究研究了政府补贴背景下电动汽车供应链的运营管理问题, 其中补贴形式是多个国家采用的‘百分比的零售价, 但有一个上限’的情形, 结果显示, 电动汽车的期望销售量随百分比值的增加而增加, 但却可能随上限值的增加而减少。

1 Authors

- PhD - Business Administration (The University of British Columbia)
- MSc - Business Administration (The University of British Columbia)
- BSc - Applied Mathematics (South China University of Technology)
- Research interests: Operations and Supply Chain Management, Incentive Mechanism Design, Cooperative Game Theory, Data Analytics
- 4 papers in UTD journals
- Joined in 2010

Industry and Consulting Experience

Sauder School of Business, The University of British Columbia, Canada

- Project Advisor, Centre for Operations Excellence, Mar 2009 - Dec 2009
Supervised three consulting projects on healthcare operations for public and private healthcare service providers
- Project Analyst, Centre for Operations Excellence, Apr 2003 - Sept 2003
Managed and conducted two consulting projects on service operations for the Canadian Air Transport Security Authority

China Telecom Guangzhou Branch, Guangzhou, China

- Computer & Telecommunications Engineer, 1993 - 2001



Liping Liang (梁麗萍)

2 Paper

(收益分配, 不是成本分摊) \Leftarrow

$$v(S_1) + v(S_2) \geq v(S_1 \cup S_2) \Leftrightarrow \text{超可加 } (S_1 \cap S_2 = \emptyset) \Leftarrow$$

$$v(S_1) + v(S_2) \leq v(S_1 \cup S_2) \Leftrightarrow \text{次可加} \Leftarrow$$

$$v(S_1) + v(S_2) = v(S_1 \cup S_2) \Leftrightarrow \text{可加} \Leftarrow$$

次可加性意味着“合作大于部分之和”，是合作的基础，次可加博弈的核心可能不存在，此时 γ 等于 $v(s)$ 线性松弛与整数解的 gap (Jain and Mahdian, 2007)，但超可加博弈（且不可加）的核心一定不存在。 \Leftarrow

$$v(S_1) + v(S_2) \leq v(S_1 \cup S_2) + v(S_1 \cap S_2) \Leftarrow$$

$$\Leftrightarrow v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T) \quad (S \subseteq T \subseteq N \setminus \{i\}) \Leftarrow$$

$$\Leftrightarrow \text{凸博弈} \Leftarrow$$

若规定 $v(\emptyset) = 0$ ，则凸博弈可推出次可加性，凸博弈核心一定非空 (Shapley, 1971)。凸博弈在经济学上意味着边际递增。 \Leftarrow

$$v(S \cup \{j\}) - v(S) \leq v(S \cup \{j, k\}) - v(S \cup \{k\}) \Leftrightarrow \text{次模} \Leftarrow$$

$$v(S \cup \{j\}) - v(S) \geq v(S \cup \{j, k\}) - v(S \cup \{k\}) \Leftrightarrow \text{超模} \Leftarrow$$

$$v(S \cup \{j\}) - v(S) = v(S \cup \{j, k\}) - v(S \cup \{k\}) \Leftrightarrow \text{模数} \Leftarrow$$

次模博弈核心一定存在，超模博弈（不是模数的）核心一定不存在。需要注意的是，一个合作博弈很可能既不是超模的，也不是次模的，即次模/超模只是对合作博弈性质的一种观察，而非分类。 \Leftarrow

$$v(S \cup \{i\}) - v(S) \geq v(T \cup \{i\}) - v(T) \quad (S \subseteq T \subseteq N \setminus \{i\}) \Leftarrow$$

$$\Leftrightarrow \text{almost diminishing marginal contributions, ADMC (Leng et al., 2021)} \Leftarrow$$

具备 ADMC 性质的合作博弈核心可能存在，也可能不存在，核心存在的充

要条件是 $\sum_{j \in N} (v(N) - v(N \setminus \{j\})) \geq v(N)$ 。 \Leftarrow

We consider a variation of the above game, denoted by $\mathcal{G}' = (N, v')$, which differs from Schulz and Uhan's game \mathcal{G} (2013) in the following two aspects. First, when a coalition uses the machine to process its agents' jobs, it incurs a machine setup cost $s > 0$ before the jobs are processed. Second, if the job of an agent $i \in N$ is completed after its deadline, the agent still gains a profit w_i but also incurs a tardiness loss $t \in (0, \min\{w_i, i \in N\}]$. In reality, the machine setup cost may not be negligible, and an agent may still obtain a positive profit despite a loss due to job tardiness. Therefore, in game \mathcal{G}' , for any coalition $S \subseteq N$ and $S \neq \emptyset$, $v'(S) = \sum_{i \in S \setminus L_S} w_i + \sum_{i \in L_S} (w_i - t) - s$, where L_S denotes the set of delayed jobs under the optimal schedule for coalition S .

Proposition 3. *The scheduling profit game \mathcal{G}' with machine setup cost s and job tardiness loss t is an ADMC game when $t \leq s$.*

Definition 1. *A coalitional game \mathcal{G} is an almost diminishing marginal contributions (ADMC) game if $v(i) \leq v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$, $\forall i \in N$, $\forall S \subseteq T \subseteq N \setminus \{i\}$, and $S \neq \emptyset$.*

supermodular应该是比ADMC更严格的一个条件，因为ADMC只能保证对小于N-1的联盟成立，但supermodular是包括N的，所以ADMC才有可能出现核心存在的情况。那篇文章后面的application讲了他是如何将一个supermodular的game改成一个ADMC的game的过程

2 Paper

Multiplayer Allocations in the Presence of Diminishing Marginal Contributions: Cooperative Game Analysis and Applications in Management Science

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Abstract. We use cooperative game theory to investigate multiplayer allocation problems under the almost diminishing marginal contributions (ADMC) property. This property indicates that a player's marginal contribution to a non-empty coalition decreases as the size of the coalition increases. We develop ADMC games for such problems and derive a necessary and sufficient condition for the non-emptiness of the core. When the core is non-empty, at least one extreme point exists, and the maximum number of extreme points is the total number of players. The Shapley value may not be in the core, which depends on the gap of each coalition. A player can receive a higher allocation based on the Shapley value in the core than based on the nucleolus, if the gap of the player is no greater than the gap of the complementary coalition. We also investigate the least core value for ADMC games with an empty core. To illustrate the applications of our results, we analyze a code-sharing game, a group buying game, and a scheduling profit game.

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Keywords: coalitional games • diminishing marginal contributions • the core • the nucleolus • the Shapley value

3 Insights

- A new perspective of interaction of cooperative game and optimization.
- A theoretical tool for us to apply cooperative game.
- A good example for us to illustrate the theoretical framework of cooperative game.