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1. (a). yes, if they are all satisfied, $z_i(\underline{w}^T\underline{u}_i + w_0) \geq 1 > 0$, so they are correctly classified.

(b). $L(\underline{w}, w_0, \underline{\lambda}) = \frac{1}{2}\|\underline{w}\|^2 - \sum_{i=1}^{N}\lambda_i\left[z_i(\underline{w}^T\underline{u}_i + w_0) - 1\right]$

KKT conditions: $\begin{cases} z_i(\underline{w}^T\underline{u}_i + w_0) - 1 \geq 0, & \forall i \\ \lambda_i \geq 0, & \forall i \\ \lambda_i\left[z_i(\underline{w}^{*T}\underline{u}_i + w_0^*) - 1\right] = 0, & \forall i \end{cases}$

(c). $\frac{\partial L}{\partial \underline{w}} = 0 \Rightarrow \underline{w}^* - \sum_{i=1}^{N}\lambda_i z_i \underline{u}_i = 0, \Rightarrow \underline{w}^* = \sum_{i=1}^{N}\lambda_i z_i \underline{u}_i$

$\frac{\partial L}{\partial w_0} = 0 \Rightarrow -\sum_{i=1}^{N}\lambda_i z_i = 0$

So $L_D(\underline{\lambda}) = \frac{1}{2}\|\sum_{i=1}^{N}\lambda_i z_i \underline{u}_i\|^2 - \sum_{i=1}^{N}\lambda_i\left[z_i\left(\sum_{j=1}^{N}\lambda_j z_j \underline{u}_j^T\underline{u}_i + w_0\right) - 1\right]$

$= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j z_i z_j \underline{u}_i^T\underline{u}_j - \sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j z_i z_j \underline{u}_i^T\underline{u}_j - w_0\sum_{i=1}^{N}\lambda_i z_i + \sum_{i=1}^{N}\lambda_i$

$= -\frac{1}{2}\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j z_i z_j \underline{u}_i^T\underline{u}_j\right] + \sum_{i=1}^{N}\lambda_i$. s.t. $\sum_{i=1}^{N}\lambda_i z_i = 0$.

~~subject~~ substitute $\underline{w}^*$ into KKT in (b), so the KKT conditions are:

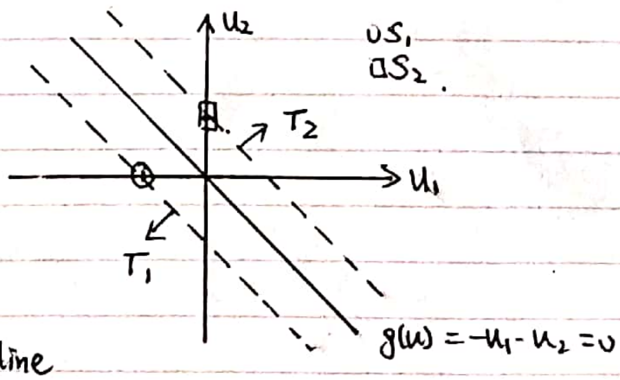
$\begin{cases} \lambda_i \geq 0, & \forall i. \\ \lambda_i\left[z_i\left(\sum_{j=1}^{N}\lambda_j z_j \underline{u}_j^T\underline{u}_i + w_0^*\right) - 1\right] = 0, & \forall i \\ z_i(\underline{w}^T\underline{u}_i + w_0) - 1 \geq 0, & \forall i. \end{cases}$

2. (a). substitute $\underline{u}_1, \underline{u}_2$ into $L_D'(\underline{\lambda}, \mu) = \sum_{i=1}^{N}\lambda_i - \frac{1}{2}\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j z_i z_j \underline{u}_i^T\underline{u}_j\right] + \mu\left(\sum_{i=1}^{N} z_i\lambda_i\right)$.

$L_D'(\underline{\lambda}, \mu) = -\frac{1}{2}\lambda_1^2 + \lambda_1 - \frac{1}{2}\lambda_2^2 + \lambda_2 + \mu(\lambda_1 - \lambda_2)$.

$\begin{cases} \partial L_D'/\partial\lambda_1 = 0 \\ \partial L_D'/\partial\lambda_2 = 0 \\ \partial L_D'/\partial\mu = 0 \end{cases} \Rightarrow \begin{cases} -\lambda_1 + 1 + \mu = 0 \\ -\lambda_2 + 1 - \mu = 0 \\ \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = 1 \\ \lambda_2 = 1 \\ \mu = 0 \end{cases}$ substitute these into the 2nd KKT condition and get $w_0 = 0$

$\underline{w} = \sum_{i=1}^{N}\lambda_i z_i \underline{u}_i = \begin{bmatrix}-1\\0\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$ so $g(\underline{u}) = -u_1 - u_2$



decision rule:

$g(\underline{u}) = \begin{cases} -u_1 - u_2 \geq 1 > 0, & \forall \underline{u} = \begin{bmatrix}u_1\\u_2\end{bmatrix} \in S_1 \\ -u_1 - u_2 \leq -1 < 0. & \forall \underline{u} = \begin{bmatrix}u_1\\u_2\end{bmatrix} \in S_2 \end{cases}$

(b). $d(\underline{u}_1, H) = \left|\frac{g(\underline{u}_1)}{\|\underline{w}\|}\right| = \left|\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$.

$d(\underline{u}_2, H) = \left|\frac{g(\underline{u}_2)}{\|\underline{w}\|}\right| = \left|-\frac{1}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

No, because now H is vertical to the line connecting $\underline{u}_1$ and $\underline{u}_2$ and cut ~~at~~ it equally.