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An Exponential Cone Programming Approach for Managing Electric Vehicle Charging

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2023/8/2

Introduction

EV Charging

- A key to the mass adoption of EVs is the ease of charging, where public charging will play an increasingly important role

Customers:

- stochastic arrivals of customers:
- arrival time, desired departure time, charging requirements

Introduction

Company A			
Billing Element	Meter Reading	Rate	Charge
Energy Usage	60,000 kWhr	$0.06/kWhr	$3,600
Demand Charge	**200 kW**	$13/kW	**$2,600**
Total Charges			**$6,200**

Company B			
Billing Element	Meter Reading	Rate	Charge
Energy Usage	60,000 kWhr	$0.06/kWhr	$3,600
Demand Charge	**490 kW**	$13/kW	**$6,370**
Total Charges			**$9,970**

TELD 特来电

国家电网

Pay the bill

research subject

The tariff structure

- Demand charges
- Energy charges

EV charging service provider:

- This total demand charge for an EV charging service provider can be as high as 70% of its total electricity cost.

Introduction

- **Research Subject**： EV charging service provider

- **Goal**： minimizing the total expected cost

- **Optimization approach**： scheduling EV charging (joint pricing and scheduling)

- **Model**： model it as a stochastic program (SP) and characterize the random number of arriving customers to follow Poisson distributions

Summary of innovation

- **Modeling fits the scene well** (Complex but solvable)
  - arrival time, desired departure time, charging requirements
  - tariff structure in electric power

- **solving method** (80,000 random variables and 700 ,000 decision variables)
  - moment-generating functions (MGFs) +exponential cone programming(ECP) approximations
  - Upper (Lower) bound of SP and a performance guarantee
  - Entropic dominance constraints (ambiguity set) + ECP

From the numerical experiment, ECP is fast and good

Summary of innovation

- **Differences from existing literature**
  - Focus on the operational level rather than infrastructure planning
  - Takes into account the setup of the supplier's purchase of electricity from the utility (including its tariff structure)
- **This research is very comprehensive, including**
  - Benchmark with SAA and DRO
  - Uncapacitated/ Capacitated case (a limited number of chargers)
  - charging scheduling /joint pricing and scheduling (optimal price)
  - Poisson's estimate is inaccurate
  - Time Discretization(15min/period —— 1min/period)
  - All-period, on peak, and mid-peak demand charge

➢ EV Arrivals

We classify EV customers into V types according to the triple (s_v, τ_v, u_v)

s_v: arrival time

τ_v: desired departure time ($s_v \leqslant \tau_v$)

u_v: charging requirement for customer type $v \in [V]$.

Note: customers are heterogeneous in all three dimensions (v is very large)

we assume the arrivals of customer types $v \in [V]$ with infinite chargers are independent Poisson random variables, and each has an arrival rate λ_v. (denoted by $\tilde{z}_v \sim \lambda_v$)

$$\mathcal{Z} \triangleq \left\{ z \geq 0 \,\middle|\, \sum_{v \in \mathcal{V}_t} z_v \leq C, \ \ \forall t \in [T] \right\}.$$

$\mathcal{V}_t \triangleq \{v \in [V] | s_v \leq t \leq \tau_v\}$ denote the set of customer types at the station in period t

$\tilde{z} \triangleq (\tilde{z}_v)_{v \in [V]}$ denote the vector of these truncated Poisson random variables ($\mathbb{P}^C$)

➢ Decisions

Decision variable:

the menu-based charging schedule $\boldsymbol{x} \triangleq (x_{v,t})_{v\in[V],\, t\in\mathcal{T}_v}$

where $x_{v,t}$denotes in period $t \in\ T_v$ the charging speed for customer type v;
$T_v \triangleq \{s_v, \ldots, \tau_v\}$ is the set of periods within the charging window of customer type v.

The feasible set of x, denoted by X, is given as follows:

$$\mathcal{X} \triangleq \left\{ x \;\middle|\; \begin{array}{ll} \sum_{t\in\mathcal{T}_v} \eta x_{v,t} = u_v & \forall v \in [V] \\ 0 \le x_{v,t} \le K/\eta & \forall v \in [V], t \in \mathcal{T}_v \end{array} \right\}$$

- customer type v needs to fulfill the charging requirement u_v, $\eta \in (0,1]$ is the ratio of the quantity of electricity increased in the battery to the quantity of electricity used to charge the battery.;
- charging speed is within the limit.

Related Models

➢ Total Cost

$$c(x,\tilde{z}) \triangleq \underbrace{d\max_{t\in[T]}\{f_t(x,\tilde{z})\}}_{\text{demand charge}} + \underbrace{\sum_{s\in[T]} e_s f_s(x,\tilde{z})}_{\text{energy charge}},$$

$f_t(x,\tilde{z})$ denote the total electricity used to charge EVs of all customers in period t, that is

$$f_t(x,\tilde{z}) = \sum_{v\in\mathcal{V}_t} x_{v,t}\tilde{z}_v.$$

➢ Model Formulation

We formulate the problem of scheduling EV charging as an SP:

$$\min_{x\in\mathcal{X}} \mathbb{E}_{\mathbb{P}^c}[c(x,\tilde{z})], \qquad (3)$$

where the expectation is over $\tilde{z}$. We denote an optimal solution to (3) by $x^*$and the optimal value of (3) by π^*, that is, $\pi^* = \mathbb{E}_{\mathbb{P}^c}[c(x^*,\tilde{z})]$.

Related Models

➢ ECP Approximations

Uncapacitated Case

(Method: MGF+ECP)

假设我们有一个随机变量X，它表示某个产品的寿命（以小时为单位），服从指数分布，参数为λ。

我们可以通过矩生成函数推导出X的各阶矩。首先，我们计算矩生成函数M(t)。对于指数分布的随机变量X，其概率密度函数为f(x) = λe^(-λx)，则矩生成函数为：

M(t) = E[e^(tX)] = ∫(0 to ∞) λe^(-λx)e^(tx) dx

通过对上述积分进行计算，我们可以得到矩生成函数的表达式。具体地，对于指数分布的随机变量X，其矩生成函数为：

M(t) = λ / (λ - t)

通过对矩生成函数进行不同阶数的导数，我们可以得到随机变量X的各阶矩。例如，对于一阶矩（均值）：

E[X] = M'(0) = (λ) / (λ - 0) = 1/λ

对于二阶矩（方差）：

E[X^2] = M''(0) = 2 / (λ^2)

通过类似的计算，我们可以得到更高阶的矩。

在这个例子中，矩生成函数帮助我们推导出指数分布随机变量X的均值和方差，以及其他阶数的矩。这些矩是对该随机变量性质的重要描述，通过矩生成函数，我们可以方便地计算它们的准确值。

Related Models

Introduction | Innovation | Model | Algorithm | Experiments | Conclusion

➢ ECP Approximations

demand charge $\boxed{f_t(x,\tilde{z}) = \sum_{v\in\mathcal{V}_t} x_{v,t}\tilde{z}_v.}$

Let $\mathbb{P}^\infty$ denote the joint distribution of $\tilde{z}$ for $C=\infty$. Note that $\tilde{z}_v$s for $v\in[V]$ are independent Poisson random variables. Given any $\boldsymbol{\theta} \triangleq (\theta_v)_{v\in[V]}$, the MGF of $\tilde{z}\sim\mathbb{P}^\infty$ is

$$\begin{aligned}\mathbb{E}_{\mathbb{P}^\infty}\left[\exp\left(\sum_{v\in[V]}\theta_v\tilde{z}_v\right)\right] &= \prod_{v\in[V]}\mathbb{E}_{\mathbb{P}^\infty}[\exp(\theta_v\tilde{z}_v)] \\ &= \prod_{v\in[V]}\exp\left(\lambda_v(e^{\theta_v}-1)\right),\end{aligned} \tag{4}$$

where the first equality is due to the independence of $\tilde{z}_v$'s and the second equality follows from the closed-form MGF expression of a Poisson random variable $\tilde{z}\sim$

Proof. To obtain an upper bound of π^*, we first obtain an upper bound of $\mathbb{E}_{\mathbb{P}^\infty}[\max_{t\in[T]} f_t(x,\tilde{z})]$:

$$\begin{aligned}&\mathbb{E}_{\mathbb{P}^\infty}\left[\max_{t\in[T]} f_t(x,\tilde{z})\right] \\ &= \mathbb{E}_{\mathbb{P}^\infty}\left[\max_{t\in[T]}\left(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda}) + f_t(x,\boldsymbol{\lambda})\right)\right] \\ &\le \mathbb{E}_{\mathbb{P}^\infty}\left[\max_{t\in[T]}\left(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda})\right) + \max_{t\in[T]} f_t(x,\boldsymbol{\lambda})\right] \\ &= \mathbb{E}_{\mathbb{P}^\infty}\left[\max_{t\in[T]}(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda}))\right] + \max_{t\in[T]} f_t(x,\boldsymbol{\lambda}).\end{aligned} \tag{6}$$

We then obtain an upper bound of the first term in (6) given any $\mu>0$ as follows:

$$\begin{aligned}&\mathbb{E}_{\mathbb{P}^\infty}\left[\max_{t\in[T]}(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda}))\right] \\ &\le \mu\ln\mathbb{E}_{\mathbb{P}^\infty}\left[\exp\left(\max_{t\in[T]}\left(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda})\right)/\mu\right)\right] \\ &\le \mu\ln\mathbb{E}_{\mathbb{P}^\infty}\left[\sum_{t\in[T]}\exp\left(\left(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda})\right)/\mu\right)\right] \\ &= \mu\ln\sum_{t\in[T]}\mathbb{E}_{\mathbb{P}^\infty}\left[\exp\left(\left(f_t(x,\tilde{z}) - f_t(x,\boldsymbol{\lambda})\right)/\mu\right)\right] \\ &= \mu\ln\sum_{t\in[T]}\mathbb{E}_{\mathbb{P}^\infty}\left[\exp\left(\sum_{v\in\mathcal{V}_t}\frac{x_{v,t}}{\mu}(\tilde{z}_v-\lambda_v)\right)\right] \\ &= \mu\ln\sum_{t\in[T]}\prod_{v\in\mathcal{V}_t}\mathbb{E}_{\mathbb{P}^\infty}\left[\exp\left(\frac{x_{v,t}}{\mu}\tilde{z}_v\right)\right]\exp\left(-\frac{x_{v,t}}{\mu}\lambda_v\right) \\ &= \mu\ln\sum_{t\in[T]}\exp\left(\sum_{v\in\mathcal{V}_t}\lambda_v(e^{x_{v,t}/\mu} - 1 - x_{v,t}/\mu)\right),\end{aligned} \tag{7}$$

➢ ECP Approximations

Uncapacitated Case

(Method: MGF+ECP)

Proposition 1. *When $C=\infty$, the optimal value of ECP-U gives an upper bound of π^*:*

$$\inf_{x\in\mathcal{X},\kappa,\gamma,\mu>0,\xi,\zeta} d(\kappa+\gamma)+\sum_{s\in[T]} e_s f_s(x,\boldsymbol{\lambda})$$

$$\text{s.t.} \sum_{v\in\mathcal{V}_t} x_{v,t}\lambda_v \le \gamma \qquad \forall t\in[T], \qquad (5a)$$

$$(\text{ECP-U}) \qquad \mu\exp(x_{v,t}/\mu)\le \xi_{v,t} \qquad \forall t\in[T], v\in\mathcal{V}_t, \qquad (5b)$$

$$u\exp\left(\left(-\kappa+\sum_{v\in\mathcal{V}_t}\lambda_v(\xi_{v,t}-x_{v,t}-\mu)\right)\Big/\mu\right)\le\zeta_t \qquad (5c)$$

$$\forall t\in[T],$$

$$\sum_{t\in[T]}\zeta_t\le\mu. \qquad (5d)$$

All the constraints in this model involve either linear or exponential functions and thus can be expressed as exponential cone constraints. Hence, ECP-U is an ECP, and thus can be solved via MOSEK.

Related Models

➢ ECP Approximations

Capacitated Case ($\tilde{z}\sim\mathbb{P}^C$ does not follow independent Poisson distributions)

(Method: Entropic dominance constraints +ECP)

We use the infinitely constrained "entropic dominance" ambiguity set, adapted from Chen et al. (2019):

$$\mathcal{F} \triangleq \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^V) \;\middle|\; \begin{array}{l} \tilde{z} \sim \mathbb{P} \\ \boxed{\ln \mathbb{E}_{\mathbb{P}}[\exp(\boldsymbol{\theta}'\tilde{z})] \leq \sum_{v\in[V]} \lambda_v(e^{\theta_v} - 1), \quad \forall \boldsymbol{\theta} \geq \mathbf{0}} \\ \mathbb{P}[\tilde{z} \in \mathcal{Z}] = 1 \end{array} \right\},$$

Therefore, we can obtain an upper bound of the optimal value of (3) by considering the worst-case expected total cost over the ambiguity set $\mathcal{F}$:

$$\min_{x\in\mathcal{X}} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}[c(x,\tilde{z})], \qquad \text{(DRO-Ent)}$$

Proposition 3. *When $C < \infty$, the following ECP-C gives an upper bound of π^*:*

$$\text{(ECP-C)} \quad \inf_{\substack{x\in\mathcal{X}, a, b\geq 0, \nu\geq 0, y\geq 0, \mu>0,\\ \kappa,\gamma,\alpha,\beta\geq 0, \xi,\zeta,\rho\geq 0}} d(\kappa+\gamma+\alpha+\boldsymbol{\beta}'\boldsymbol{\lambda}) + a + \boldsymbol{b}'\boldsymbol{\lambda}$$

$$\text{s.t.} \sum_{v\in\mathcal{V}_t} y_{v,t}\lambda_v \leq \gamma \quad \forall t\in[T], \qquad (13a)$$

$$(\xi_{v,t}, \mu, y_{v,t}) \in \mathcal{K}_{\exp} \quad \forall t\in[T], v\in\mathcal{V}_t, \qquad (13b)$$

$$\left(\zeta_t, \mu, -\kappa + \sum_{v\in\mathcal{V}_t} \lambda_v(\xi_{v,t} - y_{v,t} - \mu)\right) \in \mathcal{K}_{\exp} \quad \forall t\in[T], \qquad (13c)$$

$$\sum_{t\in[T]} \zeta_t \leq \mu, \qquad (13d)$$

$$C\sum_{k\in[T]} \rho_t^k \leq \alpha \quad \forall t\in[T], \qquad (13e)$$

$$x_{v,t} - y_{v,t} - \beta_v \leq \sum_{k\in\mathcal{T}_v} \rho_t^k \quad \forall t\in[T], v\in\mathcal{V}_t, \qquad (13f)$$

$$C\sum_{t\in[T]} \nu_t \leq a, \qquad (13g)$$

$$\sum_{s\in\mathcal{T}_v} x_{v,s}e_s - b_v \leq \sum_{t\in\mathcal{T}_v} \nu_t \quad \forall v\in[V]. \qquad (13h)$$

Joint Pricing and Scheduling

Price-Dependent Arrival Rate. For ease of exposition and computational tractability, we assume that the customer arrival rate is linearly decreasing in the price:

$$\lambda_v = \overline{\lambda}_v(1 - r_v p), \qquad \forall v \in [V], \tag{16}$$

Objective Function

$$\max_{p \in [\underline{p}, \overline{p}], x \in \mathcal{X}} \quad \mathbb{E}_{\mathbb{P}^C}\left[\sum_{v \in [V]} p u_v \tilde{z}_v - c(x, \tilde{z})\right]. \tag{17}$$

Problem (17) is much more challenging than the scheduling problem (3) because the underlying distribution $\mathbb{P}^C$ in (17) depends on the pricing decision p. To solve (17) efficiently, we leverage our ECP approximations for both the uncapacitated and capacitated cases.

Approximation cost allocation algorithm

Proposition 4. *When $C=\infty$, the optimal value of JPS-U is a lower bound of (17):*

$$\sup_{x\in\mathcal{X},\, p\in[\underline{p},\overline{p}],\, \mu>0,\, \boldsymbol{\lambda},\kappa,\gamma} \sum_{v\in[V]} p u_v \lambda_v - d(\kappa+\gamma) - \sum_{s\in[T]} e_s f_s(\boldsymbol{x},\boldsymbol{\lambda}) \qquad \text{(JPS-U)}$$

$$\text{s.t.} \quad (5a),(8),(16).$$

- Unlike ECP-U, which is convex, JPS-U is nonconvex because p (and equivalently λ_v) is a decision variable.
- Therefore, we propose an optimization procedure to solve JSP-U efficiently by alternating between fixing p and fixing x and μ.

Algorithm 1 (Alternating Optimization for JPS-U)

1. **Initialization** Set initial price $p^{(0)} \in [\underline{p},\overline{p}]$, iteration counter $i \leftarrow 1$
2. **Scheduling optimization** Solve Model (18) with input $p^{(i-1)}$ and let $\boldsymbol{x}^*$ and μ^* be the optimal solution; set $\boldsymbol{x}^{(i-1)} \leftarrow \boldsymbol{x}^*$ and $\mu^{(i-1)} \leftarrow \mu^*$. Store optimal value $val_1^{(i)}$;
3. **Pricing optimization** Solve Model (19) with inputs $\boldsymbol{x}^{(i-1)}$ and $\mu^{(i-1)}$, and let p^* be the optimal solution; set $p^{(i)} \leftarrow p^*$. Store optimal value $val_2^{(i)}$;
4. **Termination** If $|val_2^{(i)} - val_1^{(i)}| < \delta$ (where $\delta > 0$ is a given small tolerance), set $p^* \leftarrow p^{(i)}$, solve Model (18) with input $p^{(i)}$ to obtain $\boldsymbol{x}^*$, output p^* and $\boldsymbol{x}^*$, and then stop. Otherwise, set $i \leftarrow i+1$ and go back to Step 2.

Output: Pricing decision p^* and scheduling decision $\boldsymbol{x}^*$

Proposition 5. *The sequence of optimal values $\{val_1^{(i)}, val_2^{(i)}\}$ in Algorithm 1 is nondecreasing and converges to a finite value.*

Numerical experiment

Settings

- t=15min, T=96
- 14,284 customer types (v)
- U=62 kWh;
- K=10.75kWh/period
- η=0.9

Unit charge

$$\hat{e}_t = \begin{cases} \$0.1466/\text{kWh} & \text{if } 13 \le \lceil t/4 \rceil \le 18 \text{ (on-peak hours)} \\ \$0.0895/\text{kWh} & \text{if } 9 \le \lceil t/4 \rceil \le 12 \text{ or } 19 \le \lceil t/4 \rceil \le 23 \\ & \text{(mid-peak hours)} \\ \$0.0582/\text{kWh} & \text{otherwise (off-peak hours),} \end{cases}$$

$$\hat{d}_t = \begin{cases} \$0.465/\text{kW} & \forall t \in [T] \doteq [96] \text{ (all-period)} \\ \$0.540/\text{kW} & \text{if } 13 \le \lceil t/4 \rceil \le 18 \text{ (on-peak hours)} \\ \$0.165/\text{kW} & \text{if } 9 \le \lceil t/4 \rceil \le 12 \text{ or } 19 \le \lceil t/4 \rceil \le 23 \\ & \text{(mid-peak hours),} \end{cases}$$

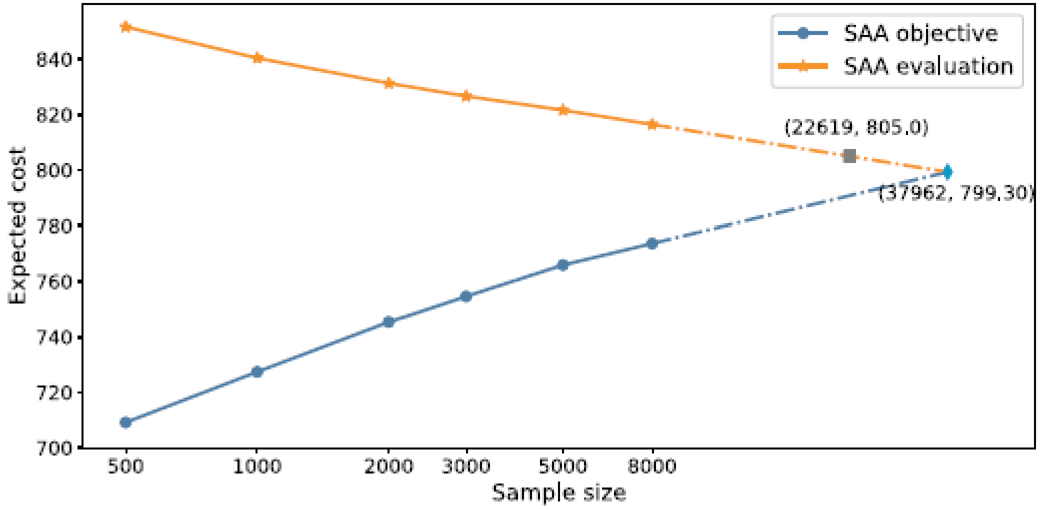
Numerical experiment

Table 1. Performance Comparison Between ECP-C (or ECP-U) and SAA

C	Method	Total cost: Mean	Total cost: Standard deviation	CPU Time (s)
15	ECP-C	600.40	35.20	312.33
	SAA	608.91	36.76	13,015.66
20	ECP-C	717.73	44.42	354.66
	SAA	727.38	46.22	15,806.33
25	ECP-C	783.66	57.13	445.66
	SAA	794.04	59.44	16,207.17
30	ECP-C	805.00	67.55	320.56
	SAA	816.39	69.94	16,005.50
∞	ECP-U	808.90	71.30	194.22
	SAA	819.80	73.64	17,673.80

SAA (8000 samples)

Figure 3. (Color Online) Expected Cost Under SAA at Different Sample Sizes Given $C = 30$



- ECP is faster and better than SAA, DRO and other methods;
- ECP is not limited by sample size.

Numerical experiment

- **Opt-15 min To 1 min policy**: the 1-minute charging policy transformed via Algorithm 2 from the optimal charging policy solved on 15-minute discretization.

- **Opt-1-min policy**: the optimal charging policy solved directly on 1-minute discretization. Using the optimal policies allows

Table 3. Performance over a One-Hour Time Horizon (5 a.m. to 6 a.m.) with 1-Minute Discretization

Hour	Policy	Total cost Mean	Total cost Standard deviation	E[Demand charge/ total cost]
5 a.m. to 6 a.m.	Opt-1-min	1.046	1.144	0.361
	Opt-15minTo1min	1.061	1.163	0.370

- **Algorithm 2 is very efficient and greatly reduces the computation time.**

Algorithm 2 (Transform a 15-Minute Charging Policy $\boldsymbol{x}$ into a 1-Minute Implementable Policy $\boldsymbol{y}$)

Input: 15-minute charging policy $x_{v,t}$

for *1-minute customer type* $v' \in [V']$ **do**
  set $s_v \leftarrow \lceil s_{v'}/15 \rceil$, $\tau_v \leftarrow \lceil \tau_{v'}/15 \rceil$, $v \leftarrow (s_v, \tau_v, u_{v'})$;
  for *15-minute period* $t \in \{s_v, \ldots, \tau_v\}$ **do**
    compute the actual 1-minute stay duration S_t of EV of type v' in period t;
    compute $\overline{x}_{v,t} \leftarrow K/(15\eta) \cdot S_t$;
  end
  if $s_v < \tau_v$ **then**
    compute the excess charging quantity in *arrival* period $E \leftarrow \max\{0, x_{v,s_v} - \overline{x}_{v,s_v}\}$;
    update $x_{v,s_v} \leftarrow x_{v,s_v} - E$; set $t \leftarrow s_v + 1$;
    while $E > 0$ **do**
      update $x_{v,t} \leftarrow x_{v,t} + E$; compute $E \leftarrow \max\{0, x_{v,t} - \overline{x}_{v,t}\}$;
      update $x_{v,t} \leftarrow x_{v,t} - E$; set $t \leftarrow t + 1$;
    end
    compute the excess charging quantity in *departure* period $E \leftarrow \max\{0, x_{v,\tau_v} - \overline{x}_{v,\tau_v}\}$;
    update $x_{v,\tau_v} \leftarrow x_{v,\tau_v} - E$; set $t \leftarrow \tau_v - 1$;
    while $E > 0$ **do**
      update $x_{v,t} \leftarrow x_{v,t} + E$; compute $E \leftarrow \max\{0, x_{v,t} - \overline{x}_{v,t}\}$;
      update $x_{v,t} \leftarrow x_{v,t} - E$; set $t \leftarrow t - 1$;
    end
  end
  for *1-minute period* $t' \in \{s_{v'}, \ldots, \tau_{v'}\}$ **do**
    set $t \leftarrow \lceil t'/15 \rceil$; set $y_{v',t'} \leftarrow x_{v,t}/S_t$;
  end
end

Output: 15minTo1min implementable charging policy $y_{v',t'}$

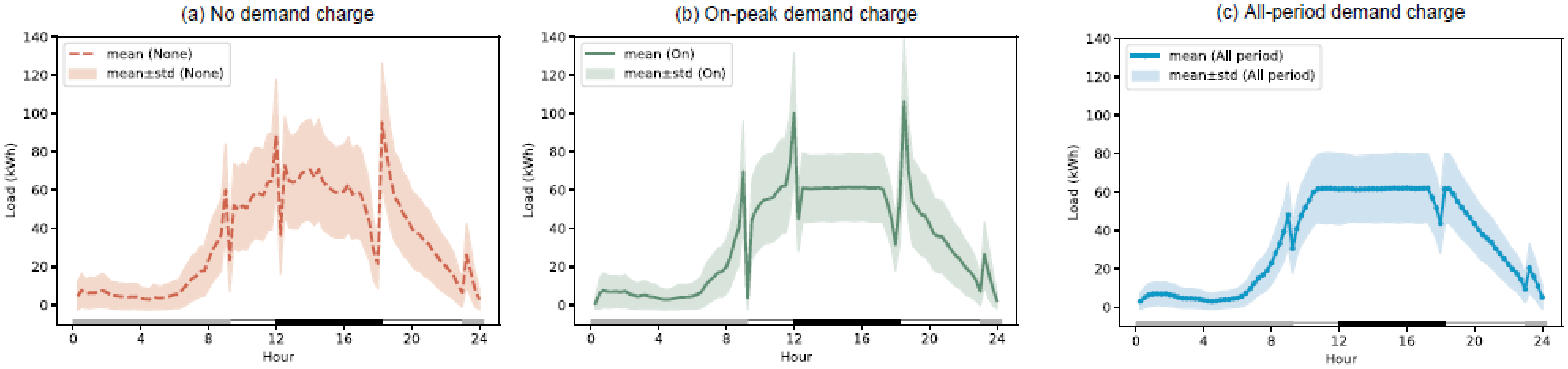
Numerical experiment

Table 11. Performance Improvement of ECP-C over the Maximum-Speed Charging Policy

	Total cost		Expected maximum load			
C	Mean	Standard deviation	Overall	On-peak	Mid-peak	Off-peak
15	13.60%	27.87%	23.64%	23.56%	21.43%	10.33%
20	12.12%	25.49%	20.99%	20.68%	18.56%	8.41%
25	11.22%	21.80%	18.91%	19.00%	16.03%	8.70%
30	10.99%	19.02%	17.94%	18.89%	13.65%	8.59%

- Without making decisions about charging speed, the cost increases a lot.
- The charging load of the ECP is smoother, which is conducive to the battery maintenance of the EV.

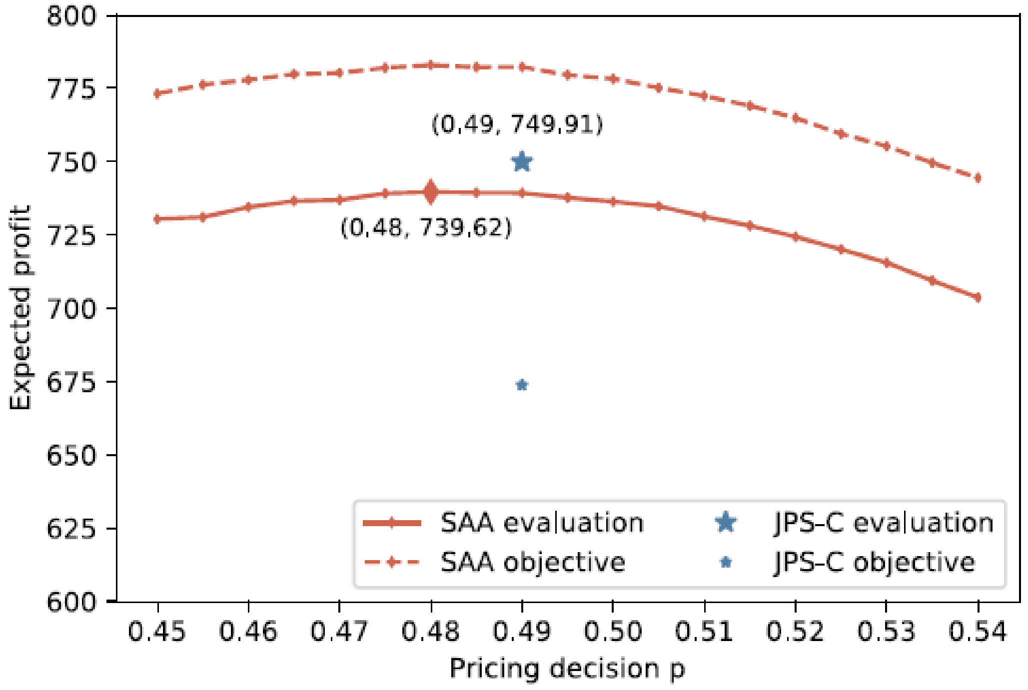
Figure 5. (Color Online) Electricity Load Under ECP-C for Different Compositions of the Demand Charge Given $C = 30$



- Electricity load becomes smooth when consider all-period demand charge.

Numerical experiment

Figure 7. (Color Online) Expected Profits Under SAA and JPS-C



- The optimal price $p = 0.48$ under SAA leads to an evaluation of the expected profits under SAA to be 739.62, lower than the expected profits under JPSC, which is 749.91.

Conclusion

◆ **Main Contribution**

- Model fits well
- ECP performs well
- Extension studies well

Thanks for

Your Listening