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# Cost allocation of cooperative autonomous truck platooning: Efficiency and stability analysis

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# Paper Information

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## ■ Research Areas

- Transportation network modelling
- Autonomous vehicles
- Container drayage

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- **Research Areas**
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  - Liner container shipping
  - Electric-vehicle infrastructure planning

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  - BS, East China Normal University, 1984
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| TITLE | CITED BY | YEAR |
|---|---|---|
| Sailing speed optimization for container ships in a liner shipping network<br>S Wang, Q Meng<br>Transportation Research Part E: Logistics and Transportation Review 48 (3 ... | 472 | 2012 |
| Containership routing and scheduling in liner shipping: overview and future research directions<br>Q Meng, S Wang, H Andersson, K Thun<br>Transportation Science 48 (2), 265-280 | 446 | 2014 |

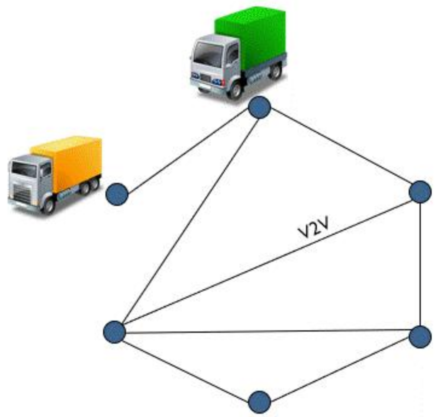
- **Research Areas**
  - Transportation network modelling
  - Liner container shipping
  - Electric-vehicle infrastructure planning

✓ Co-Editor-in-Chief, TRE

✓ Associate Editor, TRB

- **Vehicle platooning**: an autonomous vehicle (AV) technology

  - A group of vehicles traveling with small headway by means of advanced automated driving systems and vehicle-to-vehicle communication.




https://xiaotongsun.com/

- **Some important facts:**

(i) The leading vehicle may save less fuel consumption than the following vehicle in a platoon.

(ii) The formation of platoons requires a reasonable synchronization of AT departure time.

## Cooperative AT platooning

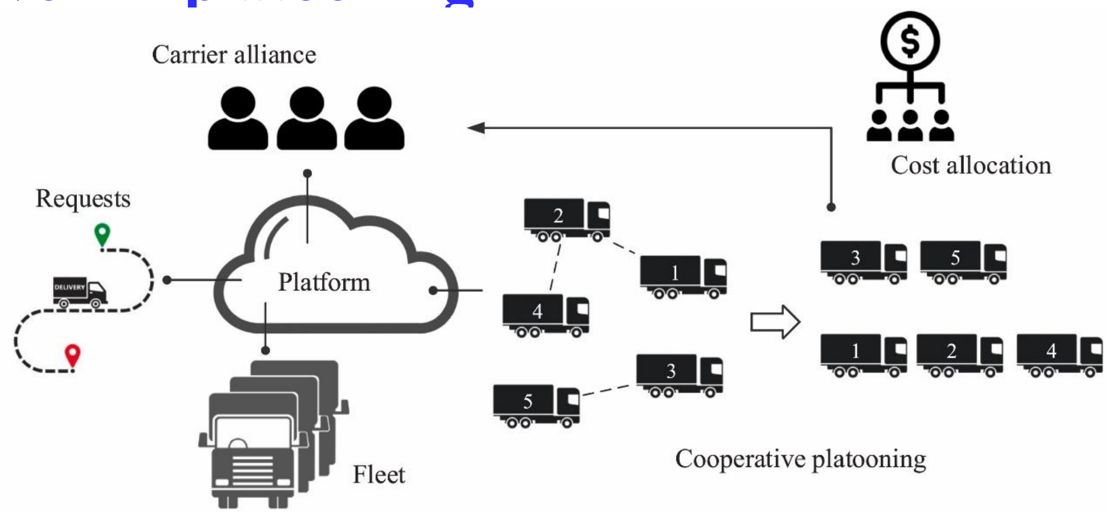



Fig. 1. The platform of carrier alliance for cooperative AT platooning.

- *Budget balance constraint:* $\alpha(N) = \pi(N)$

(i) Efficiency: the total cost is allocated to each carrier without a budget deficit.

- *Coalition stability constraints:* $\alpha(S) \le \pi(S)$

(ii) Stability: all carriers form a grand alliance and do not form sub-alliance.

## ■ Basic problem

My understanding: Fuel consumption cost VS Schedule deviation cost

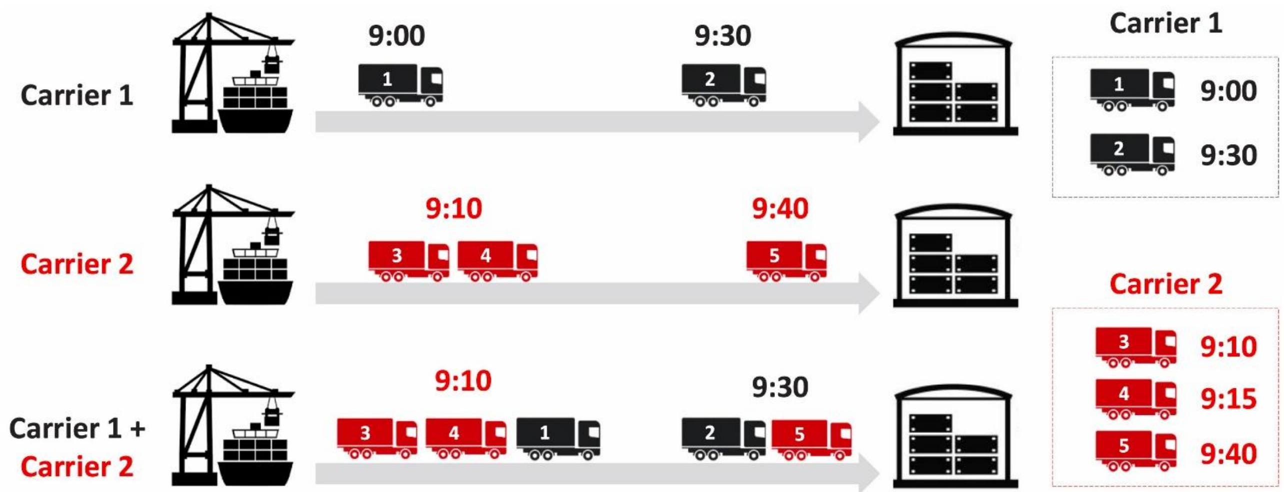



Fig. 2. An illustrative example.

The fuel consumption cost is set as $c_i^l = 50$ and $c_i^f = 40$.
The schedule deviation coefficients for earliness and lateness are set as $p_i^l = p_i^e = 0.5$.

The total cost of carriers 1 and 2 is 50+50=100 and 90+50+5*0.5=142.5.
The total cost of the alliance is 130+90+0.5*(10+5+10)=232.5<100+142.5

## ■ Research Gap

| Related area | Research content | Published research | Research Gap |
|---|---|---|---|
| **Vehicle platooning** | The truck platooning on a given route. | Boysen et al. (2018)<br>Zhang et al. (2017)<br>Chen et al. (2019, 2020) | Multiple carriers |
| | Vehicle platooning in networks. | Larson et al. (2016)<br>Abdolmaleki et al. (2021)<br>Pei et al. (2021) | |
| **Cost/profit allocation** | near-optimal cost allocation | Faigle and Kern (1993)<br>Shapley and Shubik (1966)<br>Caprara and Letchford (2010)<br>Liu et al. (2018) | Stability and efficiency |
| | profit/cost allocation for truck platooning. | Johansson and Mårtensson (2019)<br>Sun and Yin (2019)<br>Bouchery et al. (2022) | |

## ■ Cooperative AT platooning game

fuel consumption cost  schedule deviation cost

$$\text{(P1)} \quad v_1(S) = \min \boxed{\sum_{i,j\in\mathcal{N}_S} z_{ij}c_j^f + \sum_{i\in\mathcal{N}_S} z_{ii}(c_i^l - c_i^f)} + \boxed{\sum_{i\in\mathcal{N}_S} p_i^l\delta_i^l + p_i^e\delta_i^e}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{N}_S} z_{ji} = 1, \quad \forall i \in \mathcal{N}_S,$$

Every request should be assigned to one AT platoon.

$$\sum_{j\in\mathcal{N}_S} z_{ij} \le Q \cdot z_{ii}, \quad \forall i \in \mathcal{N}_S,$$

The length of platoons is limited.

$$s_i - s_j \ge (z_{ij} - 1)\cdot M, \quad \forall i,j \in \mathcal{N}_S,$$

All ATs in one platoon depart simultaneously.

$$s_i - s_j \le (1 - z_{ij})\cdot M, \quad \forall i,j \in \mathcal{N}_S,$$

$$\delta_i^e \ge d_i - s_i, \quad \forall i \in \mathcal{N}_S,$$

The schedule earliness and lateness.

$$\delta_i^l \ge s_i - d_i, \quad \forall i \in \mathcal{N}_S,$$

$$z_{ij} \in \{0,1\}, \quad \forall i,j \in \mathcal{N}_S,$$

$$s_i, \delta_i^e, \delta_i^l \in \mathbb{R}_+, \quad \forall i \in \mathcal{N}_S.$$

**Proposition 1**

$$\boxed{r_{ij} = p_i^e \cdot \max\{d_j - d_i, 0\} + p_i^l \cdot \max\{d_i - d_j, 0\}.}$$

$$\text{(P2)} \quad v_1(S) = \min \sum_{i,j\in\mathcal{N}_S} y_{ij}(r_{ij} + c_j^f) + \sum_{i,j\in\mathcal{N}_S} w_{ij}(c_j^l - c_j^f)$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{N}_S} y_{ji} = 1, \quad \forall i \in \mathcal{N}_S,$$

$$\sum_{j\in\mathcal{N}_S} y_{ij} \le Q \cdot y_{ii}, \quad \forall i \in \mathcal{N}_S,$$

$$\sum_{j\in\mathcal{N}_S} w_{ij} = y_{ii}, \quad \forall i \in \mathcal{N}_S,$$

$$w_{ij} \le y_{ij}, \quad \forall i,j \in \mathcal{N}_S,$$

$$y_{ij}, w_{ij} \in \{0,1\}, \quad \forall i,j \in \mathcal{N}_S.$$

**Proposition 1.** *Consider the optimal departure time $s^*$ to model P1. For an AT platoon serving a set of requests $\Gamma$, there exists a request $j \in \Gamma$ such that the optimal platoon departure time is at the scheduled departure time of request $j$, namely, $s_i^* = d_j, \forall i \in \Gamma$.*

## ■ Cooperative AT platooning game Goemans and Skutella (2004)

**Proposition 2.** *There is no integrality gap for the linear relaxation of model P2 if the following two conditions are satisfied, namely,*
*(i) Homogeneous cost coefficients,* i.e., $c^f = c_i^f$, $c^l = c_i^l$, $p^l = p_i^l$, *and* $p^e = p_i^e$ *for each* $i \in \mathcal{N}$;
*(ii) The number of requests whose scheduled departure time falls into the interval* $[d_i - \frac{c^l - c^f}{p_\mu^l}, d_i + \frac{c^l - c^f}{p_\mu^e}]$ *for any* $i \in \mathcal{N}$ *is less or equal to Q.*

- *homogeneous cooperative AT platooning game*

(i) The schedule deviation coefficients are zero, namely, $p_i^e = p_i^l = 0$.

(ii) The coefficients of fuel consumption are homogeneous, i.e., $c^l = c_i^l$ and $c^f = c_i^f$ for each $i$.

$$v_2(S) = \alpha_S \cdot c^l + (N_S - \alpha_S) \cdot c^f,$$

where $N_S$ is the number of requests in the coalition $S$ and $\alpha_S = \left\lceil \frac{N_S}{Q} \right\rceil$ represents the number of AT platoons (leading vehicles).

The closed-form formula of characteristic function allows us to derive the analytical results.

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## ■ Extension

(i) A common origin and multiple destinations.

(ii) A longer platoon saves more fuel consumption.

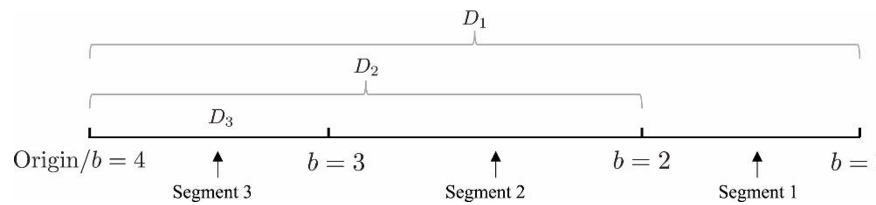


Fig. 3. Corridor with one origin and three destinations.

$$\text{(P3)} \quad v_3(S) = \min \sum_{i,j \in \mathcal{N}_S} r_{ij} y_{ij} + \sum_{b \in \mathcal{B}} \sum_{i \in \mathcal{N}_S} \sum_{j \in \mathcal{N}_S(b)} \sum_{q=1}^{Q} (D_b - D_{b+1})(c^l_{j,q} \cdot \sigma^{l,q}_{ij,b} + c^f_{j,q} \cdot \sigma^{f,q}_{ij,b})$$

s.t. (11)–(12),

$$\sum_{j \in \mathcal{N}_S(b)} y_{ij} = \sum_{q=1}^{Q} \pi^q_{i,b} \cdot q, \quad \forall i \in \mathcal{N}_S, b \in \mathcal{B},$$ The length of the platoon

$$\sigma^{l,q}_{ij,b} + \sigma^{f,q}_{ij,b} \geq \pi^q_{i,b} + y_{ij} - 1, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, j \in \mathcal{N}_S(b), q = 1, 2, \ldots, Q,$$

$$\sigma^{l,q}_{ij,b} + \sigma^{f,q}_{ij,b} \leq \pi^q_{i,b}, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, j \in \mathcal{N}_S(b), q = 1, 2, \ldots, Q,$$

$$\sigma^{l,q}_{ij,b} + \sigma^{f,q}_{ij,b} \leq y_{ij}, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, j \in \mathcal{N}_S(b), q = 1, 2, \ldots, Q,$$

$$\sigma^{l,q}_{ij,b} + \sigma^{f,q}_{ij,b} = y_{ij} \cdot \pi^q_{i,b},$$

$$\sum_{j \in \mathcal{N}_S(b)} \sigma^{l,q}_{ij,b} = \pi^q_{i,b}, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, q = 1, 2, \ldots, Q,$$ There exists a leading AT in the specified platoon.

$$y_{ij} \in \{0, 1\}, \quad \forall i, j \in \mathcal{N}_S,$$

$$\pi^q_{i,b} \in \{0, 1\}, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, q = 1, 2, \ldots, Q,$$

$$\sigma^{l,q}_{ij,b}, \sigma^{f,q}_{ij,b} \in \{0, 1\}, \quad \forall b \in \mathcal{B}, i \in \mathcal{N}_S, j \in \mathcal{N}_S(b), q = 1, 2, \ldots, Q.$$

## Cost allocations models

- The approximate core

$$\sum_{k\in S} x_k \le v(S)+\epsilon, \quad \forall S\subset \mathcal{K},$$

*stability violation*

$$\sum_{k\in \mathcal{K}} x_k = v(\mathcal{K})-\gamma,$$

*efficiency violation*

- **Single-objective models**

$$\text{(CAM-1)} \quad h(\gamma,v)=\min_{\epsilon,\mathbf{x}}\{\epsilon \ :\ \mathbf{x}\in \Pi(\epsilon,\gamma,v), \epsilon\in\mathbb{R}_+, \mathbf{x}\in\mathbb{R}^K\},$$

$$\text{(CAM-2)} \quad \mu(\epsilon,v)=\min_{\gamma,\mathbf{x}}\{\gamma \ :\ \mathbf{x}\in \Pi(\epsilon,\gamma,v), \gamma\in\mathbb{R}_+, \mathbf{x}\in\mathbb{R}^K\},$$

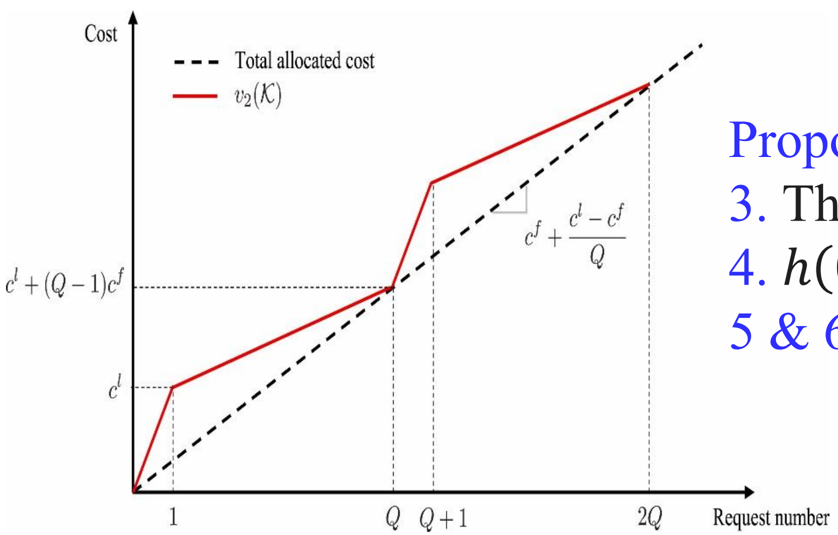


Fig. 4. Illustration of lower bound of $v_2(K)$

Propositions:

3. The function $h(\gamma, v)$ is decreasing with respect to $\gamma$.

4. $h(0,v) \le \mu(0,v)$.

5 & 6. The upper bounds of $\mu(0,v_2)$ and $h(0,v_2)$.

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## Cost allocations models

- **Bi-objective models**

$$\text{(CAM-3)} \quad \min_{\mathbf{x},\gamma,\epsilon} \begin{pmatrix} \gamma \\ \epsilon \end{pmatrix}$$

If the cost allocation $\hat{\mathbf{x}}$ is *Pareto-optimal* to model CAM-3, then there does not exist another cost allocation $\mathbf{x}'$ such that $\epsilon(\mathbf{x}') \leq \epsilon(\hat{\mathbf{x}})$ and $\gamma(\mathbf{x}') \leq \gamma(\hat{\mathbf{x}})$ with at least one inequality strictly holding.

**Proposition 7.** *Suppose* $\hat{\mathbf{x}} = \{\hat{x}_k, k \in \mathcal{K}\}$ *is a Pareto-optimal cost allocation for any cooperative AT platooning game* $(\mathcal{K}, v)$*, then the cost allocation* $\hat{\mathbf{x}}$ *is individually rational, i.e.,* $\hat{x}_k \leq v(\{k\})$ *for each player* $k \in \mathcal{K}$.

**Proposition 8.** *Suppose* $\hat{\mathbf{x}} = \{\hat{x}_k, k \in \mathcal{K}\}$ *is a Pareto-optimal cost allocation for the game* $(\mathcal{K}, v_2)$*, then we have* $\hat{x}_k \geq \bar{\alpha}_{\{k\}} c^l + (N_{\{k\}} - \bar{\alpha}_{\{k\}}) c^f$ *for each player* $k \in \mathcal{K}$ *where* $\bar{\alpha}_{\{k\}} = \lceil \frac{N_{\{k\}} - (Q-1)}{Q} \rceil$.

## ■ Row-generation

**Algorithm 1:** Row-generation-based solution method.

**Input:** Initial restricted coalition set $\Xi = \{\{1\}, \{2\}, ..., \{K\}\}$.

**Output:** Optimal solution to model CAM-1 or CAM-2.

1 Solve the initial master problem and separation problem;

2 **while** $\Phi < 0$ **do**

3 Update the restricted coalition set: $\Xi \leftarrow \Xi \cup \{k \in \mathcal{K} \mid \phi_k = 1\}$;

4 Given the coalition set $\Xi$, solve the master problem and obtain the optimal solution $\bar{x}_k \; \forall k \in \mathcal{K}$;

5 Solve the separation problem and obtain the optimal solution $\phi_k \; \forall k \in \mathcal{K}$;

6 Record the optimal cost allocation as $x_k = \bar{x}_k \; \forall k \in \mathcal{K}$

- separation problem

$$\text{(SP)} \quad \Phi = \min_{S \subset \mathcal{K}} \; v(S) + \bar{\epsilon} - \sum_{k \in S} \bar{x}_k,$$

$$\Rightarrow$$

$$\text{(SP-IP)} \quad \Phi = \min \sum_{i,j \in \mathcal{N}} y_{ij}(r_{ij} + c_j^f) + \sum_{i,j \in \mathcal{N}} w_{ij}(c_j^l - c_j^f) + \bar{\epsilon} - \sum_{k \in \mathcal{K}} \bar{x}_k \phi_k$$

$$\text{s.t.} \quad (12)\text{–}(15),$$

$$\phi_k = \sum_{j \in \mathcal{N}} y_{ji}, \quad \forall k \in \mathcal{K}, i \in \mathcal{N}_k,$$

$$\sum_{k \in \mathcal{K}} \phi_k \le K - 1,$$

$$\phi_k \in \{0, 1\}.$$

# 5. Numerical experiments

## ■ Optimal solution

**Table 4**
Cost allocation solution from different methods.

| $S$ | $v_1(S)$ | CAM-1 | | CAM-2 | | Shapley | |
|---|---|---|---|---|---|---|---|
| | | Cost | Diff | Cost | Diff | Cost | Diff |
| {1} | 447.00 | 440.76 | −6.24 | 442.76 | −4.24 | 441.18 | −5.82 |
| {1, 2} | 651.20 | 648.38 | −2.82 | 648.00 | −3.20 | 648.82 | −2.38 |
| {1, 2, 3} | 1056.74 | **1057.50** | **0.76** | 1056.74 | 0.00 | **1057.49** | **0.75** |
| {1, 2, 3, 4} | 1355.50 | **1355.50** | **0.00** | **1354.36** | **−1.14** | **1355.50** | **0.00** |
| {1, 2, 4} | 945.62 | **946.38** | **0.76** | 945.62 | 0.00 | **946.83** | **1.21** |
| {1, 3} | 851.50 | 849.88 | −1.62 | 851.50 | 0.00 | 849.85 | −1.65 |
| {1, 3, 4} | 1150.30 | 1147.88 | −2.42 | 1149.12 | −1.18 | 1147.86 | −2.44 |
| {1, 4} | 741.46 | 738.76 | −2.70 | 740.38 | −1.08 | 739.19 | −2.27 |
| {2} | 213.50 | 207.62 | −5.88 | 205.24 | −8.26 | 207.64 | −5.86 |
| {2, 3} | 621.50 | 616.74 | −4.76 | 613.98 | −7.52 | 616.31 | −5.19 |
| {2, 3, 4} | 913.98 | **914.74** | **0.76** | 911.60 | −2.38 | **914.32** | **0.34** |
| {2, 4} | 509.82 | 505.62 | −4.20 | 502.86 | −6.96 | 505.65 | −4.17 |
| {3} | 413.50 | 409.12 | −4.38 | 408.74 | −4.76 | 408.67 | −4.83 |
| {3, 4} | 706.36 | **707.12** | **0.76** | 706.36 | 0.00 | **706.68** | **0.32** |
| {4} | 303.50 | 298.00 | −5.50 | 297.62 | −5.88 | 298.01 | −5.49 |

$(\mathcal{K}, v_1)$

0.76<1.14, which is consistent with Proposition 4.

Shapley method fulfills the efficiency condition but causes a larger stability violation.

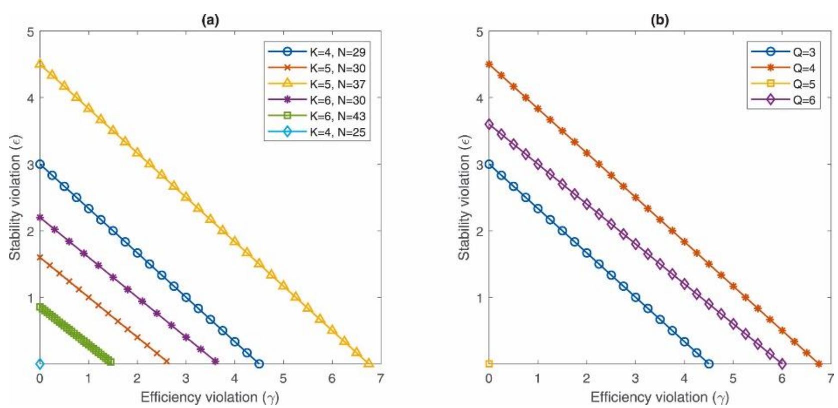


Fig. 5. Pareto frontiers of instances under different (a) players/requests; and (b) maximum platoon length.

$(\mathcal{K}, v_2)$

Pareto frontier

# 5. Numerical experiments

## ■ Optimal solution

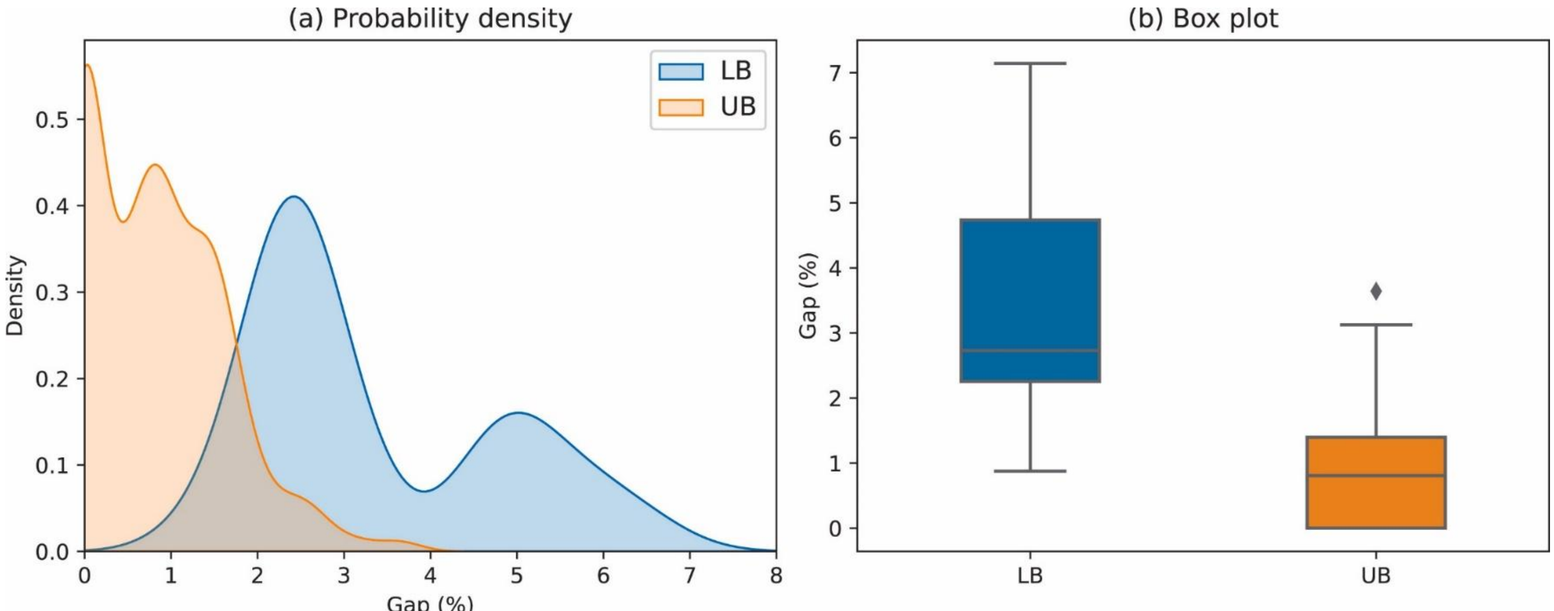


Fig. 6. Gaps between allocated cost and bounds.

The UB is tighter than LB.

## Computational performance

Table 7
Computational performance of medium instances for the game $(\mathcal{K}, v_1)$.

| Instance | Core | CAM-1 | | | CAM-2 | | |
|---|---|---|---|---|---|---|---|
| | | Obj | #Row | T | Obj | #Row | T |
| K15-N45-L-1 | E | 1.97 | 78 | 6971 | 2.63 | 76 | 1393 |
| K15-N45-L-2 | E | 1.44 | 69 | 302 | 1.87 | 68 | 195 |
| K15-N45-L-3 | E | 2.76 | 71 | 1929 | 3.27 | 76 | 2742 |
| K15-N45-L-4 | E | 2.37 | 71 | 441 | 2.77 | 73 | 376 |
| K15-N45-L-5 | E | 2.26 | 68 | 2074 | 2.83 | 69 | 1995 |
| K15-N45-M-1 | NE | 0 | 409 | 324 | 0 | 390 | 296 |
| K15-N45-M-2 | NE | 0 | 205 | 134 | 0 | 232 | 165 |
| K15-N45-M-3 | NE | 0 | 357 | 237 | 0 | 322 | 216 |
| K15-N45-M-4 | NE | 0 | 136 | 71 | 0 | 147 | 80 |
| K15-N45-M-5 | NE | 0 | 529 | 438 | 0 | 397 | 306 |
| K15-N60-L-1 | – | – | 19 | 7200 | – | 20 | 7200 |
| K15-N60-L-2 | – | – | 17 | 7200 | – | 26 | 7200 |
| K15-N60-L-3 | – | – | 26 | 7200 | – | 18 | 7200 |
| K15-N60-L-4 | – | – | 23 | 7200 | – | 19 | 7200 |
| K15-N60-L-5 | – | – | 21 | 7200 | – | 19 | 7200 |
| K15-N60-M-1 | NE | 0 | 203 | 319 | 0 | 194 | 311 |
| K15-N60-M-2 | NE | 0 | 321 | 503 | 0 | 335 | 526 |
| K15-N60-M-3 | NE | 0 | 125 | 228 | 0 | 112 | 178 |
| K15-N60-M-4 | NE | 0 | 139 | 179 | 0 | 192 | 266 |
| K15-N60-M-5 | NE | 0 | 166 | 263 | 0 | 175 | 292 |

The schedule deviation coefficients impose a significant impact on computational performance.

# 5. Comments

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## ■ Negative comments:

- ✓ To solve a larger size of instances, an efficient heuristic solution method needs to be developed in the future study.
- ✓ Multiple origins and destinations.

## ■ My comments:

- ✓ 没有写*Core*的概念，对非合作博弈领域的读者更友善，同时概念迁移，可解释性强。
- ✓ 应用类的文章，更关注问题和模型的性质，而不是算法的可行性和复杂度。但相比于纯OM的文章，管理启示的笔墨少。
- ✓ 技术难度小，框架可模仿性强。做了模型的Extension，和大量的数值实验。
- ✓ 一些想法：外卖配送？

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# Thank You!

Reporter： Zhao Junzhe