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Data-driven platelet inventory management under uncertainty in the remaining shelf life of units

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Introduction

Background:

- Platelet transfusions are required for the treatment of patients with blood disorders as well as a variety of other clinical treatments.

- In most North American hospitals, the required platelets are procured from a central supplier serving multiple hospitals, for example, the Red Cross in the United States or the Canadian Blood Services (CBS) in Canada.

- Determining the ordering quantities at the hospital involves managing a trade-off between wastage and shortage costs over time.

Introduction

- Problem description:
  - Propose data-driven approaches to inventory management of platelets for hospitals that order their blood from a central supplier.
  - The shelf life of units is fixed, but there is uncertainty in the initial age.
  - Demand is satisfied according to the oldest-unit, first-out (OUFO) allocation policy, that is, using the oldest units available. (Note that the OUFO policy is equivalent to first-in, first-out (FIFO) when all units have the same remaining age.)
  - The objective is to determine the daily ordering quantities that minimize a weighted sum of expiry and shortage rates incurred over a finite horizon.

Uncertain remaining shelf life

- Characteristic 1: Uncertain remaining shelf life
  - Consider the uncertainty in the remaining shelf life (or initial age) of the delivered units.
  - The historical initial age values are realized under the ordering decisions of the hospital and could be endogenous, that is, depend on the order quantity.

Daily demand

Characteristic 2: Daily demand

- The required inventory levels are assumed to be a linear function of the observed values of the features in that period and are estimated through an empirical risk minimization (ERM) framework.

- More specifically, the coefficients of the linear model are obtained by minimizing an empirical estimate of the inventory costs (with regularization), comprising a weighted sum of wastage and shortage costs. (OR 2019, PNAS 2017)

Contents

Big data newsvendor (OR 2019)

Big data platelet usage (PNAS 2017)

Contents

Data

Data

- Transfusion Registry for Utilization, Surveillance, and Tracking (TRUST) database.
- Platelet inventory data and clinical data for the transfused patients.
- Hamilton General Hospital (HGH) and Juravinski Hospital (JH).

Inventory		Patient
Hospital identifier	⇒	Gender
Product name		Age
Product unit number		Patient ABO group
Product ABO group		Patient Rh type
Product Rh type		Admission date
Collection date		Admission time
Receipt date		Admission type
Receipt time		Discharge date
Expiry date		Discharge time
Issue date		Discharge type
Status change date		Location (e.g., ward or ICU)
Final disposition		Special requirements
		Lab results (e.g., platelet count)

FIGURE 1 Structure of the TRUST dataset

Variability in the remaining age

- Uncertainty:
  - Remaining age (or equivalently initial age) of delivered units is subject to significant uncertainty at both hospitals.

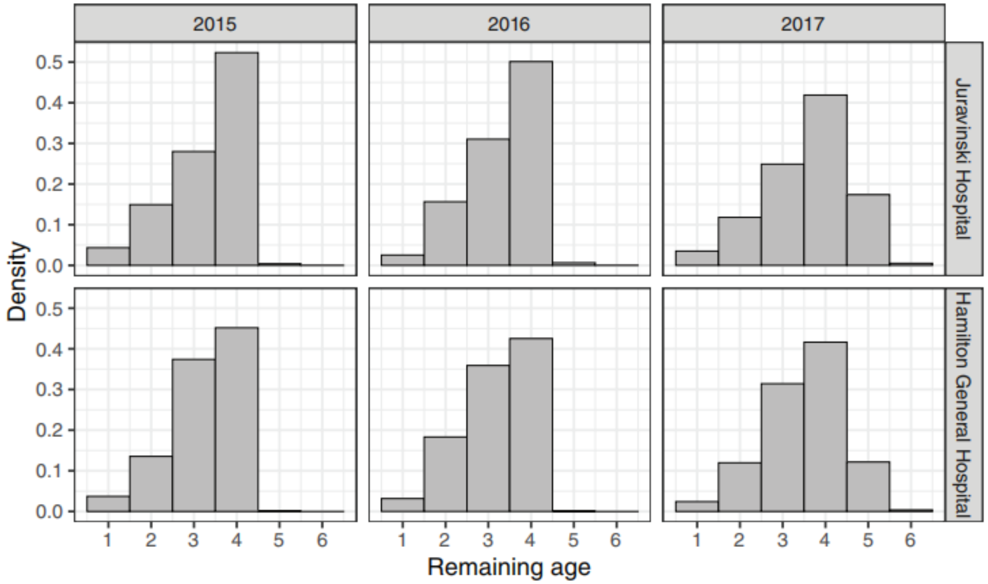

FIGURE 2 Remaining age distribution of received units at the two hospitals

Variability in the remaining age

Multinomial logistic regression:

- Multinomial Logistic Regression是一种广义线性模型，用于多分类问题。在该模型中，假设有k个可能的输出类别，使用它来建立一个模型，根据输入变量来预测每个类别的概率。

- Assuming that the log odds ratio of received units with remaining ages 2, 3, 4, and 5 days versus 1 day is a linear function of certain explanatory.

$$\log\left(\frac{P(R_i = k)}{P(R_i = 1)}\right) = c_0^k + c_1^k x_i + \sum_{i=1}^{6} d_i^k A_\alpha + \sum_{j=1}^{11} m_j^k M_j \, , \forall k \in \{2,3,4,5\},$$

where $P(R_t = k)$ is the probability of receiving a unit with the remaining age of k days on day t; x_t is the order size on day t, and A_{it} and M_{jt} are the indicators of day-of-week and month of-year, respectively.

Variability in the remaining age

- Multinomial logistic regression:
  - Time-dependent
  - Order size has a statistically significant effect on the age of delivered units, suggesting that the age distribution of the delivered units could be endogenous.

Table EC.1 **Fitted multinomial logistic regression to the remaining age data for HGH in 2017.**

	Remaining Age			
	2	3	4	5
c_0	5.661***	6.352***	4.310***	−11.896***
	(1.220)	(1.204)	(1.207)	(0.687)
c_1	−0.078*	−0.112***	−0.027	−0.044
	(0.043)	(0.041)	(0.041)	(0.046)
Tue	−1.359**	−1.302**	−1.971***	−1.484**
	(0.538)	(0.520)	(0.536)	(0.644)
Wed	−0.792	−2.689***	−0.039	−3.696***
	(0.562)	(0.568)	(0.547)	(1.174)
Thu	−2.707***	−1.409***	−0.187	1.637***
	(0.616)	(0.537)	(0.535)	(0.622)
Fri	−1.380*	−0.152	1.358*	3.384***
	(0.791)	(0.732)	(0.729)	(0.797)
Sat	17.124***	18.493***	20.070***	21.616***
	(0.324)	(0.183)	(0.171)	(0.295)
Sun	−1.553	−3.160**	−0.577	2.562*
	(1.247)	(1.265)	(1.192)	(1.318)
Feb	−1.542	−1.062	−0.777	−7.644***
	(1.184)	(1.172)	(1.178)	(0.00000)
Mar	−2.522**	−1.891*	−1.176	−9.739***
	(1.094)	(1.079)	(1.082)	(0.00000)
Apr	−2.027	−0.382	−0.322	−7.041***
	(1.273)	(1.243)	(1.247)	(0.00000)
May	−2.951***	−2.474**	−1.670	−8.317***
	(1.092)	(1.076)	(1.077)	(0.00000)
Jun	−2.609**	−2.588**	−1.858*	−8.580***
	(1.105)	(1.094)	(1.098)	(0.00000)
Jul	−2.372**	−1.499	−1.561	−5.687***
	(1.158)	(1.140)	(1.146)	(0.00000)
Aug	−1.936	−0.788	−0.197	14.308***
	(1.268)	(1.242)	(1.247)	(0.680)
Sep	−2.632*	−1.416	0.832	16.644***
	(1.552)	(1.469)	(1.451)	(0.916)
Oct	12.219***	12.865***	14.803***	30.577***
	(0.394)	(0.332)	(0.310)	(0.702)
Nov	−3.798***	−2.856***	−2.077*	13.020***
	(1.100)	(1.074)	(1.075)	(0.427)
Dec	−1.704	−1.775	−1.097	13.213***
	(1.145)	(1.138)	(1.142)	(0.540)
Akaike Inf. Crit.	4,199.189	4,199.189	4,199.189	4,199.189

Note: *p<0.1; **p<0.05; ***p<0.01

Variability in demand

■ Transfusion records for platelets:

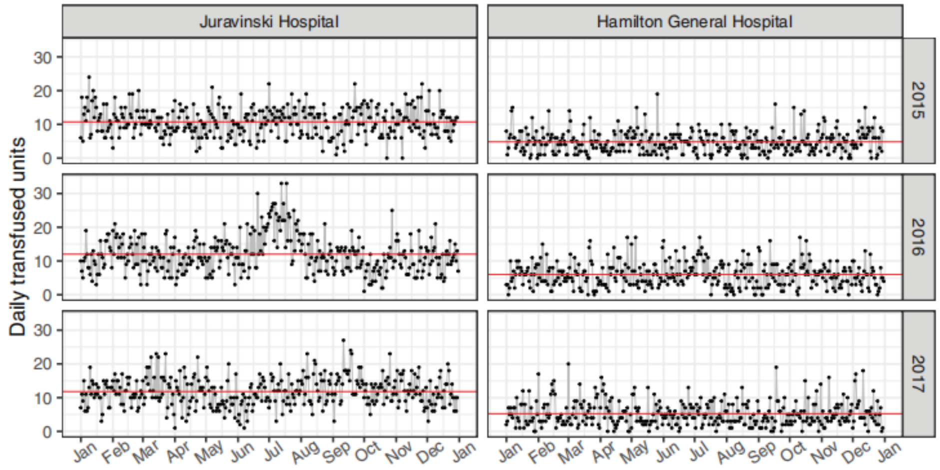

FIGURE 3 Daily usage of platelet units at HGH and JH. The red line is the estimated average daily usage [Color figure can be viewed at wileyonlinelibrary.com]

- Daily transfusions (demand) at HGH were on average 5.4 units with a standard deviation of 3.6, while JH had a daily average demand of 11.5 units with a standard deviation of 4.7.

- Daily transfusions (demand)? Equivalent?

Variability in demand

- Construction of features:
  - 23 time-dependent binary variables: the day-of-week (6/7?), month-of-year (11/12?), and national-holidays status (1/4?). One day before and up to two days after national holidays are considered as well.
  - Average daily usage during the last 7 days.
  - Number of patients with abnormal platelet count.
  - 27 covariates are then created based on the available information in the Admission data and Inventory data.

Variability in demand

Linear regression model for daily demand:

- Our basic regression model is:

$$D_t = c_0 + c_1 z_{1t} + c_2 z_{2t} + \sum_{i=1}^{6} d_i A_{it} + hH_t + \epsilon_t,$$

- The estimates are presented:

Table EC.2 Estimated coefficients of the linear demand regression model based on data for HGH in 2015-2016.

	Dependent variable:	
	demand	Std. Error
c_0	3.256***	0.686
c_1	0.288***	0.080
c_2	0.047**	0.023
d_1	0.684	0.450
d_2	0.335	0.447
d_3	−0.128	0.454
d_4	−0.402	0.460
d_5	−3.062***	0.469
d_6	−2.773***	0.448
h	−3.444***	0.719
R^2	0.204	
F Statistic	20.307***	
Note:	*p<0.1; **p<0.05; ***p<0.01	

Variability in demand

- Linear regression including all 52 features:
  - Linear least-squares regression model (with or without regularization) has modest predictive power, even with all 52 features.
  - As such, determining the ordering quantities using the predictions leads to poor performance.
  - As we illustrate in Section 8, our proposed models can achieve high performance by directly taking the downstream ordering decision problem into account when predicting the demand.

Hospital	Perf. Measure	Lasso		Least square	
		Training	Test	Training	Test
HGH	MSE	9.53	11.43	8.98	12.31
	R^2	0.22	$\boxed{0.17}$	0.26	0.11
JH	MSE	12.62	13.04	11.36	15.48
	R^2	0.47	$\boxed{0.32}$	0.52	0.20

Table EC.3 Prediction accuracy of the linear regression model with all 52 covariates.

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Notations

Notations	Meaning	Notations	Meaning
m	maximum shelf (e.g., days)	$X_{t,i}$	inventory level of units with a remaining shelf life of $i \in \{1, \dots, m-1\}$ days.
T	planning horizon	$\mathbf{Z} = (\mathbf{Z}_1, \dots, \mathbf{Z}_T)$	information state
$\mathbf{D} = (D_1, \dots, D_T)$	demand	$\mathbf{Q} = (Q_1, \dots, Q_T)$	the starting inventory (after receiving the order)
$\mathbf{X}_t = (X_{t,1}, \dots, X_{t,m-1})$	The inventory state at the beginning of period t (before receiving the order)	$\mathbf{Y}_t = (Y_{t,1}, \dots, Y_{t,m})$	number of received units whose remaining shelf life is $i \in \{1, \dots, m\}$

$$\sum_{i=1}^{m} Y_{t,i} = Q_i - \sum_{i=1}^{m-1} X_{t,i} \, ,$$

Sequence of events in period *t*

- Inventory dynamics:
  - 1. At the beginning of period t, the order $\sum_{i=1}^{m} Y_{t,i} = Q_t - \sum_{i=1}^{m-1} X_{t,i}$ arrives.
  - 2. Demand in period t is realized and satisfied according to the OUFO allocation policy, that is, using the oldest units available in the inventory.
  - 3. Unsatisfied demand is lost. Units with a remaining shelf life of one are expired, and the remaining units are carried over to the next period with their remaining shelf life decreased by one:

$$X_{t+1,j} = (X_{t,j+1} + Y_{t,j+1} - (D_t - \sum_{i=1}^{j}(X_{t,i} + Y_{t,i}))^+)^+$$
$$j \in \{1, \ldots, m-2\},$$
$$X_{t+1,m-1} = (Y_{t,m} - (D_t - \sum_{i=1}^{m-1}(X_{t,i} + Y_{t,i}))^+)^+.$$

  - 4. The order for period $t+1$ is placed at the end of period t.

Sequence of events in period t

- Two types of costs:
  - Base-stock ordering policies in the form of $\pi = (Q_1, \ldots, Q_T)$
  - Each period t an order of size $Q_t - \sum_{i=1}^{m-1} X_{t,i}$
  - $C(\pi) = \sum_{t=1}^{T} (X_{t,1}^{\pi} + Y_{t,1}^{\pi} - D_t)^+ + c(D_t - Q_t^{\pi})^+$,

Approach

- ERM:

  - Required inventory (critical level) for each period is a linear function of the observed information state z_t in that period: $Q := \{q : \mathcal{Z} \to \mathbb{R}_+ ; q(\mathbf{z}_t) = \mathbf{z}_t \cdot \boldsymbol{\beta}\}$

  - Finding the best linear decision rule that minimizes the regularized empirical risk with a sparsity penalty similar to the Lasso:

$$\min_{\boldsymbol{\beta}} C_n(\boldsymbol{\beta}) + \lambda \parallel \boldsymbol{\beta} \parallel_1,$$

where $C_n(\boldsymbol{\beta})$ is the empirical estimate of the cost based on n periods of training data.

Contents

Fixed initial age model

- Considering the case:
  - Remaining age of delivered units is assumed to be fixed and equal to $\mu \in \{1, \dots, m\}$.
  - OUFO allocation policy coincides with FIFO.
  - The obtained decision rule serves as an approximation for the case where the remaining age is subject to variability.
  - In the following, we assume without loss of generality that $\mu = m$.

Fixed initial age model

- Model:
  - $s_{t,i}$ the number of units with the remaining shelf life of i days used to satisfy the demand in period t.
  - Primarily concerned with the case where the initial age is not fixed, we treat μ as a "hyperparameter" and choose its value together with λ through cross-validation.
  - Linearize the sparsity penalty by replacing the term $\lambda \parallel \beta \parallel_1 = \lambda \sum_{k=1}^{p} \mid \beta_k \mid$ in the objective with the constraint $\sum_{k=1}^{p} \mid \beta_k \mid \leq \epsilon$, and instead choose ϵ through cross-validation.

$$\min_{\beta,\{s_{t,i}\}} \sum_{t=1}^{n} (w_t + c \times l_t) + \lambda \|\beta\|_1, \tag{7}$$

subject to,

$$q_t = \mathbf{z}_t \cdot \beta \quad \forall t \in \mathcal{N}, \tag{8}$$

$$y_t = q_t - \sum_{i=1}^{m-1} x_{t,i} \quad \forall t \in \mathcal{N}, \tag{9}$$

$$w_t = x_{t,1} - s_{t,1} \quad \forall t \in \mathcal{N}, \tag{10}$$

$$x_{t+1,j} = x_{t,j+1} - s_{t,j+1} \quad \forall j \in \{1, \ldots, m-2\}, t \in \mathcal{N}, \tag{11}$$

$$x_{t+1,m-1} = y_t - s_{t,m} \quad \forall t \in \mathcal{N}, \tag{12}$$

$$\sum_{i=1}^{m} s_{t,i} + l_t = d_t \quad \forall t \in \mathcal{N}, \tag{13}$$

$$l_t \geq 0 \quad \forall t \in \mathcal{N}, \tag{14}$$

$$w_t \geq 0 \quad \forall t \in \mathcal{N}, \tag{15}$$

$$s_{t,i} \geq 0 \quad \forall i \in \{1, \ldots, m\}, t \in \mathcal{N}, \tag{16}$$

$$x_{t+1,j} \geq 0 \quad \forall j \in \{1, \ldots, m-1\}, t \in \mathcal{N}. \tag{17}$$

Fixed initial age model

- Optimal policy :

Proposition 1. For a given $\boldsymbol{\beta}$, if the FIFO allocation policy is feasible for (7)–(17), then it must also be optimal.

- The result holds since regardless of the value of $\boldsymbol{\beta}$ allocating the oldest units minimizes both the number of units wasted and lost demand.

Fixed initial age model

- Note:

  - For a given $\boldsymbol{\beta}$, the FIFO policy does not necessarily satisfy the nonnegativity of the order quantity y_t in (9).

$$y_t = q_t - \sum_{i=1}^{m-1} x_{t,i} \quad \forall t \in \mathcal{N}, \tag{9}$$

  - This can be addressed by replacing (9) with $y_t = (q_t - \sum_{i=1}^{m-1} x_{t,i})^+$ to ensure that order quantities remain nonnegative under FIFO. However, doing so results in a Mixed Integer Program (MIP) that is computationally hard to solve.

  - Instead, we assume that if the required inventory determined for a period is smaller than the inventory level, no orders are placed in that period. Empirically, in all of our numerical experiments, this approach results in the same cost as the optimal cost of the fixed initial age problem.

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Worst-Case cost under remaining age uncertainty

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- Setting:
  - Examine the worst-case cost of ignoring the variability in the remaining age of orders for a given linear decision.
  - Given the optimal coefficients of the fixed initial age model, $\bar{\boldsymbol{\beta}}$, we compute the corresponding worst-case out-of-sample performance over a family of remaining age distributions including those that are decision dependent.
  - Uncertainty set:

$$\mathcal{Y} := \left\{ \mathbf{y} \in \mathbb{R}_+^{T\times m};\ \sum_{t=1}^{T} y_{t,i} = \hat{p}_i \times \sum_{t=1}^{T} y_t, \quad \forall i \in \{1, \ldots, m\}, \right.$$
$$\left. \sum_{i=1}^{m} y_{t,i} = y_t, \quad \forall t \in \{1, \ldots, T\} \right\}. \qquad (18)$$

where $\hat{p}_i$ is an estimate of the fraction of delivered units with remaining age $i \in \{1, 2, \ldots, m\}$ over the planning horizon, which can, for example, be taken as the empirical distribution of the initial age values observed in historical data.

Worst-Case cost under remaining age uncertainty

- The adversary's problem:
  - Select the remaining age values of the ordered units over the planning horizon from the uncertainty set, so as to maximize the total cost:
  - The adversary's problem is nonconvex but can be reformulated as a MIP.

$$W_T(\bar{\beta}) := \max_{\mathbf{y}} \sum_{t=1}^{T} (w_t + c \times l_t) \tag{19}$$

subject to,

$$y_t = \left(\mathbf{z}_t \cdot \bar{\beta} - \sum_{i=1}^{m-1} x_{t,i} \right)^+ \quad \forall t \in \mathcal{T}, \tag{20}$$

$$l_t = \left(d_t - \sum_{i=1}^{m-1} x_{t,i} - y_t \right)^+ \quad \forall t \in \mathcal{T}, \tag{21}$$

$$w_t = (x_{t,1} + y_{t,1} - d_t)^+ \quad \forall t \in \mathcal{T}, \tag{22}$$

$$x_{t+1,j} = \left(x_{t,j+1} + y_{t,j+1} - \left(d_t - \sum_{i=1}^{j} (x_{t,i} + y_{t,i}) \right)^+ \right)^+ \quad \forall j \in \{1, \dots, m-2\}, t \in \mathcal{T}, \tag{23}$$

$$x_{t+1,m-1} = \left(y_{t,m} - \left(d_t - \sum_{i=1}^{m-1} (x_{t,i} + y_{t,i}) \right)^+ \right)^+ \quad \forall t \in \mathcal{T}, \tag{24}$$

$$\mathbf{y} \in \mathcal{Y}. \tag{25}$$

Worst-Case cost under remaining age uncertainty

MIP Reformulation:

- For instance:

$$y_t = (\mathbf{z}_t \cdot \bar{\beta} - \sum_{i=1}^{m-1} x_{t,i})^+ \forall t \in \mathcal{T},$$

Can be expressed as follows:

$$y_t = \mathbf{z}_t \cdot \bar{\boldsymbol{\beta}} - \sum_{i=1}^{m-1} x_{t,i} + v_t^y, \forall t \in \mathcal{T},$$

$$y_t \le M\delta_t^y, \forall t \in \mathcal{T},$$

$$v_t^y \le M(1-\delta_t^y), \forall t \in \mathcal{T},$$

$$\delta_t^v \in \{0,1\}, \forall t \in \mathcal{T}, v_t^y, \forall t \in \mathcal{T}, y_t \ge 0, \forall t \in \mathcal{T}$$

- Can be solved using a solver (e.g., Gurobi) that supports ordered sets of Type 1 (SOS-1) constraints as well.

$$SOS1(v_t^y, \delta_t^y) \forall t \in \mathcal{T},$$

- To reduce the computational time, we further add additional logical constraints to the model Hooker et al. (1994).

Worst-Case cost under remaining age uncertainty

- MIP Reformulation:

$$W_T(\bar{\beta}) := \max_{\mathbf{y}} \sum_{t=1}^{T} (w_t + c \times l_t) \qquad (29)$$

subject to,

$$y_t = \mathbf{z}_t \cdot \bar{\beta} - \sum_{i=1}^{m-1} x_{t,i} + v_t^y \quad \forall t \in \mathcal{T}, \qquad (30)$$

$$l_t = d_t - \sum_{i=1}^{m-1} x_{t,i} - y_t + v_t^l \quad \forall t \in \mathcal{T}, \qquad (31)$$

$$w_t = x_{t,1} + y_{t,1} - d_t + v_t^w \quad \forall t \in \mathcal{T}, \qquad (32)$$

$$x_{t+1,1} = \sum_{i=1}^{2} (x_{t,i} + y_{t,i}) - d_t - w_t + v_t^{x_1} \quad \forall t \in \mathcal{T}, \qquad (33)$$

$$x_{t+1,j} = \sum_{i=1}^{j+1} (x_{t,i} + y_{t,i}) - d_t - w_t - \sum_{i=1}^{j-1} x_{t+1,i} + v_t^{x_j}$$
$$\forall j \in \{2, \dots, m-2\}, t \in \mathcal{T}, \qquad (34)$$

$$x_{t+1,m-1} = y_t + \sum_{i=1}^{m-1} x_{t,i} - d_t - w_t$$
$$- \sum_{i=1}^{m-2} x_{t+1,i} + v_t^{x_{m-1}} \quad \forall t \in \mathcal{T}, \qquad (35)$$

$$y_t \le M\delta_t^y \quad \forall t \in \mathcal{T}, \qquad (36)$$

$$v_t^y \le M(1 - \delta_t^y) \quad \forall t \in \mathcal{T}, \qquad (37)$$

$$l_t \le M\delta_t^l \quad \forall t \in \mathcal{T}, \qquad (38)$$

$$v_t^l \le M(1 - \delta_t^l) \quad \forall t \in \mathcal{T}, \qquad (39)$$

$$w_t \le M\delta_t^w \quad \forall t \in \mathcal{T}, \qquad (40)$$

$$v_t^w \le M(1 - \delta_t^w) \quad \forall t \in \mathcal{T}, \qquad (41)$$

$$x_{t+1,j} \le M\delta_t^{x_j} \quad \forall j \in \{1, \dots, m-1\}, t \in \mathcal{T}, \qquad (42)$$

$$v_t^{x_j} \le M(1 - \delta_t^{x_j}) \quad \forall j \in \{1, \dots, m-1\}, t \in \mathcal{T}, \qquad (43)$$

$$\delta_t^y, \delta_t^l, \delta_t^w, \delta_t^{x_j} \in \{0, 1\} \quad \forall j \in \{1, \dots, m-1\}, t \in \mathcal{T}, \qquad (44)$$

$$v_t^y, v_t^l, v_t^w, v_t^{x_j} \ge 0, \quad \forall j \in \{1, \dots, m-1\}, t \in \mathcal{T}, \qquad (45)$$

$$y_t, l_t, w_t, x_{t+1,j} \ge 0 \quad \forall j \in \{1, \dots, m-1\}, t \in \mathcal{T}, \qquad (46)$$

$$\mathbf{y} \in \mathcal{Y}. \qquad (47)$$

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Contents

The Robust Model

Robust setting:

- Obtain decision rules that are robust to the uncertainty in the initial age of delivered units. We do so by finding coefficients that minimize the in-sample worst case cost.

- Uncertainty set over n periods of training data:

$$\mathcal{Y}^{(k)} := \left\{ \begin{array}{c} \boldsymbol{y} \in \mathbb{R}_+^{T*m}, \sum_{t=kT+1}^{kT+T} y_{t,i} = \widehat{p}_i \times \sum_{t=kT+1}^{kT+T} y_t \,, \forall i \in \{1,\dots,m\}, \\ \sum_{i=1}^{m} y_{t,i} = y_i, \forall i \in \{kT+1,\dots,kT+T\} \end{array} \right\}$$

- Let $l := n/T$. (For instance, in our experiments the planning period is T = 1 year and the training set includes data from $n = 2$ years and hence $l = 2$.) For $k \in \{0,\dots,l-1\}$ denoted by $\boldsymbol{y}_k$ the matrix of the number of units delivered with different remaining ages in periods $t \in \{kT+1,\dots,kT+T\}$.

The Robust Model

- Robust problem:

$$\min_{\boldsymbol{\beta}} W_n(\boldsymbol{\beta}) + \lambda \parallel \boldsymbol{\beta} \parallel_1$$

where $W_n(\boldsymbol{\beta})$ is the worst-case (in-sample) cost corresponding to coefficient vector $\boldsymbol{\beta}$:

$$W_n(\boldsymbol{\beta}) := \max_{\mathbf{y}_0,\cdots,\mathbf{y}_{(l-1)}} \sum_{t=1}^{n} (w_t + c \times l_t)$$

subject to,

$$(20) - (24),$$

$$\mathbf{y}_0 \in \mathcal{Y}^{(0)}, \ldots, \mathbf{y}_{(l-1)} \in \mathcal{Y}^{(l-1)}.$$

The Robust Model

■ Robust problem:

- Rewrite the problem:

$$\min_{\gamma,\beta} \gamma + \lambda \|\beta\|_1$$

subject to,

$$\begin{cases} \gamma \geq \sum_{t=1}^{n} (w_t + c \times l_t), \\ y_t = \left(\mathbf{z}_t \cdot \boldsymbol{\beta} - \sum_{i=1}^{m-1} x_{t,i} \right)^+ \forall t \in \mathcal{N}, \\ \mathit{for\ all}\ \mathbf{y}_0 \in \mathcal{Y}^{(0)}, \dots, \mathbf{y}_{(l-1)} \in \mathcal{Y}^{(l-1)} \\ (21)\text{-}(24), \end{cases}$$

- The main challenge in solving the robustness problem is that the feasible set of the remaining age values depends on the value of $\boldsymbol{\beta}$, which itself is a decision variable.

The Robust Model

- Challenge:

  - A common approach is to sequentially fix a set of decision variables and optimize over the other to obtain a lower bound and then utilize the lower bound to obtain an upper bound and repeat this procedure until convergence to the optimal solution.

  - In our problem, fixing the initial age values (and hence γ) fully determines the order sizes and hence $\boldsymbol{\beta}$, making this approach inapplicable.

$$\min_{\gamma,\beta} \gamma + \lambda\|\boldsymbol{\beta}\|_1$$

subject to,

$$\begin{cases} \gamma \geq \sum_{t=1}^{n} (w_t + c \times I_t), \\ y_t = \left(\mathbf{z}_t \cdot \boldsymbol{\beta} - \sum_{i=1}^{m-1} x_{t,i} \right)^+ \forall t \in \mathcal{N}, \\ \textit{for all } \mathbf{y}_0 \in \mathcal{Y}^{(0)}, \ldots, \mathbf{y}_{(l-1)} \in \mathcal{Y}^{(l-1)} \\ (21)\text{-}(24), \end{cases}$$

Iterative cutting-plane algorithm

- Subproblem (Upper bound):

  - Starting with an initial coefficient vector, say $\bar{\boldsymbol{\beta}}$, we first solve the adversary's problem to obtain $\{y_{t,i}^*\}$.

$$W_n(\beta) := \max_{y_0, \ldots, y_{(l-1)}} \sum_{t=1}^{n} (w_t + c \times l_t)$$

subject to,

$$(20) - (24),$$

$$\mathbf{y}_0 \in \mathcal{Y}^{(0)}, \ldots, \mathbf{y}_{(l-1)} \in \mathcal{Y}^{(l-1)}.$$

  - Define $f_{t,i}^*(\bar{\boldsymbol{\beta}}) := y_{t,i}^*(\bar{\boldsymbol{\beta}}) / y_t^*(\bar{\boldsymbol{\beta}})$ when $y_t^*(\bar{\boldsymbol{\beta}}) \neq 0$ and $f_{t,\mu}^*(\bar{\beta}) := 1$ for some μ, otherwise.

Iterative cutting-plane algorithm

- Master problem (Lower bound):

$$\min_{\gamma,\beta} \gamma + \lambda\|\beta\|_1$$

subject to,

$$\gamma \geq \alpha_{\bar{\beta}}(\beta),$$

where

$$\alpha_{\bar{\beta}}(\beta) := \min_{\{s_{t,i}\}} \sum_{t=1}^{n} (w_t + c \times l_t)$$

subject to

$$y_t = \mathbf{z}_t \cdot \beta - \sum_{i=1}^{m-1} x_{t,i},$$

$$w_t = x_{t,1} + f^*_{t,1}(\bar{\beta})y_t - s_{t,1},$$

$$x_{t+1,j} = x_{t,j+1} + f^*_{t,j+1}(\bar{\beta})y_t - s_{t,j+1} \quad \forall j \in \{1,\dots,m-2\}, t \in \mathcal{N},$$

$$x_{t+1,m-1} = f^*_{t,m}(\bar{\beta})y_t - s_{t,m} \quad \forall t \in \mathcal{N},$$

$$\sum_{i=1}^{m} s_{t,i} + l_t = d_t \quad \forall t \in \mathcal{N},$$

$$y_t \geq 0 \quad \forall t \in \mathcal{N},$$

$$l_t \geq 0 \quad \forall t \in \mathcal{N},$$

$$w_t \geq 0 \quad \forall t \in \mathcal{N},$$

$$s_{t,i} \geq 0, \quad \forall i \in \{1,\dots,m\}, t \in \mathcal{N},$$

$$x_{t+1,j} \geq 0, \quad \forall j \in \{1,\dots,m-1\}, t \in \mathcal{N}.$$

  - Fixing $f^*_{t,i}(\bar{\boldsymbol{\beta}})$.
  - Note that $\alpha_{\bar{\boldsymbol{\beta}}}(\boldsymbol{\beta})$ is the minimum cost under coefficient vector $\boldsymbol{\beta}$ and the remaining ages of delivered units are determined based on the fractions $f^*_{t,i}(\bar{\boldsymbol{\beta}})$.

Iterative cutting-plane algorithm

- Detailed outline:
  - After solving the master problem and finding the new coefficients, we repeat the same process by generating and adding new cuts to the master problem until the subproblem generates an upper-bound lower than or equal to the master problem's objective.

ALGORITHM 1 Iterative cutting-plane method to solve (56)–(57)

Output: approximate robust coefficients $\boldsymbol{\beta}^*$
Input: initial $\boldsymbol{\beta}^0$
$\bar{\gamma} \leftarrow -\infty$, $\eta \leftarrow 0$
while $W_n(\boldsymbol{\beta}^\eta) > \bar{\gamma}$ **do**
  solve

$$\min_{\gamma,\boldsymbol{\beta}} \gamma \tag{71}$$

subject to,

$$\gamma \geq \alpha_{\boldsymbol{\beta}^\rho}(\boldsymbol{\beta}), \quad \forall \rho \in \{0,\ldots,\eta\}, \tag{72}$$

$$\beta_k \leq u_k, \quad \forall k \in \{1,\ldots,p\}, \tag{73}$$

$$-\beta_k \leq u_k, \quad \forall k \in \{1,\ldots,p\}, \tag{74}$$

$$\sum_{k=1}^{p} u_k \leq \epsilon, \tag{75}$$

  $\eta \leftarrow \eta + 1$
  let $\tilde{\gamma}$ and $\tilde{\boldsymbol{\beta}}$ be an optimal solution to (71)-(75)
  $\bar{\gamma} \leftarrow \tilde{\gamma}$
  $\boldsymbol{\beta}^\eta \leftarrow \tilde{\boldsymbol{\beta}}$
end
$\boldsymbol{\beta}^* \leftarrow \boldsymbol{\beta}^\eta$

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■ Setting:

- We use data from 2015 to 2016 to train our models and test their performance using data from 2017.

- Specifically, after solving for $\boldsymbol{\beta}$ through the fixed initial age or the robust model, we find the required inventory levels for each period q_t in the test set using the feature data available in that period.

- The ordering decisions are then determined through $y_t = \lceil (q_t - \sum_{i=1}^{m-1} x_{t,i})^+ \rceil$, where $\lceil x \rceil$ gives the least integer value greater than or equal to x.

Comparison with historical performance

HGH:

- The historical number of orders, emergency order rate, and the expiry rate: 2099, 13.9%, and 8.7% for HGH in 2017.
- Both models, achieve a significant improvement with respect to out-of-sample performance for all performance measures.
- For all cost parameters, the robust model leads to a smaller total number of orders.

TABLE 1 Summary of the case study results for HGH. The values in brackets correspond to the half-width of a 95% confidence interval for the corresponding estimates

	Fixed initial age model								Robust model						
	In-sample			Out-of-sample					In-sample		Out-of-sample				
c	Obj.	Cost	WC Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost	Obj.	Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost
1	150.70	108.36	388.70	1970.11	4.21	2.44	131.12	422.14	369.38	127.44	1950.66	6.20	1.57	151.66	406.33
		(±0.64)		(±0.87)		(±0.04)	(±0.87)			(±0.50)	(±0.68)		(±0.03)	(±0.68)	
2	207.00	145.82	484. 55	2003.95	2.54	4.05	183.06	531.92	468.22	161.75	1975.06	3.90	2.69	207.09	520.36
		(±0.66)		(±1.08)		(±0.05)	(±1.08)			(±0.55)	(±0.92)		(±0.05)	(±0.91)	
6	251.45	163.41	569.53	2081.14	1.20	7.48	305.58	736.09	564.13	182.08	2052.20	1.41	6.18	300.83	738.52
		(±0.95)		(±1.49)		(±0.07)	(±1.50)			(±0.96)	(±1.35)		(±0.07)	(±1.37)	

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Comparison with historical performance

- HGH:
  - The robust model significantly reduces the worst-case (in sample) costs.
  - For $c = 1, 2$, the (out-of-sample) worst-case cost under the robust model is lower than that of the fixed initial age model, whereas for $c = 6$ the robust model leads to a slightly higher worst-case cost.

TABLE 1 Summary of the case study results for HGH. The values in brackets correspond to the half-width of a 95% confidence interval for the corresponding estimates

	Fixed initial age model								Robust model						
	In-sample			Out-of-sample					In-sample		Out-of-sample				
c	Obj.	Cost	WC Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost	Obj.	Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost
1	150.70	108.36	388.70	1970.11	4.21	2.44	131.12	422.14	369.38	127.44	1950.66	6.20	1.57	151.66	406.33
		(±0.64)		(±0.87)		(±0.04)	(±0.87)			(±0.50)	(±0.68)		(±0.03)	(±0.68)	
2	207.00	145.82	484. 55	2003.95	2.54	4.05	183.06	531.92	468.22	161.75	1975.06	3.90	2.69	207.09	520.36
		(±0.66)		(±1.08)		(±0.05)	(±1.08)			(±0.55)	(±0.92)		(±0.05)	(±0.91)	
6	251.45	163.41	569.53	2081.14	1.20	7.48	305.58	736.09	564.13	182.08	2052.20	1.41	6.18	300.83	738.52
		(±0.95)		(±1.49)		(±0.07)	(±1.50)			(±0.96)	(±1.35)		(±0.07)	(±1.37)	

Comparison with historical performance

- JH:
  - The historical number of orders, emergency order rate, and expiry rate were, respectively, 4481, 8.48%, and 2.28% for JH in 2017.
  - Again we observe a significant reduction in all performance measures for both models and under the three cost ratios.

TABLE 1 Summary of the case study results for HGH. The values in brackets correspond to the half-width of a 95% confidence interval for the corresponding estimates

	Fixed initial age model								Robust model						
	In-sample			Out-of-sample					In-sample		Out-of-sample				
c	Obj.	Cost	WC Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost	Obj.	Cost	Orders	% Emerg.	% Expiry	Cost	WC Cost
1	150.70	108.36	388.70	1970.11	4.21	2.44	131.12	422.14	369.38	127.44	1950.66	6.20	1.57	151.66	406.33
		(±0.64)		(±0.87)		(±0.04)	(±0.87)			(±0.50)	(±0.68)		(±0.03)	(±0.68)	
2	207.00	145.82	484. 55	2003.95	2.54	4.05	183.06	531.92	468.22	161.75	1975.06	3.90	2.69	207.09	520.36
		(±0.66)		(±1.08)		(±0.05)	(±1.08)			(±0.55)	(±0.92)		(±0.05)	(±0.91)	
6	251.45	163.41	569.53	2081.14	1.20	7.48	305.58	736.09	564.13	182.08	2052.20	1.41	6.18	300.83	738.52
		(±0.95)		(±1.49)		(±0.07)	(±1.50)			(±0.96)	(±1.35)		(±0.07)	(±1.37)	

Sensitivity analysis

- Sensitivity to the remaining age distribution:
  - Changing the coefficients $c_1^4 = -0.027$ and $c_1^5 = -0.044$ to -0.7
  - The cost estimate for the fixed initial age model (with different c values) increases significantly by 80.8%, 86.2%, and 84.1% for the three cost ratios, respectively, whereas the robust observes lower increases of 44.6%, 51.2%, and 81.5%, respectively。

Table EC.1 Fitted multinomial logistic regression to the remaining age data for HGH in 2017.

	Remaining Age 2	3	4	5
c_0	5.661*** (1.220)	6.352*** (1.204)	4.310*** (1.207)	−11.896*** (0.687)
c_1	−0.078* (0.043)	−0.112*** (0.041)	−0.027 (0.041)	−0.044 (0.046)
Tue	−1.359** (0.538)	−1.302** (0.520)	−1.971*** (0.536)	−1.484** (0.644)

TABLE 3 Performance sensitivity to the remaining age distribution for HGH. In the modified multinomial model, the odds of receiving fresh units decrease more rapidly in the size of orders

	Fixed initial age model			Robust model		
c	% Emerg.	% Expiry	Cost	% Emerg.	% Expiry	Cost
1	4.00	7.42	237.01	6.00	4.87	219.30
		(±0.06)	(±1.28)		(±0.05)	(±1.01)
2	2.36	11.06	340.93	3.70	7.65	313.19
		(±0.08)	(±1.67)		(±0.07)	(±1.50)
6	1.07	17.69	562.71	1.31	16.04	546.05
		(±0.09)	(±2.21)		(±0.09)	(±2.03)

Sensitivity analysis

Sensitivity to additional features:

- Investigate the value of adding additional features available in our data that have a weaker correlation with demand.
- Although the additional features can slightly reduce the in-samples objectives, these savings do not always translate to improved out-of-sample performance, in particular when c is large.
- This is consistent with our previous observations on the lower generalizability of the fixed initial age model due to increased sensitivity to large in-sample demand values.

TABLE 4 Performance sensitivity to adding more covariates for HGH. A negative difference value means that the performance improves by adding more covariates

	Fixed initial age model				Robust model			
	In-sample		Out-of-sample		In-sample		Out-of-sample	
c	Obj.	Cost	Cost	WC Cost	Obj.	Cost	Cost	WC Cost
1	−9.58	−4.50	−2.10	−4.69	−11.50	0.20	−0.51	−12.82
		(±0.95)	(±1.13)			(±0.66)	(±1.00)	
2	−17.50	−14.41	4.10	−6.89	−23.17	−6.55	−7.00	−16.02
		(±0.94)	(±1.58)			(±0.83)	(±1.10)	
6	−16.58	0.37	35.81	26.08	−15.30	−10.80	19.18	29.93
		(±1.28)	(±1.90)			(±1.24)	(±1.87)	

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Conclusions

- The ordering quantities are set according to base-stock levels where the required inventory in each period is a linear function of a set of observed features in that period.
- We find that the hospitals could have achieved a significant reduction in order size, expiry, and shortage rates by guiding their ordering decisions with our models.
- The key characteristic of the problem we study is the variability in the remaining age of received units, which can also be endogenous (i.e., depending on the order size).
- Ignoring the uncertainty could lead to poor out-of-sample performance.

Comments

- Positive comments:
  - One-step prediction framework : ERM
  - Remaining age of received units is endogenous

- Negative comments:
  - Is daily transfusion equivalent to demand? May be correct.

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Thank You!

Reporter：于成成