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Project Evaluation and Selection with Task Failures

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Production and Operations Management, 2020

Received: August 2018;

Accepted: August 2019 by Chelliah Sriskandarajah, after 2 revisions

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2023.10.04

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Research Interest:

- Supply Chain Finance and Risk Management
- Game Theory
- Combinatorial Optimization

Papers:

- 6 UTD 24 papers (1 MS; 1 OR; 2 MSOM; 2 POM), 1 Math Programming paper;
- Google Scholar citation: 1350.

Education:

- PhD in Operations Research, The Ohio State University, 2002.04-2007.08
- M.S. in Transportation Engineering, The Ohio State University, 2000.09-2002.01
- M.S. in Structural Engineering, Tsinghua University, 1996.09-1999.07
- B.S. in Civil Engineering/Computer Science, Tsinghua University, 1991.09-1996.07

Work:

- 2018.12-Now： Professor with Tenure, Antai College of Economics and Management, Shanghai Jiao Tong University;
- 2018.08-2018.12： Associate Professor with Tenure, Antai College of Economics and Management, Shanghai Jiao Tong University;
- 2012.08-2018.07： Associate Professor, Antai College of Economics and Management, Shanghai Jiao Tong University;
- 2010.08-2012.07： Assistant Professor, Antai College of Economics and Management, Shanghai Jiao Tong University
- 2007.09-2010.06： Postdoc Research Fellow, Washington University in St. Louis

Nicholas G. Hall

- Education:
  - PhD in Management Science, University of California, Berkeley (1986)
  - MA in Economics, University of Cambridge
  - BA in Economics, University of Cambridge
- Work:
  - Berry Family Professor in the Department of Operations and Business Analytics at Fisher College of Business, The Ohio State University.
- Awards:
  - **President, INFORMS, 2018;**
  - A 2021 bibliometric study ranked him **first** among all scholars in the research field of **scheduling.**

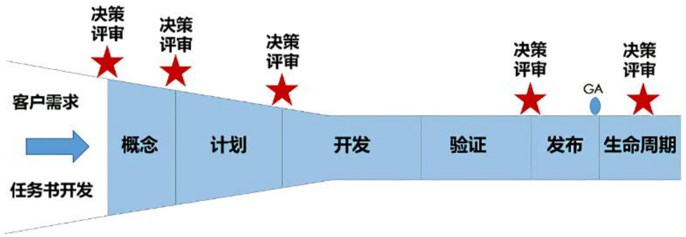
- **Research Interest:**
  - Project Management
  - Scheduling
  - Behavioral Issues in Operations
  - Sports Analytics
- **Papers:**
  - 98 articles in the leading journals (OR\MS\MOR\MP\M&SOM);
  - Google Scholar citation: 7445.

Zhixin Liu

- Education:
  - PhD degree in Operations Management from Ohio State University
  - MS degree in Operations Research from Tsinghua University
  - BS degree in Computational Mathematics from Nankai University
- Work:
  - Professor, University of Michigan.
- Research Interest:
  - Modeling and Optimization
  - Supply chain management
- Papers:
  - 1 OR, 3 POM, 1 JOC;
  - Google Scholar citation: 786.

Project management: How to evaluate and select?

- **Project management** is a highly important, global business process.
  - Examples: New product development, pharmaceutical development projects.



- Various estimates for the global impact of project management within the world's economic activity range from 20% to 30% , implying an annual value of about $27 trillion.
- "Doing the right projects is a big factor in doing projects right".

Problem:

1. How to evaluate their available projects individually?
   - Expected net present value, **ENPV**;
2. Given their limited resources, how to select their available projects to run?
   - Trade-off: **risk of failure** vs **cost**.
3. How to sequence the tasks within the project? How to make the timing decision?

Contributions

- This work contributes to the extensive project management literature by modeling and solving the problem of **maximizing the ENPV of a project that is subject to failure**.
- We formulate a mathematical model that:

1. Recognizes the **reduction in project risk each time a task is completed**;
    - Example: Development of new pharmaceuticals;
2. Implements this **risk reduction** into the ENPV evaluation of the project and permits optimization of the ENPV through **sequencing and timing decisions** for the tasks within the project;
3. Design an **exact algorithm** based on branch and bound.

Fundamental Model

- Let n denote the number of tasks in the project.
- Task i has a cash flow F_i at its completion time C_i , where $F_i > 0$ for cash inflows and $F_i < 0$ for cash outflows.
- Let Δ denote the deadline of the project, after which the project is worthless. ($C_{n+1} \leq \Delta$)
- Let $D_i \geq 0$ denote the duration of task i.
- Let S_i denote the set of immediate successors of task i under the given precedence constraints, which are:

$$C_k - C_i \geq D_k, \qquad k \in S_i, i = 0, \ldots, n. \,(1)$$

Fundamental Model

- Let r_f denote the exogenous failure rate of the common risk to the project.
- Let r_i denote the task-specific failure rate of task i. (independent)

- **Classical ENPV Maximization:**

$$\max_{C_1,C_2,\cdots,C_n} \sum_{i=1}^{n} F_i \exp(-r_f C_i) \quad (3)$$

$$\text{s.t.} \quad \text{Constraints}(1),$$

$$C_0 = 0, \quad (4)$$

$$C_{n+1} \leq \Delta. \quad (5)$$

Linearization

$v_i = \exp(-r_f C_i)$

$$\max_{v_1,v_2,\cdots,v_n} \sum_{i=1}^{n} F_i v_i$$

$$\text{s.t.} \quad \exp(r_f D_k) v_k - v_i \leq 0, \qquad k \in S_i, \quad i = 0, \ldots, n,$$

$$v_0 = 1,$$

$$v_{n+1} \geq \exp(-r_f \Delta).$$

- **Limitation:** this model does not describe that the project risk decreases as tasks are completed in practice,.

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ENPV Maximization with Task-Specific Risk

- First, consider a given task completion time sequence: $C_1 \leq C_2 \leq \cdots \leq C_n$;
- Divide the time $[0, C_1]$ into m equal-length intervals: $\delta = \frac{C_1}{m} \rightarrow 0$ as $m \rightarrow \infty$;
- For interval $[0, \delta]$, the probability for tasks $1, \ldots, n$ to succeed is $(1 - r_f\delta) \prod_{i=1}^{n}(1 - r_i\delta)$;
- The probability for the project not to fail by time C_1 as:

$$
\begin{aligned}
&\lim_{m\to\infty} (1 - r_f\delta)^m \Pi_{i=1}^{n} (1 - r_i\delta)^m \\
&= \lim_{m\to\infty} \left(1 - \frac{r_f C_1}{m}\right)^m \left(1 - \frac{r_1 C_1}{m}\right)^m \left(1 - \frac{r_2 C_1}{m}\right)^m \\
&\cdots \left(1 - \frac{r_n C_1}{m}\right)^m \\
&= \exp(-r_f C_1) \exp(-r_1 C_1) \exp(-r_2 C_1) \cdots \exp(-r_n C_1) \\
&= \exp(-(r_f + r_1 + r_2 + \cdots + r_n) C_1) = \exp(-R_1 C_1),
\end{aligned}
$$

$$R_i \equiv r_f + \sum_{j=i}^{n} r_j, \quad i = 1, \ldots, n,$$

- Therefore, the ENPV of cash flow F_1 is: $ENPV_1 = F_1 \exp(-R_1 C_1)$

ENPV Maximization with Task-Specific Risk

- Given that tasks $1, \ldots, n$ have not failed by C_1, the probability for tasks $2, \ldots, n$ not to fail during the interval $[C_1, C_2)$ is $exp(-(r_f + r_2 + \cdots + r_n)(C_2 - C_1)) = exp(-R_2(C_2 - C_1))$.
- $ENPV_2 = F_2 \cdot exp(-R_1(C_1 - C_0)) \cdot exp(-R_2(C_2 - C_1))$.
- ……
- $ENPV_i = F_i \cdot exp[-\sum_{j=1}^{i} R_j (C_j - C_{j-1})]$.
- $ENPV = \sum_{i=1}^{n} ENPV_i$

- The risk level of the project declines during its execution: $R_1 \geq R_2 \geq \cdots \geq R_n$.

$$R_i \equiv r_f + \sum_{j=i}^{n} r_j, \quad i = 1, \ldots, n,$$

ENPV Maximization with Task-Specific Risk

- Overall optimization model:
  - *Binary variable:* x_{ij} (if task i is scheduled as the jth task);
  - $R_j = r_f + \sum_{k=j}^{n} \sum_{l=1}^{n} r_l x_{lk}, j = 1, \dots, n$

(MIP)

$$\max_{C_i, x_{ij}} \sum_{i=1}^{n} \left\{ \left[\sum_{k=1}^{n} F_k x_{ki} \right] \exp\left(-\sum_{j=1}^{i} \left[(r_f + \sum_{k=j}^{n} \sum_{l=1}^{n} r_l x_{lk})(C_j - C_{j-1}) \right] \right) \right\} \quad (10)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_{ij} = 1, \quad 1 \le j \le n, \quad (11)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad 1 \le i \le n, \quad (12)$$

$$\sum_{j=1}^{n} x_{kj} C_j - \sum_{j=1}^{n} x_{ij} C_j \ge D_k, \quad 0 \le i \le n, k \in S_i, \quad (13)$$

$$x_{ij} \in \{0, 1\}, \quad 1 \le i, j \le n, \quad (14)$$

(4) and (5).

$$C_0 = 0, \quad (4)$$

$$C_{n+1} \le \Delta. \quad (5)$$

Approximating the Maximum ENPV

- **Bounds for a Given Full Sequence**: $C_1 \le C_2 \le \cdots \le C_n$

(SSP) $$\max_{C_1,\cdots,C_n} \sum_{i=1}^{n} F_i \exp\left(-\sum_{j=1}^{i} R_j (C_j - C_{j-1})\right)$$

s.t. constraints(1), (4), (5),

$$C_{i+1} - C_i \ge 0, \qquad 0 \le i \le n,$$

$$y_i = \exp\left(-\sum_{j=1}^{i} R_j (C_j - C_{j-1})\right) \Rightarrow \ln(y_i)$$

$$= -\sum_{j=1}^{i} R_j (C_j - C_{j-1}),$$

(SSP0) $$\max_{y_0, y_1, \cdots, y_{n+1}} \sum_{i=1}^{n} F_i y_i$$

s.t. $$\sum_{j=i+1}^{k} \frac{\ln(y_j) - \ln(y_{j-1})}{R_j} \le -D_k, \quad \text{for } 0 \le i \le n,\ k \in S_i, \tag{17}$$

$$\sum_{j=1}^{n+1} \frac{\ln(y_j) - \ln(y_{j-1})}{R_j} \ge -\Delta, \tag{18}$$

$$y_{i+1} - y_i \le 0, \qquad 0 \le i \le n, \tag{19}$$

$$y_0 = 1 \text{ and } y_{n+1} \ge 0. \tag{20}$$

Simplified Model —Eliminate exp(.)→ **Reformulated Model**

Approximating the Maximum ENPV

- **Bounds for a Given Full Sequence**: $C_1 \le C_2 \le \cdots \le C_n$

Table 1 Accuracy of Bounds for a Full Sequence

n_c	10	12	14	16	18	20	22	24	26	28	30
Gap%	0.005	0.004	0.005	0.003	0.005	0.005	0.005	0.008	0.011	0.013	0.015

(SSP0) $$\max_{y_0, y_1, \cdots, y_{n+1}} \sum_{i=1}^{n} F_i y_i$$

$$\text{s.t.} \sum_{j=i+1}^{k} \frac{\ln(y_j) - \ln(y_{j-1})}{R_j} \le -D_k, \quad \text{for } 0 \le i \le n,\ k \in S_i, \tag{17}$$

$$\sum_{j=1}^{n+1} \frac{\ln(y_j) - \ln(y_{j-1})}{R_j} \ge -\Delta, \tag{18}$$

$$y_{i+1} - y_i \le 0, \qquad 0 \le i \le n, \tag{19}$$

$$y_0 = 1 \text{ and } y_{n+1} \ge 0. \tag{20}$$

Reformulated Model

(SSP1) $$\max_{0 \le y_i \le 1} \sum_{i=1}^{n} F_i y_i$$

$$\text{s.t.} \sum_{j=m+1}^{k-1} \frac{R_k}{R_j}(y_{j-1} - y_j) + y_{k-1} - y_k \exp(R_k D_k) \ge 0,$$

$$\text{for } 0 \le m \le n, k \in S_m, \tag{21}$$

$$\left[1 - \frac{R_{n+1}}{R_1} - \exp(-R_{n+1}\Delta)\right] y_0 - \sum_{j=1}^{n} \left(\frac{R_{n+1}}{R_{j+1}} - \frac{R_{n+1}}{R_j}\right) y_j + y_{n+1} \ge 0, \tag{22}$$

(19) and (20).

Upper Bound

(THEOREM 1)

(SSP2) $$\max_{0 \le y_i \le 1} \sum_{i=1}^{n} F_i y_i$$

$$\text{s.t.} \left[1 - \frac{R_k}{R_{m+1}} - \exp(-R_k D_k)\right] y_m - \sum_{j=m+1}^{k-1} \left(\frac{R_k}{R_{j+1}} - \frac{R_k}{R_j}\right) y_j + y_k \le 0, \text{ for } 0 \le m \le n,\ k \in S_m, \tag{23}$$

$$\sum_{j=1}^{n} \frac{R_{n+1}}{R_j}(y_{j-1} - y_j) + y_n - y_{n+1} \exp(R_{n+1}\Delta) \le 0, \tag{24}$$

(19) and (20).

Lower Bound

(THEOREM 2)

Approximating the Maximum ENPV

- **Bounds for a Given Partial Sequence**: $C_1 \le C_2 \le \cdots \le C_l$ **known**, task $i = l + 1, \ldots, n$ **unknown**.

Upper Bound

(THEOREM 3)

$$\text{(PSSP1)} \quad \max_{0 \le y_i \le 1} \sum_{i=1}^{n} F_i y_i$$

$$\text{s.t.} \quad \sum_{j=i+1}^{k-1} \frac{R_k}{R_j}(y_{j-1} - y_j) + y_{k-1} - y_k \exp(R_k D_k) \ge 0, \quad i \in \sigma', \ k \in S_i \cap \sigma', \tag{26}$$

$$\sum_{j=i+1}^{l} \frac{R_{k,\max}}{R_j}(y_{j-1} - y_j) + y_l - y_k \exp(R_{k,\min} D_k) \ge 0, \quad i \in \sigma', \ k \in S_i \cap \sigma'', \tag{27}$$

$$y_i - y_k \exp(R_{k,\min} D_k) \ge 0, \quad i \in \sigma'', \ k \in S_i \cap \sigma'', \tag{28}$$

$$\left[1 - \frac{R_{n+1}}{R_1} - \exp(-R_{n+1}\Delta)\right] y_0 - \sum_{j=1}^{l} \left(\frac{R_{n+1}}{R_{j+1}} - \frac{R_{n+1}}{R_j}\right) y_j + \frac{R_{n+1}}{R_{i,\min}} y_i \ge 0, \quad i \in \sigma'', \tag{29}$$

$$y_{i+1} - y_i \le 0, \quad 0 \le i \le l-1, \tag{30}$$

$$y_i - y_l \le 0, \quad i \in \sigma'', \tag{31}$$

$$y_0 = 1, \quad y_i \ge 0, \quad i \in \sigma''.$$

Lower Bound

(THEOREM 4)

$$\text{(PSSP2)} \quad \max_{0 \le y_i \le 1} \sum_{i=1}^{n} F_i y_i$$

$$\text{s.t.} \quad \left[1 - \frac{R_k}{R_{i+1}} - \exp(-R_k D_k)\right] y_i - \sum_{j=i+1}^{k-1} \left(\frac{R_k}{R_{j+1}} - \frac{R_k}{R_j}\right) y_j + y_k \le 0, \quad i \in \sigma', k \in S_i \cap \sigma', \tag{33}$$

$$\left[1 - \frac{R_{k,\min}}{R_{i+1}} - \exp(-R_{k,\min} D_k)\right] y_i - \sum_{j=i+1}^{l} \left(\frac{R_{k,\min}}{R_{j+1}} - \frac{R_{k,\min}}{R_j}\right) y_j + y_k \le 0, \quad i \in \sigma', k \in S_i \cap \sigma'', \tag{34}$$

$$\left[1 - \frac{R_{k,\min}}{R_{i,\max} - r_i} - \exp(-R_{k,\min} D_k)\right] y_i + y_k \le 0, \quad i \in \sigma'', k \in S_i \cap \sigma'', \tag{35}$$

$$\sum_{j=1}^{l} \frac{R_{n+1}}{R_j}(y_{j-1} - y_j) + y_l - y_{n+1} \exp(R_{n+1}\Delta) \le 0, \tag{36}$$

$$y_{i+1} - y_i \le 0, \quad 0 \le i \le l-1, \tag{37}$$

$$y_i - y_l \le 0, \quad i \in \sigma'', \tag{38}$$

$$y_0 = 1, \quad y_i \ge 0, \quad i \in \sigma''.$$

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Branch and Bound Algorithm

- **Heuristics：Find the initial lower bound of ENPV**
  - **Grinold Heuristic (GH):** find a feasible schedule for each constant failure rate and choose a schedule with the largest ENPV.
  - **Sequence-Based Heuristic (SH):** generate a full sequence randomly and solve problem (SSP2).
- **Branch and Bound Heuristic (BH)**
  - **Initialization**
    - Run GH and SH to find lower bound **LB**;
    - Solve (PSSP1) to find upper bound **UB**.
  - **Root Node:** create some **subnode** to constructs partial sequences, one task at a time.
  - **Nonroot Node:**
    - Solve (SSP2) to find a lower bound **LB’**, and update **LB**;
    - Solve (PSSP1) to find upper bound **UB’.**

Numerical Study

1. Study the performance of BH (GE and GH) : speed.
   - **Parameters**: the number of cash flows, depth of project network, , tightness of project deadline, size of failure rates, and pattern of cash flows.
2. BH improves project selection (GE and GH): quality.

Insights

1. Tasks with larger cash flow, that is, **larger cash inflow or smaller cash outflow**, should be processed **earlier**, holding other factors constant.
2. For a typical project with a positive ENPV, tasks with **larger risk** should be processed **earlier**, holding other factors constant.
3. For a typical project with a positive ENPV, tasks with **shorter processing time** should be processed **earlier**, holding other factors constant.

3 factors interact with each other

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Problem: ***How to sequence the tasks within the project? How to make the timing decision?***

(a) Model

I. Without Task-Specific Risk

Limitations: can't explain why project risk decreases as tasks are completed in practice .

II. With Task-Specific Risk

MIP ($\boldsymbol{C_i}$ and $\boldsymbol{x_{ij}}$)

- Many binary variables;
- Exponential terms;
- Constraints not linear;
- …….

(b) Algorithm

I. Approximating the Maximum ENPV

- Bounds for a **Full** Sequence
  - Eliminate exp(.) and reformulate.
  - (SSP1) for upper bound and (SSP2) for lower bound;
- Bounds for a **Partial** Sequence (similar as above)

II. Branch and Bound Algorithm

- **Heuristic:** GH and SH;

- **Branch and bound:**
  - Initialization—Root Node—Nonroot Node;
  - Obtain bound by **approximating method** and **heuristic**.

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Thank you for listening!

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