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The Design of Experiential Services with Acclimation and Memory Decay: Optimal Sequence and Duration

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CONTENTS

体验型服务设计：

- 顾客体验至关重要，会影响顾客忠诚度、从而影响企业利润。
- 如图所示，体验是一个会动态发生变化的过程。
- 本研究关注的是顾客从服务中获得的**记忆效用（remembered utility）**，综合考虑了一系列心理因素。
  - 适应(acclimation)
    - Adapt to states but react to changes.
  - 记忆衰退(memory decay):
- **研究问题：**

How to sequence and allocate duration to activities in a service encounter so as to maximize the remembered utility?
(Considering 2 psychological phenomena: acclimation and memory decay)

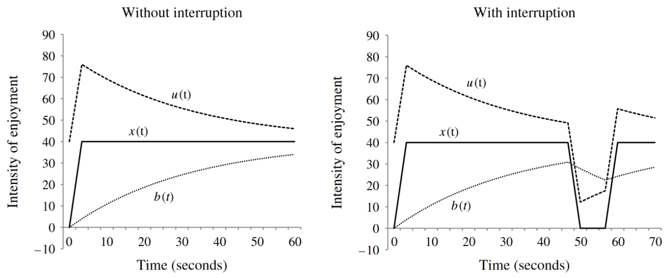
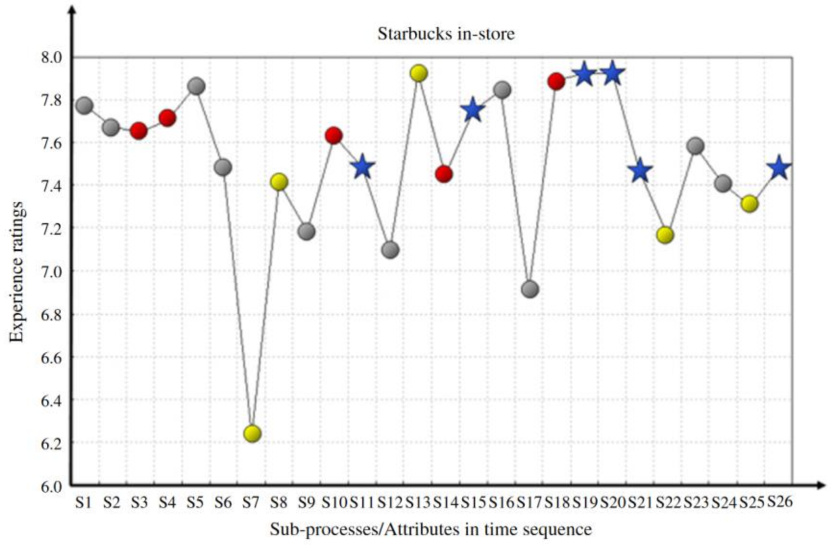


Figure 1 (Color online) Map of Starbucks' In-Store Customer Experience



Source. Lee (2008). Used with permission.
Notes. The stars represent subprocesses that are important to the customer and differentiate the brand. The circles represent subprocesses that either are not important to the customer or do not differentiate the brand.

Behavioral Aspects

- Memory Decay
- Acclimation
- Peak–End Rule
  - Capture the key features of the peak–end rule : End utility carries greater weight.

Decision Theory

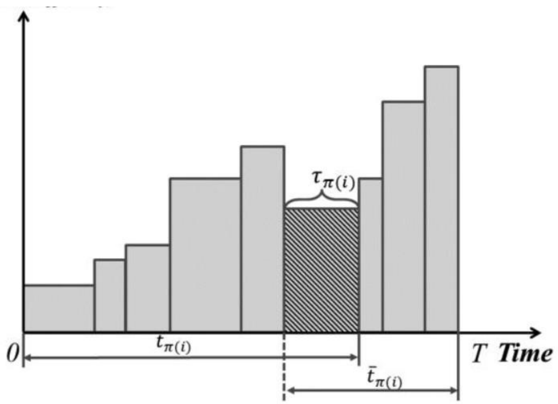
- Contribute to this literature by adopting a design perspective and by focusing on **maximizing (ex post) remembered utility** as opposed to (ex ante) total experienced utility.

Service Operations

2 most related paper:

- Aflaki and Popescu (2013) : maximize long-term customer retention.
- Dixon and Thompson (2015): study an event scheduling problem.
  - By contrast, this paper directly model the psychological phenomena, and not their outcomes.
  - This paper adopts an analytical approach, whereas theirs is computational.

模型基础设置



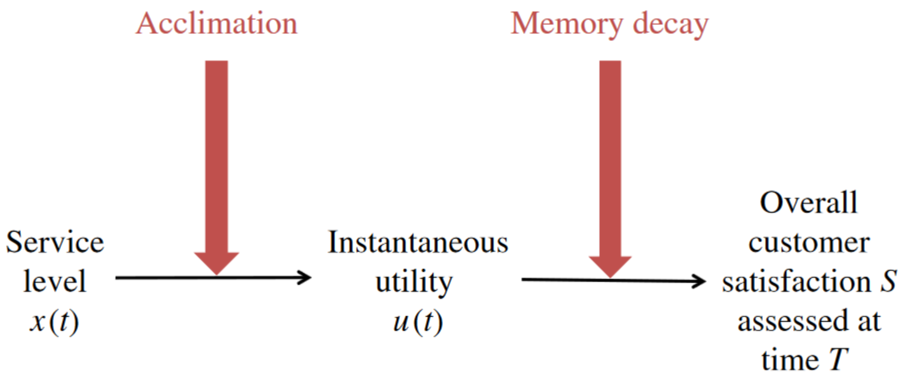
- 一个服务包括n个元素，每个元素都有服务水平x_i和时长t_i，$\sum_{i=1}^{n} t_i = T$。
- 一组服务序列表示为$(i_1, \cdots, i_n)$，其中i_k表示为这个服务序列中的第k个元素。
- T_i表示从服务开始到第i个元素结束的时间跨度，$\bar{T}_i$表示从第i个元素开始到服务结束的时间跨度。

记忆衰退过程：从瞬时效用聚合为记忆效用

$$S((i_1, \ldots, i_n), \mathbf{t}) = \sum_{k=1}^{n} \int_{T_{k-1}}^{T_k} u_{i_k}(t) e^{-w(T-t)}\, dt. \qquad (1)$$

- Reference: 艾宾浩斯遗忘曲线 (Ebbinghaus, 1913)；
- $u_{i_k}(t)$: t时刻的瞬时效用，其中$T_{k-1} \le t \le T_k$；
- w: 记忆衰退因子。

Figure 2　(Color online) Acclimation Affects Instantaneous Utility and Memory Decay Determines Its Relative Weight in the Overall Remembered Utility

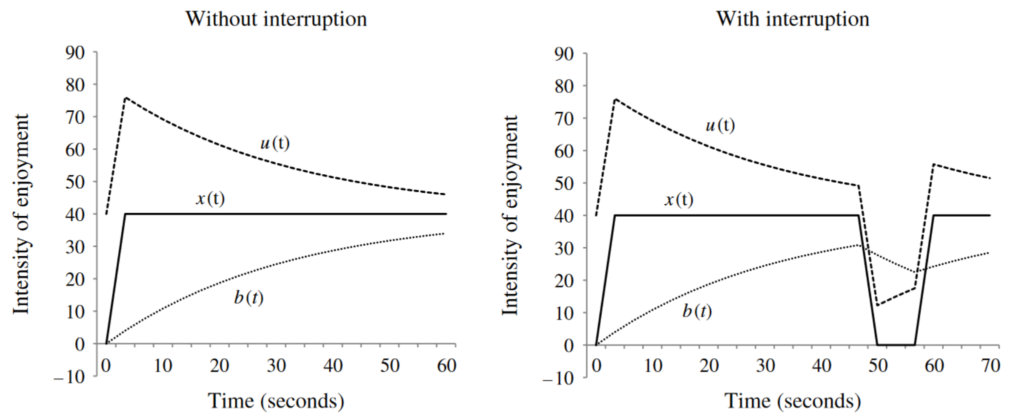


适应过程：从服务水平到瞬时效用

$$u_{i_k}(t) = U(x_{i_k} - b(t)).$$

假设是线性函数，并标准化，即:
$\boldsymbol{U(x-b) = x-b}$

- Reference: Baucells and Sarin (2013)
- $b(t)$: 顾客在t时刻的服务参考水平。



左图中的最终效用小于右图

$$\frac{db(t)}{dt} = \alpha(x(t) - b(t)),$$

- Reference: Overbosch (1986)
- 参考点的变化率与服务水平与参考点之间的差成正比，类似于牛顿冷却定律。
- α: 适应系数，越大表示顾客受之前的影响越小。

解差分方程

$$b(t) = x_{i_k} - (x_{i_k} - b(T_{k-1}))e^{-\alpha(t-T_{k-1})}, \quad T_{k-1} \le t \le T_k, \quad (2)$$

$$b(t) = x_{i_k} - \left((x_{i_1} - b(0)) + \sum_{j=2}^{k}(x_{i_j} - x_{i_{j-1}})e^{\alpha T_{j-1}}\right)e^{-\alpha t}, \quad T_{k-1} \le t \le T_k.$$

$$u_{i_k}(t) = (x_{i_1} - b(0))e^{-\alpha t} + \sum_{j=2}^{\kappa}(x_{i_j} - x_{i_{j-1}})e^{-\alpha(t-T_{j-1})}, \quad T_{k-1} \le t \le T_k. \quad (3)$$

Closed-form

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Customer Satisfaction Model

由（1）和（3）结合，可以推出：

$$S((i_1,\ldots,i_n),\mathbf{t}) = \sum_{k=1}^{n}\int_{T_{k-1}}^{T_k} u_{i_k}(t)e^{-w(T-t)}\,dt$$
$$= \sum_{k=1}^{n}(x_{i_k}-x_{i_{k-1}})\frac{e^{-\alpha\bar{T}_k}-e^{-w\bar{T}_k}}{w-\alpha}. \qquad (4)$$

为方便后续分析，引入函数Φ：

$$\Phi(\alpha,w,\bar{T}_k) \doteq \frac{e^{-\alpha\bar{T}_k}-e^{-w\bar{T}_k}}{w-\alpha} = \int_0^{\bar{T}_k} e^{-\alpha t}e^{-w(\bar{T}_k-t)}\,dt. \qquad (5)$$

Rewriting (4) as follows,

$$S = \sum_{k=1}^{n}(x_{i_k}-x_{i_{k-1}})\Phi(\alpha,w,\bar{T}_k)$$
$$= \sum_{k=1}^{n} x_{i_k}\big(\Phi(\alpha,w,\bar{T}_k)-\Phi(\alpha,w,\bar{T}_{k+1})\big), \qquad (6)$$

Service Provider’s Design Problem:

$$\max_{(i_1,\ldots,i_n),\mathbf{t}} \; S((i_1,\ldots,i_n),\mathbf{t})$$
$$= \sum_{k=1}^{n}(x_{i_k}-x_{i_{k-1}})\Phi(\alpha,w,\bar{T}_k) \qquad (7)$$
$$\text{subject to} \quad \underline{\tau}_i \le t_i \le \bar{\tau}_i, \; \forall i, \qquad (8)$$
$$\sum_{i=1}^{n} t_i = T, \qquad (9)$$
$$(i_1,\ldots,i_n) \in \mathcal{P}. \qquad (10)$$

- $\mathcal{P}$: the set of all possible feasible sequences of activities.

Table 1 **Three Stylized Models of a Service Encounter**

	Sequence	
Duration	Fixed	Variable
Fixed		Variable sequence, fixed duration (VSFD): session chair at a conference, music performance
Variable	Fixed sequence, variable duration (FSVD): spa treatment, dental procedure	Variable sequence, variable duration (VSVD): personal fitness class, museum tours, fireworks

$$\max_{(i_1,\ldots,i_n),\mathbf{t}} \quad S((i_1,\ldots,i_n),\mathbf{t}) = \sum_{k=1}^{n}(x_{i_k}-x_{i_{k-1}})\Phi(\alpha,w,\bar{T}_k)$$

$$\Rightarrow$$

$$S = \sum_{k=1}^{n}(x_{i_k}-x_{i_{k-1}})\Phi(\alpha,w,\bar{T}_k) = \sum_{k=1}^{n} x_{i_k}\left(\Phi(\alpha,w,\bar{T}_k)-\Phi(\alpha,w,\bar{T}_{k+1})\right), \qquad (6)$$

$\Phi(a,w,t)$的基础性质：

$$\Phi(\alpha,w,\bar{T}_k) \doteq \frac{e^{-\alpha\bar{T}_k}-e^{-w\bar{T}_k}}{w-\alpha}$$

1、对称性：$\Phi(a,w,t)=\Phi(w,a,t)$

2、峰值： $\Phi(a,w,t)$ is pseudoconcave in t and peaks when $t=1/m(a,w)$

3、凹凸性：$\Phi(a,w,t)$ is concave convex in t, and its inflection point occurs at $t=2/m(a,w)$;

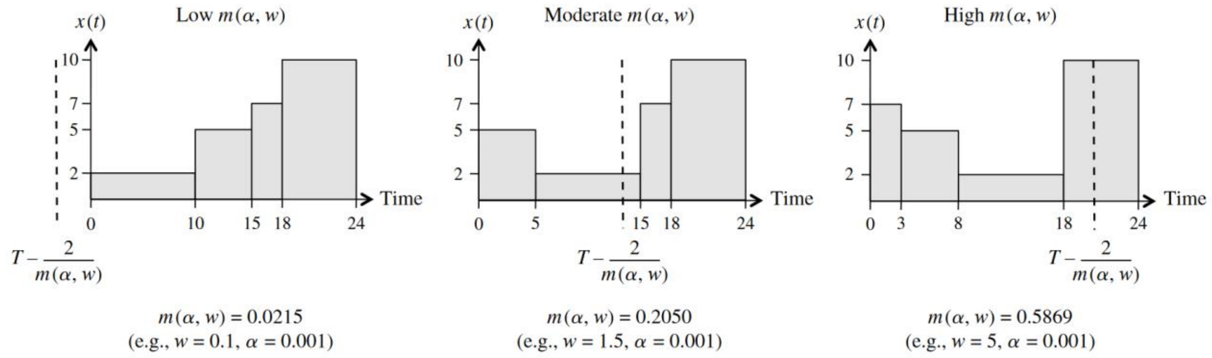
$$m(\alpha,w) \doteq \begin{cases} 0 & \text{if } \alpha=0 \text{ or } w=0, \\ \alpha & \text{if } \alpha=w, \\ \dfrac{w-\alpha}{\ln w-\ln\alpha} & \text{otherwise.} \end{cases}$$

Variable Sequence and Fixed Duration (VSFD)

Proposition 1. *In VSFD,*

1. *When $m(\alpha, w) \le 2/T$, it is optimal to sequence activities in increasing order of service levels; i.e., $x_{i_1^*} < \cdots < x_{i_n^*}$.*
2. *When $m(\alpha, w) > 2/T$, it is optimal to sequence activities in a U-shaped fashion of service levels; i.e., there exists a k such that $x_{i_1^*} > \cdots > x_{i_{k-1}^*} > x_{i_k^*} < x_{i_{k+1}^*} < \cdots < x_{i_n^*}$. In particular, either this kth activity (i_k), or its direct predecessor (i_{k-1}), or its direct successor (i_{k+1}) should take place when there remain $2/m(\alpha, w)$ time units until the end of the encounter. Furthermore, $k < n$; i.e., the last two activities are always sequenced in increasing order of service levels.*

Figure 7 The Optimal Sequence of Activities Is Either Increasing or U-shaped, Depending on the Magnitude of $m(\alpha, w)$



Note. The last two activities should always be in increasing order.

proof

Lemma A4. *In VSFD, the optimal sequence $(i_1^*, \ldots, i_n^*)$ is such that for any two consecutive activities $x_{i_k^*}$ and $x_{i_{k+1}^*}$, the following applies:*

(i) *If both activities start and finish within $[0, T - 2/m(\alpha, w)]$ then $x_{i_k^*} > x_{i_{k+1}^*}$.*

(ii) *If both activities start and finish within $[T - 2/m(\alpha, w), T]$ or if $k = n - 1$, then $x_{i_k^*} < x_{i_{k+1}^*}$.*

Proof. The proof uses an interchange argument. Let S^* be the satisfaction obtained from the optimal sequence $(i_1^*, \ldots, i_n^*)$. For any j, let S_j^* be the satisfaction obtained by interchanging i_{j-1}^* and i_j^* in the optimal sequence. Therefore, by (6),

$$S^* - S_j^* = (x_{i_{j-1}^*} - x_{i_j^*}) \cdot ((-\Phi(\bar{T}_j) + \Phi(\bar{T}_{j+1})) + (\Phi(\bar{T}_{j-1}) - \Phi(t_{i_{j-1}^*} + \bar{T}_{j+1}))).$$

We next consider three scenarios: (i) when the two activities start and end within $[0, T - 2/m(\alpha, w)]$, (ii) when the two activities start and end within $[T - 2/m(\alpha, w), T]$, and (iii) when $j = n$.

(i) If i_{j-1}^* and i_j^* start and finish within $[0, T - 2/m(\alpha, w)]$, then by Lemma A3, the corresponding $\Phi'(t)$ is increasing $\forall t \in [2/m(\alpha, w), T]$. Hence,

$$\Phi(\bar{T}_j) - \Phi(\bar{T}_{j+1}) < \Phi(t_{i_{j-1}^*} + \bar{T}_j) - \Phi(t_{i_{j-1}^*} + \bar{T}_{j+1}).$$

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Fixed Sequence and Variable Duration（FSVD）

PROOF OF PROPOSITION 2. We use Lemmas A1 and A6 for this proof. By (6), we have

$$\frac{\partial S(\mathbf{t}^*)}{\partial t_i} = \sum_{j=1}^{i}(x_j - x_{j-1})\Phi'(\bar{T}_j), \ \forall i,$$

where $x_0 = b(0)$.

1. Suppose that $x_q < \cdots < x_r$. Since $T < 1/m(\alpha, w)$, $\Phi'(t) > 0$, $\forall t \in [0, T)$ by Lemma A1, and therefore $\partial S(\mathbf{t})/\partial t_q < \cdots < \partial S(\mathbf{t})/\partial t_r$, $\forall \mathbf{t} \geq (\underline{\tau}_1, \ldots, \underline{\tau}_n)$. Hence, from condition (A2), if $t_q > \underline{\tau}_q$, $\partial S(\mathbf{t}^*)/\partial t_q = \mu$ or $\partial S(\mathbf{t}^*)/\partial t_q = \mu + \bar{\lambda}_q$. Therefore, $\partial S(\mathbf{t}^*)/\partial t_j = \mu + \bar{\lambda}_j$, $j = q+1, \ldots, r$. From Lemma A6, $t_j^* = \bar{\tau}_j$, $j = q+1, \ldots, r$. The proof for a decreasing subsequence is similar.

2. We show the result by contradiction. Suppose that $x_l < \cdots < x_r$ and that $t_l = \underline{\tau}_l$ and $t_r = \underline{\tau}_r$. Therefore, condition (A2) implies that $\partial S(\mathbf{t}^*)/\partial t_l = \mu - \underline{\lambda}_l$ and $\partial S(\mathbf{t}^*)/\partial t_r = \mu - \underline{\lambda}_r$. If for any $i, l < i < r$ we have $t_i^* > \underline{\tau}_i$, then $\partial S(\mathbf{t}^*)/\partial t_i = \mu$ or $\partial S(\mathbf{t}^*)/\partial t_i = \mu + \bar{\lambda}_i$, by condition (A2). If $\sum_{h=i}^{n} t_h^* \leq 1/m(\alpha, w)$, then $\Phi'(t) > 0$ for $t \leq \sum_{h=i+1}^{n} t_h^* < 1/m(\alpha, w)$ from Lemma A1, and therefore $\partial S(\mathbf{t}^*)/\partial t_r > \partial S(\mathbf{t}^*)/\partial t_i$, thereby a contradiction. If $\sum_{h=i}^{n} t_h^* > 1/m(\alpha, w)$, then $\Phi'(t) < 0$ for $t \geq \sum_{h=i}^{n} t_h^*$ from Lemma A1, and therefore $\partial S(\mathbf{t}^*)/\partial t_l > \partial S(\mathbf{t}^*)/\partial t_i$, thereby a contradiction. Therefore, from Lemma A6, $t_i^* = \underline{\tau}_i$, $\forall i, l < i < r$. The proof for a decreasing subsequence is similar.

3. Suppose that $x_l < \cdots < x_q$. When $\sum_{i=r}^{n} \underline{\tau}_i > 1/m(\alpha, w)$, we have $\Phi'(t) < 0$, $\forall t \in (\sum_{i=r}^{n} \underline{\tau}_i, T]$ by Lemma A1, and therefore $\partial S(\mathbf{t})/\partial t_l > \cdots > \partial S(\mathbf{t})/\partial t_q$, $\forall \mathbf{t} \geq (\underline{\tau}_1, \ldots, \underline{\tau}_n)$. From condition (A2), since $t_q > \underline{\tau}_q$, $\partial S(\mathbf{t}^*)/\partial t_q = \mu$ or $\partial S(\mathbf{t}^*)/\partial t_q = \mu + \bar{\lambda}_q$. Therefore, $\partial S(\mathbf{t}^*)/\partial t_j = \mu + \bar{\lambda}_j$, $j = l, \ldots, q-1$. Therefore, from Lemma A6, $t_j^* = \bar{\tau}_j$, $j = l, \ldots, q-1$. The proof for a decreasing subsequence is similar. □

PROPOSITION 2. *In FSVD,*

1. *When $m(\alpha, w) < 1/T$, if it is optimal to allocate more than the minimum duration to activity q, i.e., if $t_q^* > \underline{\tau}_q$, then it is optimal to allocate maximum duration to all subsequent activities if they have increasing service levels, i.e., $t_j^* = \bar{\tau}_j$, $j = q+1, \ldots, r$ if $x_q < \cdots < x_r$, and to all preceding activities if they have decreasing service levels, i.e., $t_j^* = \bar{\tau}_j$, $j = l, \ldots, q-1$ if $x_l > \cdots > x_q$.*
2. *When $1/T \leq m(\alpha, w) \leq 1/(\sum_{i=r}^{n} \underline{\tau}_i)$, for some r, it is never optimal to allocate more than the minimum duration only in the middle of an increasing sequence, i.e., if $x_l < \cdots < x_r$ and $t_l^* = \underline{\tau}_l$, $t_r^* = \underline{\tau}_r$, then $t_i^* = \underline{\tau}_i$, $\forall i, l < i < r$; additionally, it is never optimal to allocate more than the minimum duration only in the extremities of a decreasing subsequence, i.e., if $x_l > \cdots > x_r$ and $t_l^* > \underline{\tau}_l$, $t_r^* > \underline{\tau}_r$, then $t_i^* > \underline{\tau}_i$, $\forall i, l < i < r$.*
3. *When $m(\alpha, w) > 1/(\sum_{i=r}^{n} \underline{\tau}_i)$ for some r, if it is optimal to allocate more than the minimum duration to activity q, $q < r$, i.e., if $t_q^* > \underline{\tau}_q$, then it is optimal to allocate maximum duration to all preceding activities if they have increasing service levels, i.e., $t_j^* = \bar{\tau}_j$, $j = l, \ldots, q-1$ if $x_l < \cdots < x_q$, and to all subsequent activities, up to activity r, if they have decreasing service levels, i.e., $t_j^* = \bar{\tau}_j$, $j = q+1, \ldots, r$ if $x_q > \cdots > x_r$.*

$$\max_{(i_1, \ldots, i_n), \mathbf{t}} \ S((i_1, \ldots, i_n), \mathbf{t})$$

$$= \sum_{k=1}^{n}(x_{i_k} - x_{i_{k-1}})\Phi(\alpha, w, \bar{T}_k) \qquad (7)$$

$$\text{subject to} \quad \underline{\tau}_i \leq t_i \leq \bar{\tau}_i, \ \forall i, \qquad (8)$$

$$\sum_{i=1}^{n} t_i = T, \qquad (9)$$

$$(i_1, \ldots, i_n) \in \mathcal{P}. \qquad (10)$$

proof

LEMMA A6. *For FSVD the necessary conditions for optimality are given by*

$$\frac{\partial S(\mathbf{t})}{\partial t_i} = \bar{\lambda}_i + \mu, \ \forall i \in I_U,$$
$$\frac{\partial S(\mathbf{t})}{\partial t_i} = \mu, \ \forall i \in I_M, \qquad (12)$$
$$\frac{\partial S(\mathbf{t})}{\partial t_i} = \mu - \underline{\lambda}_i, \ \forall i \in I_L,$$

together with constraints (8) and (9), in which $\mu \in \Re$, $\underline{\lambda}_i \geq 0$, and $\bar{\lambda}_i \geq 0$; moreover, I_U, I_M, and I_L are three disjoint sets such that $I = I_U \cup I_M \cup I_L$ and $t_i = \bar{\tau}_i$, $\forall i \in I_U$, $t_i = \tau$, $\underline{\tau}_i < \tau < \bar{\tau}_i$, $\forall i \in I_M$, and $t_i = \underline{\tau}_i$, $\forall i \in I_L$.

PROOF. The necessary conditions for optimality of $\mathbf{t}^*$ are given by the following Karush-Kuhn-Tucker conditions (Boyd and Vandenberghe 2004):

1. The stationarity condition gives $\partial S(\mathbf{t})/\partial t_i - \mu + \underline{\lambda}_i - \bar{\lambda}_i = 0$, $\forall i$ at $\mathbf{t}^*$.
2. Complementary slackness gives $\mu(T - \sum_{i=1}^{n} t_i^*) = 0$, $\bar{\lambda}_i(\bar{\tau}_i - t_i^*) = 0$, $\forall i$, and $\underline{\lambda}_i(t_i^* - \underline{\tau}_i) = 0$, $\forall i$.
3. Primal feasibility implies that $\mathbf{t}^*$ satisfies the constraints (8) and (9).
4. Dual feasibility implies that $\mu \in \Re$ and $\underline{\lambda}_i, \bar{\lambda}_i \geq 0$, $\forall i$.

From the complementary slackness condition, $\underline{\lambda}_i = 0$, $\forall i \in I_U$, and $\bar{\lambda}_i = 0$, $\forall i \in I_L$, and both $\underline{\lambda}_i = 0$, $\bar{\lambda}_i = 0$, $\forall i \in I_M$. □

LEMMA A1. *The function $\Phi(\alpha, w, t)$ is strictly pseudoconcave in t with a stationary point at $1/m(\alpha, w)$.*

PROOF. Suppose $\alpha < w$. We have

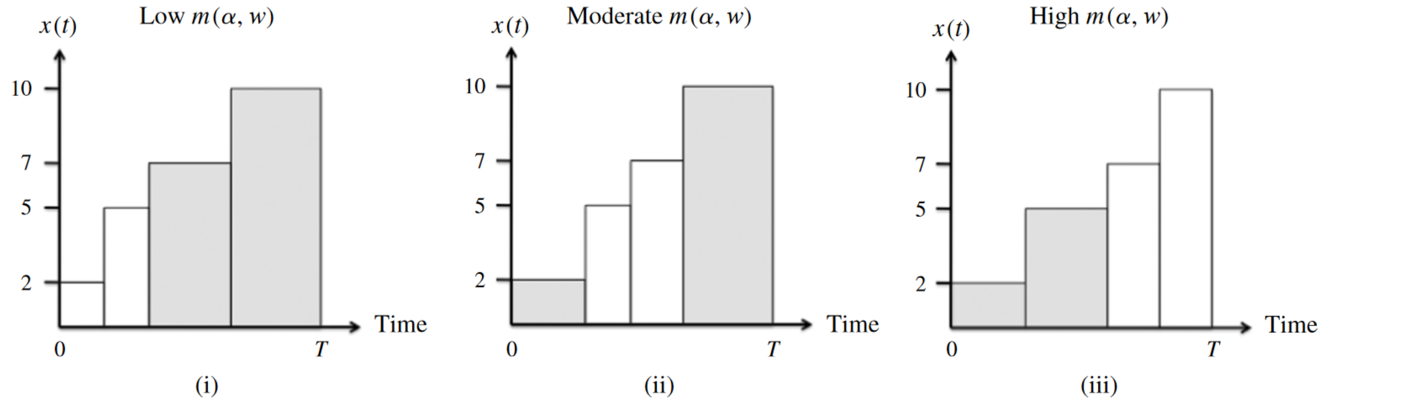
$$\Phi'(t) = \frac{e^{-\alpha t}(-\alpha) - e^{-wt}(-w)}{w - \alpha} = \frac{e^{-wt}(w - \alpha e^{t(w-\alpha)})}{w - \alpha} > 0$$

$$\Leftrightarrow w > \alpha e^{t(w-\alpha)} \Leftrightarrow \frac{\ln w - \ln \alpha}{w - \alpha} > t.$$

Since $\Phi'(t) > 0$ when $t < 1/m(\alpha, w)$, $\Phi'(t) = 0$ when $t = 1/m(\alpha, w)$, and $\Phi'(t) < 0$ when $t > 1/m(\alpha, w)$, $\Phi(t)$ is strictly pseudoconcave. Note that the result is symmetric for $w < \alpha$. □

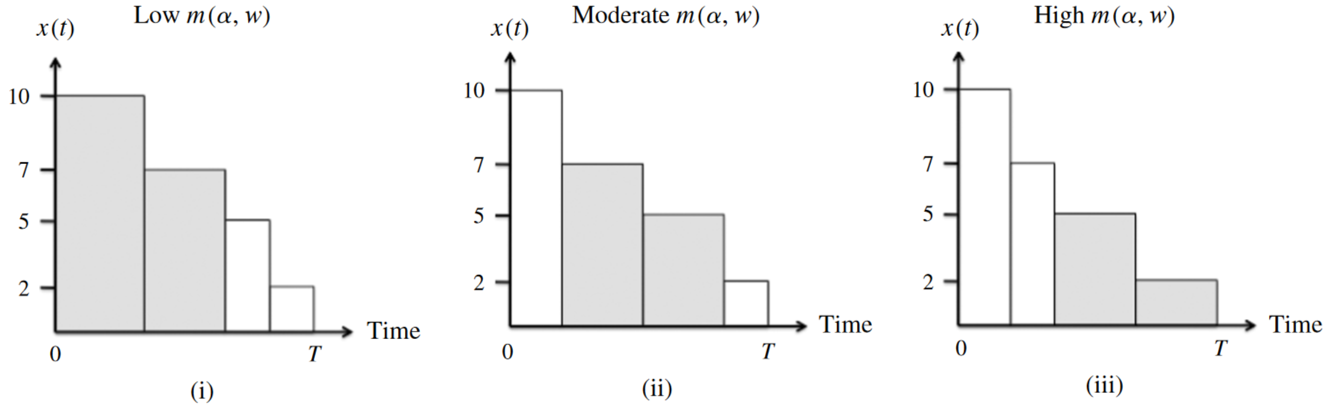
Fixed Sequence and Variable Duration（FSVD)

Figure 8 **Optimal Duration Allocation for an Increasing Sequence**



Note. The shaded activities are allocated duration above their lower bound.

Figure 9 **Optimal Duration Allocation for a Decreasing Sequence**



Note. The shaded activities are allocated duration above their lower bound.

Variable Sequence and Variable Duration（VSVD)

结合proposition1和proposition2得到

Corollary 1. *In VSVD,*

1. *When* $m(\alpha, w) < 2/T$, *a sequence with increasing service levels is optimal. Moreover,*

   (a) *when* $m(\alpha, w) < 1/T$, *there exists an index* k *such that minimum duration is allocated to the activities preceding the kth activity, i.e.,* $t_{i_j} = \underline{\tau}_{i_j}, 1 \le j \le k-1$, *and maximum duration is allocated to the activities subsequent to the kth activity, i.e.,* $t_{i_j} = \bar{\tau}_{i_j}, k < j \le n$;

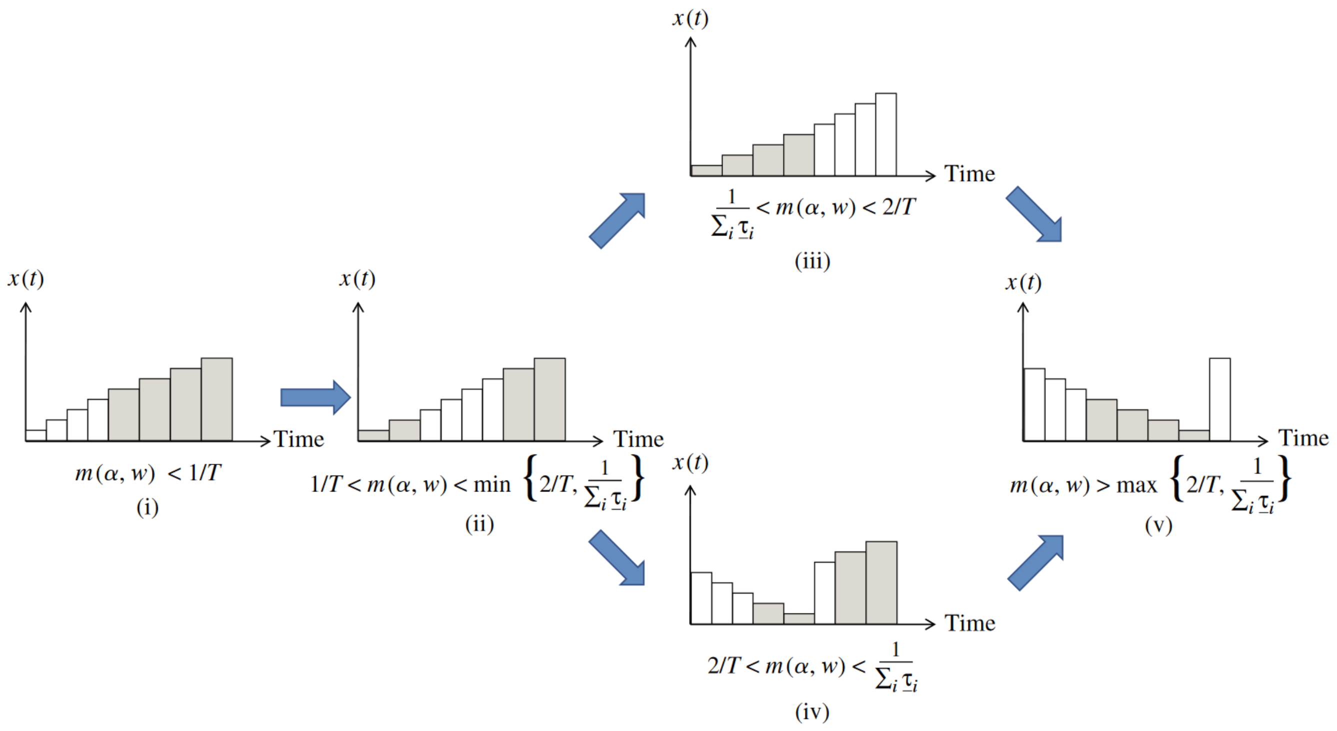
   (b) *when* $1/T < m(\alpha, w) \le 1/\underline{\tau}_{i_n}$, *the set of activities with minimum duration is contiguous; i.e., for any* l, r, *if* $t^*_{i_l} = \underline{\tau}_{i_l}$ *and* $t^*_{i_r} = \underline{\tau}_{i_r}$, *then* $t^*_{i_j} = \underline{\tau}_{i_j}, \forall j, l < j < r$; *and*

   (c) *when* $m(\alpha, w) > 1/\underline{\tau}_{i_n}$, *there exists an index* k *such that maximum duration is allocated to the activities preceding the kth activity, i.e.,* $t_{i_j} = \underline{\tau}_{i_j}, 1 \le j \le k-1$, *and minimum duration is allocated to the activities subsequent to the kth activity, i.e.,* $t_{i_j} = \bar{\tau}_{i_j}, k < j \le n$.

2. *If* $m(\alpha, w) > 2/T$, *a U-shaped sequence bottoming out at the kth activity is optimal such that*

   - *in the increasing part, the set of activities with minimum duration is contiguous; i.e., for any* $k \le l < r \le n$ *such that* $t^*_{i_l} = \underline{\tau}_{i_l}$ *and* $t^*_{i_r} = \underline{\tau}_{i_r}$, *then* $t^*_{i_j} = \underline{\tau}_{i_j}, \forall j, l < j < r$; *and*
   - *in the decreasing part, the set of activities with strictly more than the minimum duration is contiguous; i.e., for any* $1 \le l < r \le k$ *such that* $t^*_{i_l} > \underline{\tau}_{i_l}$ *and* $t^*_{i_r} > \underline{\tau}_{i_r}$, *then* $t^*_{i_j} > \underline{\tau}_{i_j}, \forall j, l < j < r$.

Figure 10 **(Color online) Optimal Sequence and Duration Allocation as $m(\alpha, w)$ Increases**



Note. The shaded activities are allocated duration above their lower bound.

3 Different Sequencing Heuristics

- the optimal sequence for that particular customer (α, w), denoted as S^*;
- a crescendo sequence, which orders activities in increasing order of service levels, i.e., $x_{i_1} \leq \cdots \leq x_{i_n}$, denoted as S^{cresc};
- a sequence that maximizes the gradient at the end, by ordering the $(n-1)$-smallest service-level activities in decreasing order of service levels and placing the activity with the highest service level at the end, i.e., $x_{i_1} \geq \cdots \geq x_{i_{n-1}} \leq x_{i_n} = \max x_i$, denoted as S^{steep}; and
- the optimal sequence if the provider ignored customer heterogeneity, i.e., based on the means (μ_α, μ_w), denoted as S^{mean}.

Table 2 Average Suboptimality Gaps for the Sequence Based on the Mean Rates and the Steep Gradient and Crescendo Sequences

		$\sigma_\alpha = \sigma_w$ (%)		
	$\mu_\alpha = \mu_w$	0.1	0.3	0.5
$1 - S^{\text{mean}}/S^*$	0.2	4.5	9.9	6.1
	0.5	1.5	9.7	16.9
	0.8	0.3	3.9	8.4
$1 - S^{\text{steep}}/S^*$	0.2	28.9	20.7	11.6
	0.5	13.1	13.6	16.9
	0.8	32.9	27.5	21.6
$1 - S^{\text{cresc}}/S^*$	0.2	16.4	8.3	4.1
	0.5	53.2	50.6	28.2
	0.8	72.1	68.8	64.0

Note. Sequences are based, respectively, on the mean rates (S^{mean}), the steep gradient sequence (S^{steep}), and the crescendo sequence (S^{cresc}) when $\mu_\alpha = \mu_w$ and $\sigma_\alpha = \sigma_w$.

Table 3 Average Suboptimality Gap for the Sequence Based on the Mean Rates and, the Steep Gradient and Crescendo Sequences

		$\mu_w = 0.2$ (%)			$\mu_w = 0.5$ (%)			$\mu_w = 0.8$ (%)		
	μ_α	$\sigma_\alpha = 0.1$	$\sigma_\alpha = 0.3$	$\sigma_\alpha = 0.5$	$\sigma_\alpha = 0.1$	$\sigma_\alpha = 0.3$	$\sigma_\alpha = 0.5$	$\sigma_\alpha = 0.1$	$\sigma_\alpha = 0.3$	$\sigma_\alpha = 0.5$
$1 - S^{\text{mean}}/S^*$	0.2	2.9	8.3	9.3	2.5	9.3	10.3	2.6	5.9	9.1
	0.5	0.4	3.8	8.1	0.6	4.8	10.4	0.5	3.4	8.5
	0.8	0.2	1.8	3.3	0.1	1.3	3.5	0.4	2.5	5.4
$1 - S^{\text{steep}}/S^*$	0.2	34.9	23.3	23.0	16.7	17.2	15.1	8.5	13.0	14.0
	0.5	17.8	24.7	20.1	8.6	16.2	17.9	21.5	18.6	17.5
	0.8	11.4	11.7	7.9	29.3	18.5	27.3	35.1	30.2	25.4
$1 - S^{\text{cresc}}/S^*$	0.2	18.0	12.7	8.4	35.3	25.2	17.4	36.6	27.8	18.2
	0.5	37.2	27.5	26.5	54.5	48.8	44.3	66.8	61.8	55.6
	0.8	44.4	40.9	41.3	71.6	63.7	65.5	72.2	69.3	64.5

Note. Sequences are based, respectively, on the mean rates (S^{mean}), the steep gradient sequence (S^{steep}), and the crescendo sequence (S^{cresc}) when $\mu_\alpha \neq \mu_w$ and $\sigma_w = 0.001$.

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