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Unsplitable:
↓
UCVRP. with limited. information.

• 随机化 VRP (CVRPSD). 开始无法知道所有需求info. 但知道location.

往往分个两阶段 decision [Bertimas, 1992]. [Ledvina et al, 2022].

① a priori route. 分配已知 location → drivers.

② recourse. 一个就测需求后做 online 的 scheduling. [Bramel et al 1992]

* UCVRP: online Bin Packing [This paper].

*. splitable: ITP heuristics [Ledvina et al, 2022].
+Next Fit.

*. VRPTW: online Generalized. Bin Packing [working]

• a priori 分配.

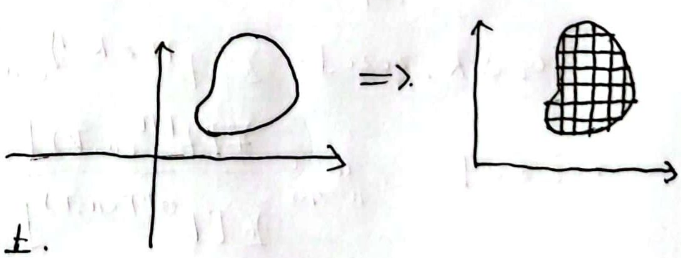
对于 distribution $\mu \in$ compact $\Omega \in \mathbb{R}^2$, 定义. simple Square Distribution (SSD)

$$f(x,y) = \sum_i k_i \mathbb{I}\{(x,y) \in S_i\} \quad \begin{cases} \sum \delta^2 \sum k_i = 1. \\ \{S_1, S_2 \dots S_U\} \text{ squares. with area } \delta^2. \end{cases}$$

cor 3. $\mu \sim f_0(x,y) \in$ compact $\Omega \in \mathbb{R}^2$.

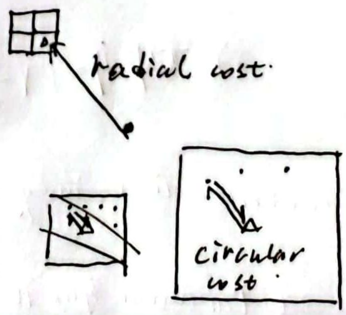
Ω 可以划分为 finite 子集, 在每个上 f_0 连续. 则 $\exists \{\varphi_n\}$ (SSD) s.t.

$$\lim \varphi_n = f_0.$$

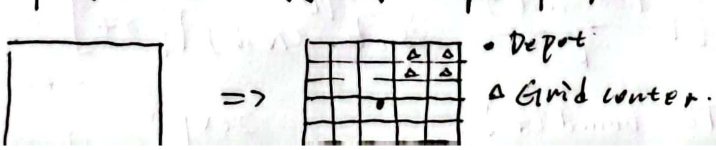


*. 接下来的讨论可以限制在 Square 上.

• Double partition.



对一个 square Ω. 将其划分为 $[n^*]^2$ 个小区域.



* Alg 3. Double partitioning with limited flexibility.

对于 region ~~M(i)~~ i. $i=1,2,\dots,\lceil n^{\frac{1}{4}}\rceil^2$.

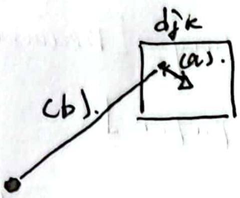
设 limited information 为 N'，将 ~~M(i)~~ i 划分为 $\lceil \frac{M(i)}{N'} \rceil$ groups.

$\{t_i^1, t_i^2, \dots, t_i^{\lceil \frac{M(i)}{N'} \rceil}\}$

对于区域 i 做 online BPP:

(a). 若 d_{jk} 被某个现有 vehicle fulfill，则让其在 Grid center 和 d_{jk} 的 location 往返.

(b) 若 ~~对~~ d_{jk} 被新的 vehicle —— ~~去到 Grid center~~ —— Depot ——.



注：这3给出了 BPP 方案是如何对应 UCVRP 的 recourse 方案的.

- online BPP.

* [Gupta and Radovanović, 2020]. PD-R-$\eta(n,D)$

* [本文] Partition-PD-R-$\eta(n,D)$ => PPD-R ——.

=

① 有 $n = kN' + s$ items.

② 将其 segment 为 $k+1$ 部分 $\begin{cases} \{d_{(i-1)N'+1}, \dots, d_{iN'}\} \triangleq G_i \\ G_{k+1} \triangleq \{d_{(kN'+1)}, \dots, d_{kN'+s}\} \end{cases}$

③ $\forall i = 1 \dots k+1$, Apply PD-R-$\eta(n,D)$ 在 G_i 上.

返回 $\sum_{i=1}^{k+1}$ ~~的~~ $b^{PPD}(G_i, D)$

* [G & R, 2020]: $\mathbb{E}[b^{PD-R-\eta}(n,D)] \le n(b(D) + \frac{1}{\eta}) + \frac{Q\eta}{2}$.

* [本文]: $\lim_{n\to N} \frac{\mathbb{E}[b^{PPD}(n,D)]}{\mathbb{E}[b^{OPT}(n,D)]} \le 1 + \frac{\sqrt{2Q}}{b(D)\sqrt{N'}} + \frac{Q}{2b(D)N'} \sim O(\frac{1}{\sqrt{N'}})$

注：其中 $\eta = \lceil \sqrt{\frac{2N'}{Q}} \rceil$.

* concentration ineq: $\mathbb{P}\left[\left| \frac{b^{PPD}(n,D)}{k} - \mathbb{E}[b^{PD}(N',D)] \right| \ge t\right] \le 2\exp\left\{\frac{-kt^2}{2N'^2(1-1/Q)^2}\right\}$

注. Partition 可以和全局 PD. bound 住!

- Connection between BPP & VRP.

Lem 2. $\{c_1 \dots c_n\}$ 为 customer 到 depot 距离.

$$\text{total travel cost} \le \sum_{i=1}^{\lceil n^{\frac14}\rceil^2} 2\frac{M(i)}{N'}\mathbb{E}[b^{PD}(N',D)]\left(\frac{\Sigma c_j}{M(i)}+\frac{\operatorname{diam}\Omega}{n^{\frac14}}\right)$$

$$+\sum_{i=1}^{\lceil n^{\frac14}\rceil^2} M(i)\frac{\operatorname{diam}\Omega}{\lceil n^{1/4}\rceil}$$

注：circular cost. $2\times\dfrac{\operatorname{diam}\Omega}{2\lceil n^{\frac14}\rceil}$

对 G_i:

radial cost = $\underbrace{\text{expected num of trips}}_{\frac{M(i)}{N'}\mathbb{E}[b^{PD}(N',D)]} \times \underbrace{\text{ub of each trip}}_{2\left(\frac{\Sigma c_j}{M(i)}+\frac{\operatorname{diam}\Omega}{n^{\frac14}}\right)}$.

Thm 8. $$\lim_{n\to\infty}\frac{Z^{PPD}(n,D)}{Z_u^*(n,D)} \le 1+\frac{\sqrt{2Q}}{b(D)}\frac{1}{\sqrt{N'}}+\frac{Q}{2b(D)}\frac{1}{N'} \sim O\left(\frac{1}{\sqrt{N'}}\right)$$

- additional overlapping routes

$$\begin{array}{lccccc}
g_1 & t_1^1 & t_1^2 & \cdots & t_1^{\lceil M(1)/N'\rceil} \\
 & \downarrow & \downarrow & & \downarrow \\
g_2 & t_2^1 & t_2^2 & \cdots & t_2^{\lceil M(2)/N'\rceil} \\
\vdots & \downarrow & \downarrow & & \downarrow \\
 & t_3^1 & t_3^2 & \cdots & t_3^{\lceil M(3)/N'\rceil} \\
g_{\lceil n^{\frac14}\rceil^2} & & & &
\end{array}$$

一共有 $\max\{\lceil M(i)/N'\rceil : i=1 \dots \lceil n^{\frac14}\rceil^2\}$ 条 overlapping

对于 t_1^1 的 remaining capacity，对于 $\{d_1 \dots d_m\} \in t_2^1$ 做 Greedy search.

k-overlap. $t_1^1 \quad t_2^1 \quad t_3^1 \quad t_4^1 \quad \cdots \quad t_k^1 \quad t_{k+1}^1$

*Thm 9. 假设 N' 足够大（ε-heavy）则

$$1 \geq \lim_{n\to\infty} \frac{Z^{k\text{-}PPD}(n,D)}{Z^{PPD}(n,D)} \geq 1-\left(\left(\frac{\max\{d(x_0,y) \mid y\in\Omega\}}{\mathbb{E}[c]} + \frac{1}{Q\,\mathbb{E}[D]}\right)\varepsilon + \frac{\sqrt{\varepsilon}}{\mathbb{E}[D]}\right)$$

注1：ε-heavy："heavy traffic"：$\varepsilon N' \geq \frac{1}{2}Q^3$　ε 很小

$\Rightarrow \sum_{j=1}^{k} u(i,i+j)^{l} \leq 2\varepsilon N'$　其中 $u(i,i+j)^{l}$ 为 t_j^{l} 和 $\{t_i^{l}: i<j\}$ 的 overlap 顾客数目.

注2：Thm 9 中的 LB，为一个与 k 无关的 const，这说明 1-overlap 和 k-overlap 效果无差别（渐近）

注3：在数值实验中，用 ~~2nbED~~ $2nb(D)\,d(x_0,y)$ 作为 $Z_u^*(n,D)$ 的估计

在 N' 较小时 $\frac{Z^{k\text{-}PPD}}{Z^{-PPD}}$ 显著，但 $N' \geq 10000$ 后无明显提升.