Optimal Robust Policy for Feature-Based Newsvendor

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Motivation

- Temporal, spatial, social, or economical features;
- Objective:

$$\min_{f:\mathcal{X}\to\mathcal{Z}} \mathbb{E}_{\mathbb{P}}[\Psi(f(X), Z)]$$

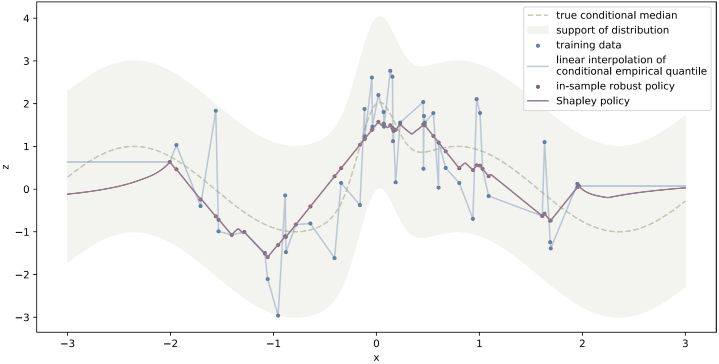
- ERM problem: pathological policy

$$\hat{\mathbb{P}} = \frac{1}{n}\sum_{k=1}^{K}\sum_{i=1}^{n_k}\delta_{(\hat{x}_k,\hat{z}_{ki})}$$

  * $n_k = 1, \hat{f}(\hat{x}_k) = \hat{z}_{k1}$ in most cases with continuous features;
  * Parameterize: trade-off between efficiency and interpretability;
  * RO: needless to restrict on a parametric family.

An Intuitive Numerical Example

- ERM: Weierstrass approximation theorem: take arbitrary values on unseen feature values;
- Shapley extension: Adjustable robust optimization, smaller Lipschitz norm.



Distributional Robust Formulation

Infinite-dimension DRO formulation:

$$v_P := \inf_{f \in \mathcal{F}} \sup_{\mathbb{P} \in \mathcal{P}_1(\mathcal{X} \times \mathcal{Z})} \{\mathbb{E}[\Psi(f(X), Z)] : \mathcal{W}(\mathbb{P}, \hat{\mathbb{P}}) \leq \rho\}$$

Solution approach: two steps.

I. Solve the in-sample primal problem $\hat{v}_P$, obtain $\hat{f}^* : \hat{\mathcal{X}} \to \mathcal{Z}$;

II. For general feature values, calculate the Shapley Extension

$$f^*(x) = \min_j \max_k A_{jk}(x)$$

$$A_{jk}(x) := \frac{\|x - \hat{x}_k\|}{\|x - \hat{x}_k\| + \|x - \hat{x}_j\|} \hat{f}^*(\hat{x}_j) + \frac{\|x - \hat{x}_j\|}{\|x - \hat{x}_k\| + \|x - \hat{x}_j\|} \hat{f}^*(\hat{x}_j)$$

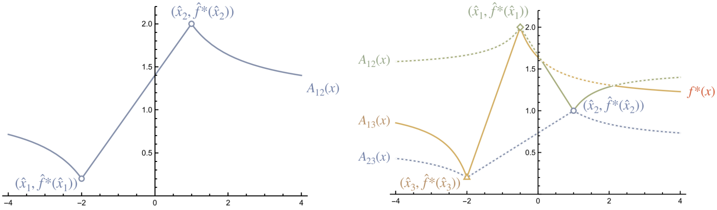
Shapley Extension

Basic properties.

- $K = 2$, line segment betweenn $\hat{x}_1$ and $\hat{x}_2$;
- $K = 2, 3$, converge to non-informative value:

$$(\max_k f(\hat{x}_k) + \min_k f(\hat{x}_k))/2$$

Figure 2. (Color online) Graph of the Shapley Policy $y = f(x)$ When $K = 2, 3$, $x \in \mathbb{R}$



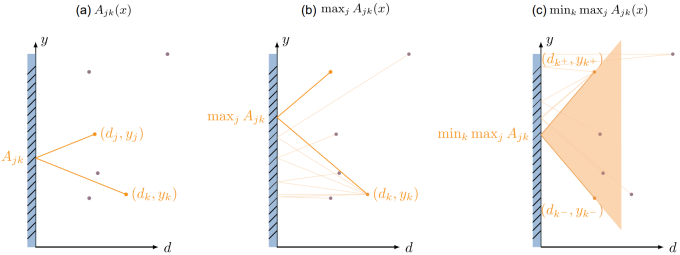
Shapley Extension

Slope Minimization. SE is the close-form solution of slope minimization problem

$$f^*(x) = \arg\min_{y\in\mathbb{R}} \left\{ \max_{k\in[K]} \frac{|\hat{f}^*(\hat{x}_k) - y|}{\|\hat{x}_k - x\|} \right\}$$

Note: $v_P = v_{\hat{P}}$ under slope minimization.

Figure 3. (Color online) Visualization of the Shapley Saddle Point



Lipschitz Regularization

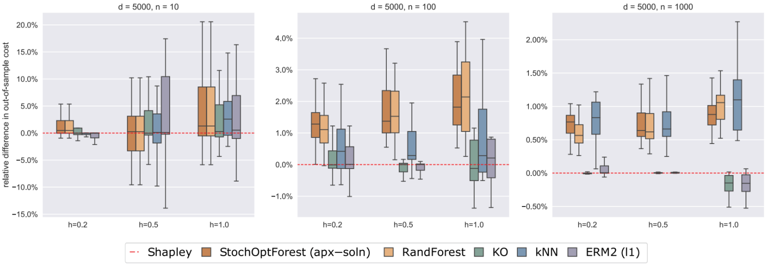
$$v_R := \min_{f \in \mathcal{F}} \left\{ (b \vee h)(1 \vee \|f\|_{\mathsf{Lip}})\rho + \mathbb{E}_{\hat{\mathbb{P}}}[\hat{\Psi}(f(\hat{X}), \hat{Z})] \right\}$$

- Equivalance between Lip-regularization & Wasser-DRO: **(Gao et al., 2022)**;
- $v_P = v_R$ under Shapley extension;

Out-of-sample Performance

- Benchmark: Forest, KO, KNN, ERM;
- Shapley outperforms other benchmarks in few-shot learning;
- KO & ERM outperforms Shapley when $n = 1000$.

Figure 5. (Color online) Box Plots of the Relative Differences in the Out-of-Sample Performance Between `Shapley` and Other Benchmarks (Arranged from Left to Right as in Legends)



Conclusion

- Motivation: balances between the variation of the ordering quantity with respect to features and the expected cost;
- Methods: Shapley extension, a novel family of policies for adjustable robust optimization.

Future Work

- Solve other problems in Adjustable-DRO manner;
    - End-to-end inventory management, an intuitive direction;
    - Finance: adjustable ambiguity set of risk aversion;
    - Transportation: traffic congestion;
    - Energy: variability of renewable energy sources.
- Shapley extension & Lipschitz regularization.

Thank You!