中国科学技术大学
University of Science and Technology of China

Inverse optimization of integer programming games for parameter estimation arising from competitive retail location selection

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  - Operations Research
  - European Journal of Operational Research
  - Transportation Research Part C
- Research Areas
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  - Hydrogen Supply Chain
  - Competition

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  - Operations Research (4)
  - Manufacturing & Service Operations Management (4)
  - Production and Operations Management (3)
  - Management Science
  - Transportation Science
  - EJOR/TRx/IISE/IJPR/IJPE (Many)
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  - Logistics and Supply Chain Management
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标题	引用次数	年份
The value of information sharing in a two-level supply chain HL Lee, KC So, CS Tang Management science 46 (5), 626-643	3128	2000
Perspectives in supply chain risk management CS Tang International journal of production economics 103 (2), 451-488	1949	2006

- Research Areas
  - Outsourcing
  - Pricing
  - Retailing
  - Supply Chain Management

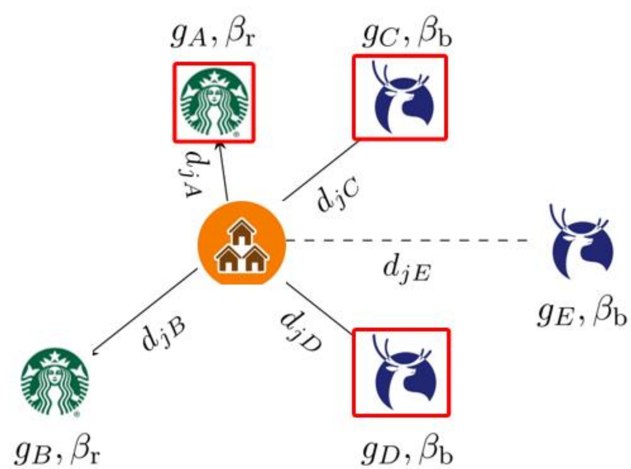
1. Background

- A realistic scenario

✓ Suppose you plan to open a coffee shop
  - There are two coffee brands, Luckin and Starbucks, in the area

✓ Know nothing about the consumer information (distance preference, price preference, brand preference, etc.)

✓ How to locate your coffee shop?
  - Estimate consumer information based on the existing layout!



1. Background

Competitive retail location selection

- ✓ Retailers choose their store locations: Need in-depth knowledge of these customer attraction parameters
  - Common approach: Customer surveys or Discrete choice models
  - A new entrant is unlikely to have access to such granular data

- ✓ Incumbent retailers: Full knowledge of each other's payoffs and the customer choice behavior.
- ✓ A new entrant: Observing (near-)optimal location selection of incumbents

Research Problem

- ✓ **New entrant's location problem**
  - Integer programming games: Competitive retail location selection
  - Inverse optimization: Parameter estimation

2. Related literature

Research Gap

Related area	Research content	Published research	Research Gap
Location choice in retail	Simultaneous competition: Assume full knowledge	Crönert & Minner, 2021b; Godinho & Dias, 2010	Need full knowledge
	Customer choice estimation: Maximum likelihood estimation or regression models	Seim, 2006; Shriver & Bollinger, 2022	Sales level data
Inverse optimization	Classic inverse optimization	Ahuja & Orlin, 2001; Wang 2009	Single decision-maker
	Inverse equilibrium problems	Bertsimas et al. 2015; Allen et al. 2022	Limited to continuous decisions

- Integer programming games (IPGs)

  - Finite action set:

$$S_i^o = \{\mathbf{x}_i \in \mathbb{Z}^{z_i} | w_q(\mathbf{x}_i) \leq 0, \forall q \in \{1, \dots, Q^i\}\}$$

  - A popular solution concept for IPGs is a Nash equilibrium:

$$\Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}) \geq \Pi_i(\tilde{\mathbf{x}}_i, \mathbf{x}_{-i}), \forall \tilde{\mathbf{x}}_i \in S_i^o, \forall i \in I$$

  - However, such pure Nash equilibria do not provably exist for general IPGs (*Carvalho et al., 2022*).
  - Approximate nash equilibrium (*Daskalakis et al., 2006*):

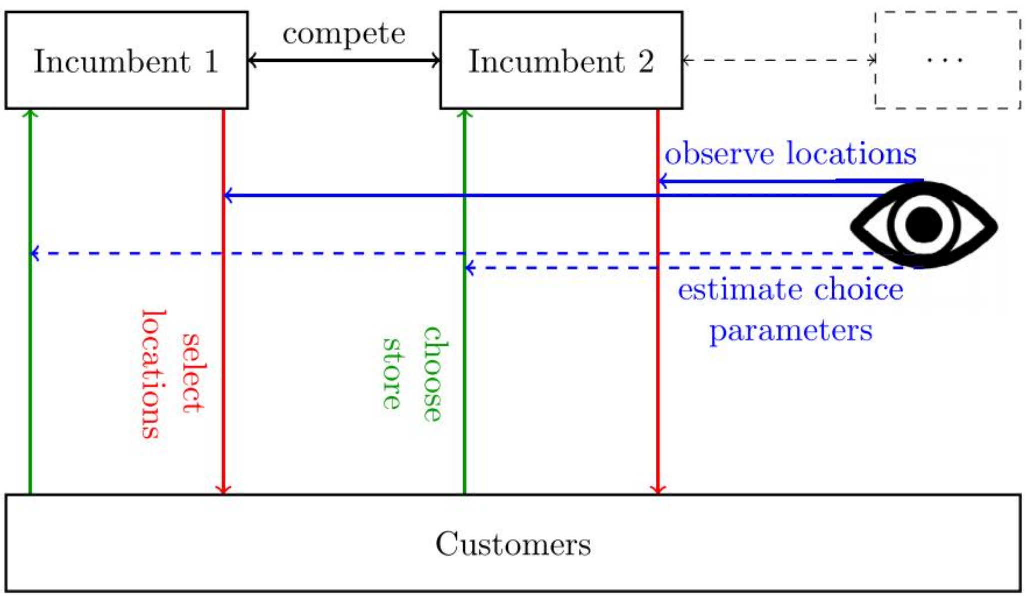
$$\Pi_i(\mathbf{x}_i, \mathbf{x}_{-i}) + \epsilon \geq \Pi_i(\tilde{\mathbf{x}}_i, \mathbf{x}_{-i}), \forall \tilde{\mathbf{x}}_i \in S_i^o, \forall i \in I$$

3. Model

- Our context

  - Incumbent retailers: Know payoff structures and customer choice behavior
  - Customers: Select certain stores they frequent

  - New entrant: Estimate the customers' choice parameters



Customer choice

✓ Utility function (*homogeneous*)

- Retail chain brand attractiveness (β_i), such as pricing and product quality,
- Accessibility ($\tilde{d}^o_{jk} = \frac{\bar{d}-d^o_{jk}}{\bar{d}}$): proximity of a store
- Convenience ($\tilde{g}^0_k = \frac{g^0_k}{\bar{g}}$): Points of interest near the location
- Utility function:

$$u^o_{ijk} = \beta_i + \alpha \tilde{d}^o_{jk} + (1-\alpha)\tilde{g}^o_k$$

✓ Fractional patronage (Luce's choice axiom, *Luce, 1959*)

$$f^o_{ij}(\mathbf{x}^o_i, \mathbf{x}^o_{-i}) = \frac{\sum_{k\in K|d_{jk}\le\bar{d}} x^o_{ik}u^o_{ijk}}{\sum_{\tilde{i}\in I}\sum_{k\in K|d_{jk}\le\bar{d}} x^o_{\tilde{i}k}u^o_{\tilde{i}jk}}, \forall i \in I, \forall j \in J, \forall 0 \in 0$$

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- Forward problem: Incumbents' location selection

✓ Profit

- annualized operating margin - annualized fixed costs

$$\prod_i^o(\boldsymbol{x}_i^o, \widehat{\boldsymbol{x}}_{-i}^o) = \sum_{j\in J} m_{ij}^o p_j^o f_{ij}^o(\boldsymbol{x}_i^o, \widehat{\boldsymbol{x}}_{-i}^o) - \sum_{k\in K} c_{ik}^o x_{ik}^o$$

- Approximate candidate locations by a grid structure

✓ Optimize their respective (annualized) profit

$$\max_{\mathbf{x}_i^o \in S_i^o} \prod_i^o(\boldsymbol{x}_i^o, \widehat{\boldsymbol{x}}_{-i}^o)$$

- Assume incumbents neglect potential entrants

  Reasons: 1. No credible threats; 2. Unattainable over a longer period

3. Model

- Inverse problem: New entrant's parameter estimation

✓ Intuitive idea

- Observing the equilibrium store locations selected by incumbents $\hat{\boldsymbol{x}}_i^0$, the new entrant aims to estimate $\boldsymbol{\alpha} \in A, \boldsymbol{\beta} \in B^n$
- ϵ - Approximate Nash equilibrium

✓ Inverse problem:

- Unilateral improvement potential

$$\delta_i^o = \boldsymbol{max}_{\mathbf{x}_i^o \in \mathcal{S}_i^o}\{\Pi_i^o(\mathbf{x}_i^o, \hat{\mathbf{x}}_{-i}^o)\} - \Pi_i^o(\hat{\mathbf{x}}_i^o, \hat{\mathbf{x}}_{-i}^o), \forall i \in I, \forall o \in O$$

- Inverse IPG

$$\boldsymbol{min}_{\alpha,\beta}||\boldsymbol{w\delta}|| = \boldsymbol{min}_{\alpha,\beta}||(w^o\delta_i^o, \forall \boldsymbol{i} \in \boldsymbol{I}, \forall \boldsymbol{o} \in \boldsymbol{O}||$$

- In an ϵ -Nash equilibrium: $\epsilon = \boldsymbol{min}_{\alpha,\beta}\boldsymbol{max\delta}$ (*Hamming distance*)
- We avoid focusing too much on a single observation

$$\epsilon \approx \boldsymbol{min}_{\alpha,\beta}||\mathbf{w}\delta||$$

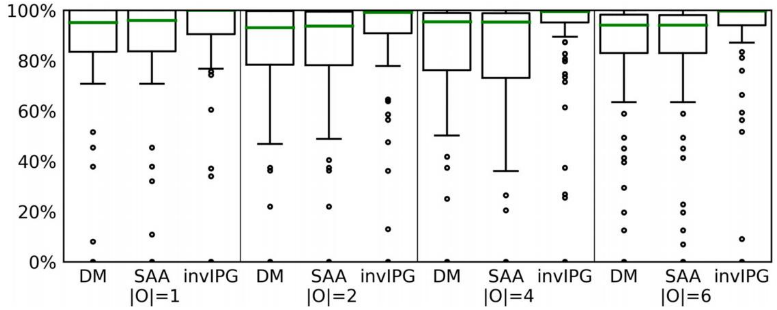
4. Numerical study

■ Comparison to full information and sampling-based benchmarks

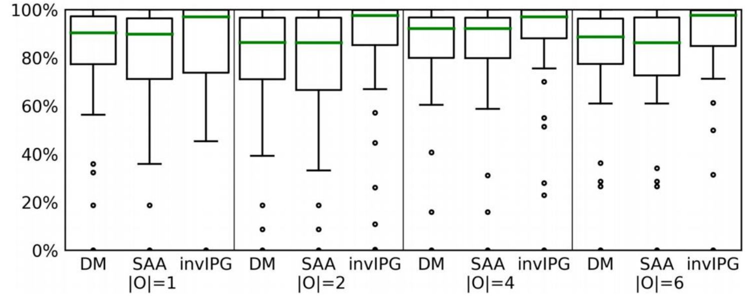
✓ Four approaches

- InvIPG
- Distribution mean (DM): $\alpha = 0.5,\ \beta_1 = 0.5, \beta_2 = 0.5$
- SAA: 64 scenarios chosen from $\{0.2,0.4,0.6,0.8\}$
- Full information: $\widehat{\alpha}, \widehat{\beta}_1, \widehat{\beta}_2 \sim \mathrm{U}(0,1)$

✓ Assume estimate (β_3) equals the ground-truth value $\widehat{\beta}_3 \sim U(0,0.5)$ in all cases. (limitations)



$|J| = |K| = 10$



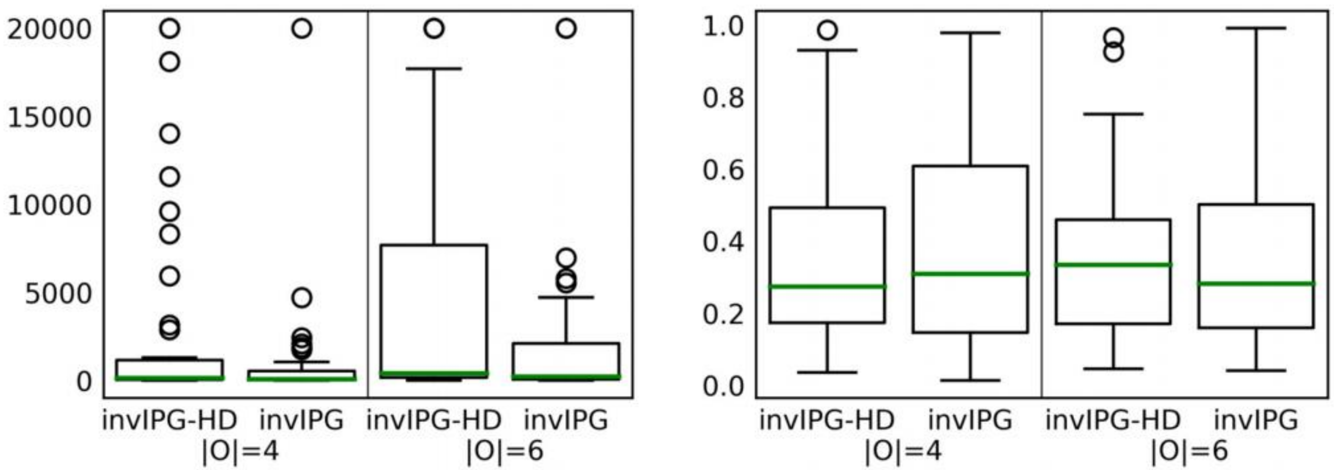
$|J| = |K| = 20$

4. Numerical study

■ Hamming distances in objective function (invIPG-HD)

✓ Two measures in objective function

- invIPG: $\epsilon \approx \boldsymbol{min}_{\alpha,\beta}||w\delta||$ (L1-norm)
- invIPG-HD: $\epsilon = \boldsymbol{min}_{\alpha,\beta}\boldsymbol{max\delta}$ **(Hamming distance)**



(a) Runtime in seconds

(b) Error (L_2-norm) between inversely estimated parameter and ground-truth values

5. Comments

- Positive comments:

  ✓ Natural and fun ideas

- Negative comments:

  ✓ Unable to estimate new entrant's own brand parameters

Thank You!

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