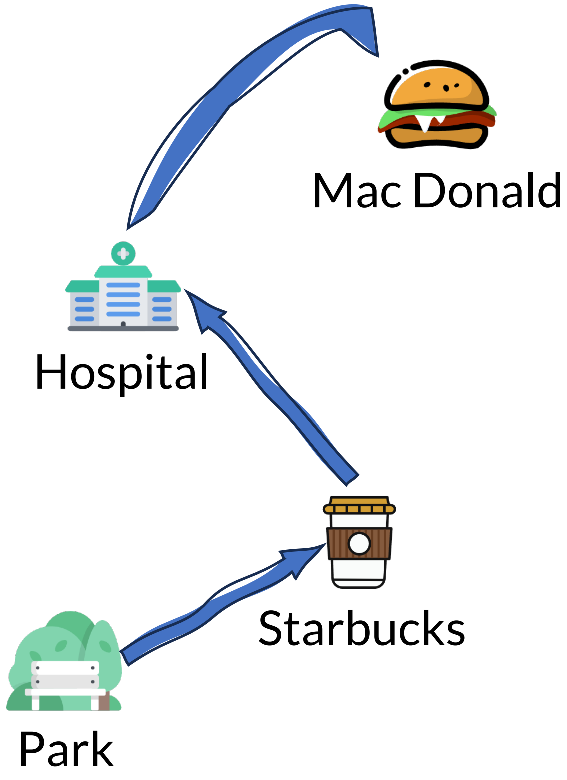
TraCS: Trajectory Collection in Continuous Space under Local Differential Privacy

Authors: Ye Zheng, Yidan Hu

Rochester Institute of Technology (RIT)

Trajectory Collection

- Private trajectories



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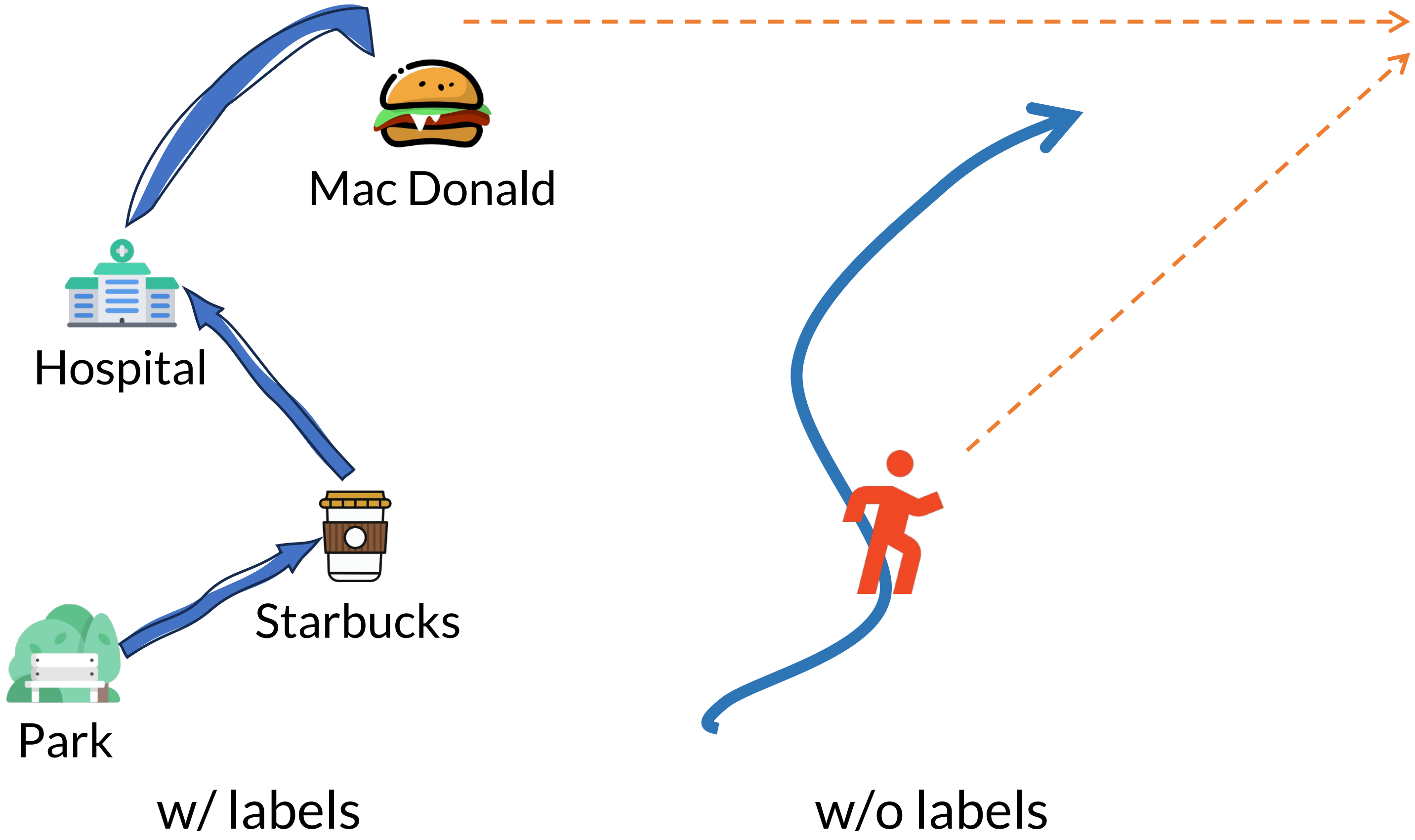

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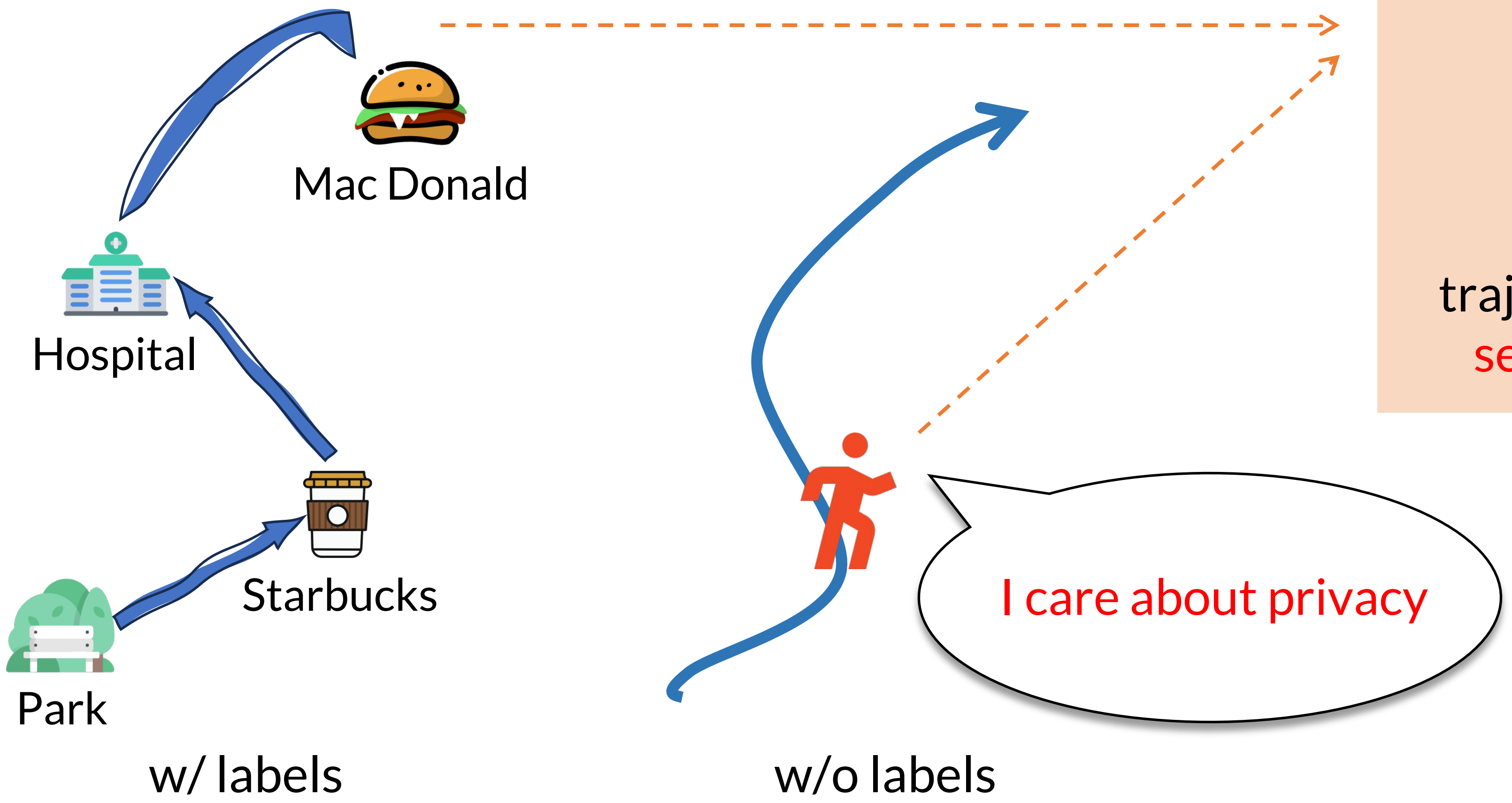

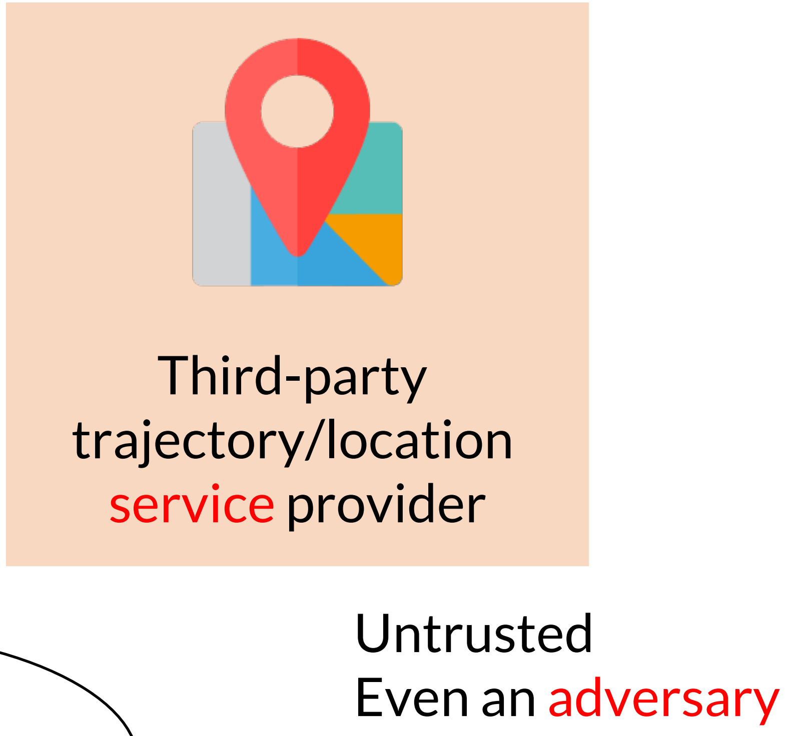

LDP-fy a Trajectory

- LDP-fy: perturb a trajectory with LDP guarantee

  - cannot distinguish location τ_1 from τ_2 with confidence level e^{ε}

$$\forall \tau_1, \tau_2, \tau_* \in \mathcal{S}: \quad \frac{\Pr[\mathcal{M}(\tau_1) = \tau_*]}{\Pr[\mathcal{M}(\tau_2) = \tau_*]} \leq e^{\varepsilon}$$

distinguish level

location space

probability difference of outputting τ_*

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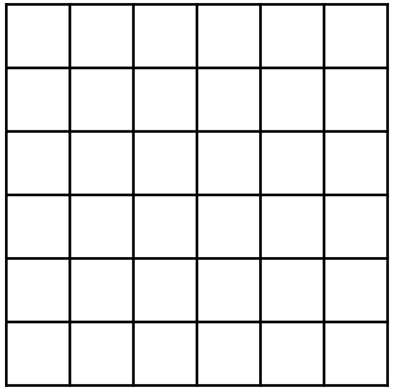
- **We want:** $\mathcal{M}$ for continuous location space

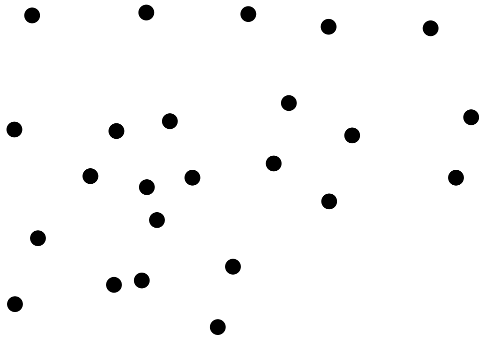
  - existing methods are all for discrete location space $\rightarrow$ not always available
  - continuous space $\supset$ discrete space $\rightarrow$ can apply to discrete space

Existing Methods

- Location space $\mathcal{S}$ is a set of cells or points



$\mathcal{S} = \{c_1, \ldots, c_n\}$



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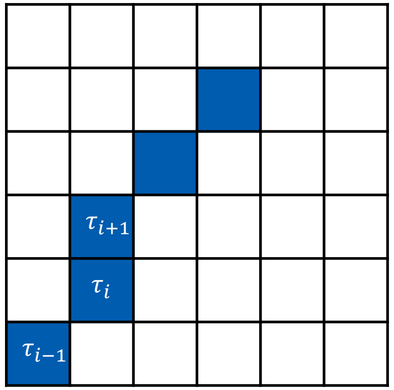

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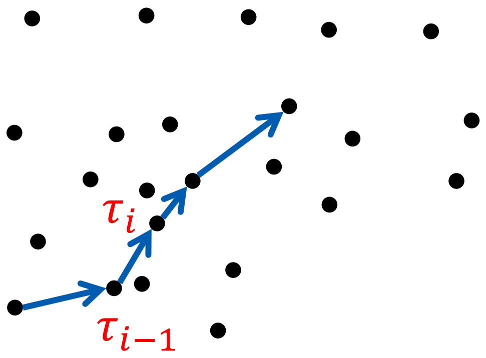

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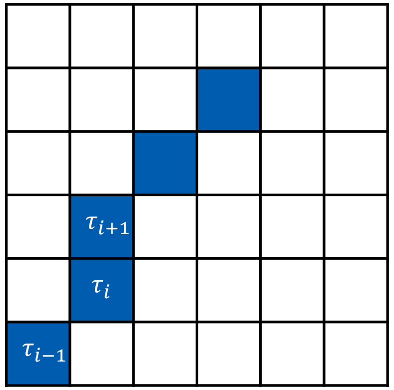

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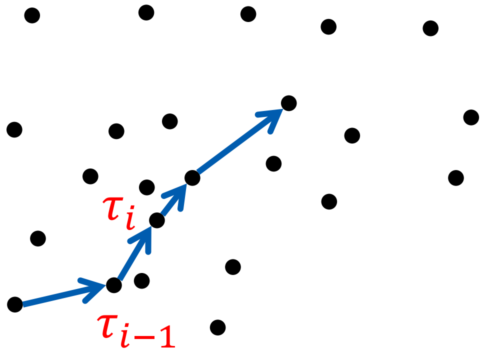

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- Perturb each location τ using the Exponential Mechanism

$$\Pr[\mathcal{M}_{\exp}(\tau) = \tau^*] = \frac{\exp\left(kd(\tau, \tau^*)\right)}{\sum_{\tau' \in S} \exp\left(kd(\tau, \tau')\right)}$$

distance

sum of distance

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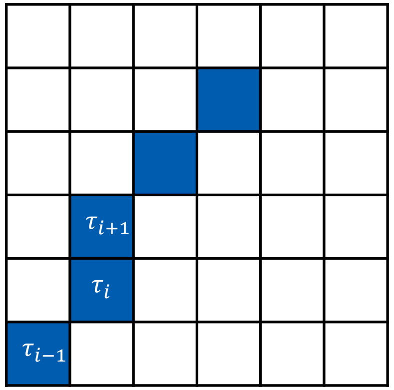

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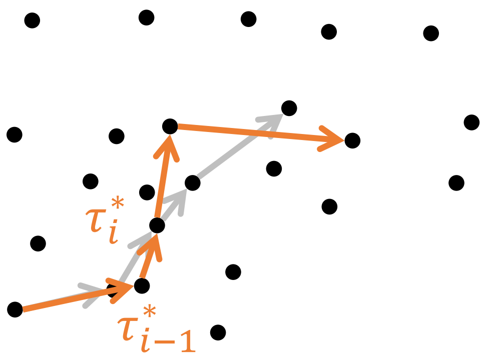

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Perturbed trajectory (ensuring LDP)

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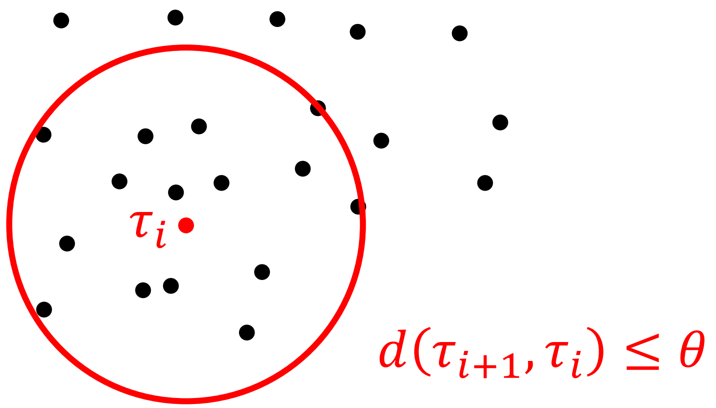
- NGram [2021]: reachability constraint from public knowledge

  - e.g. distance reachability

  - the next location cannot be too far

  - $\mathcal{M}_{\exp}(\tau)$ on reduced $\mathcal{S}$



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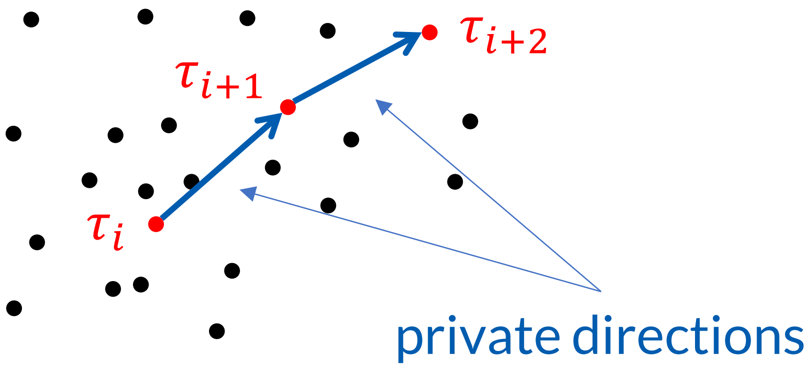

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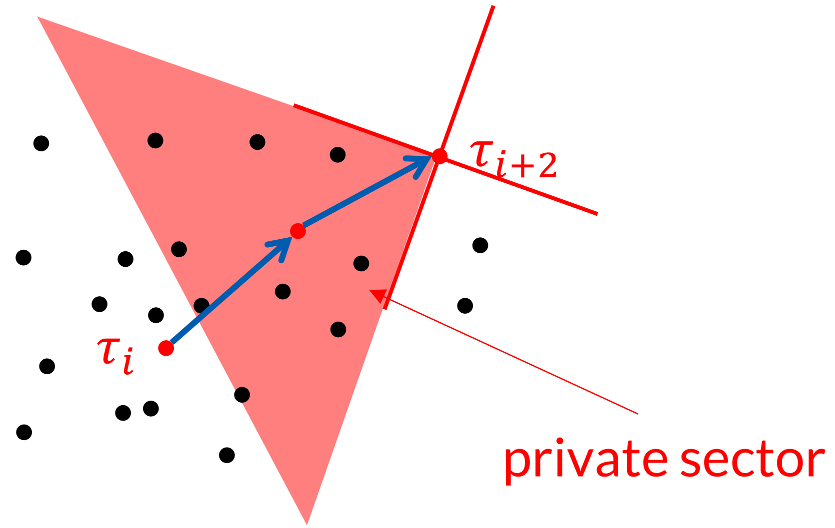

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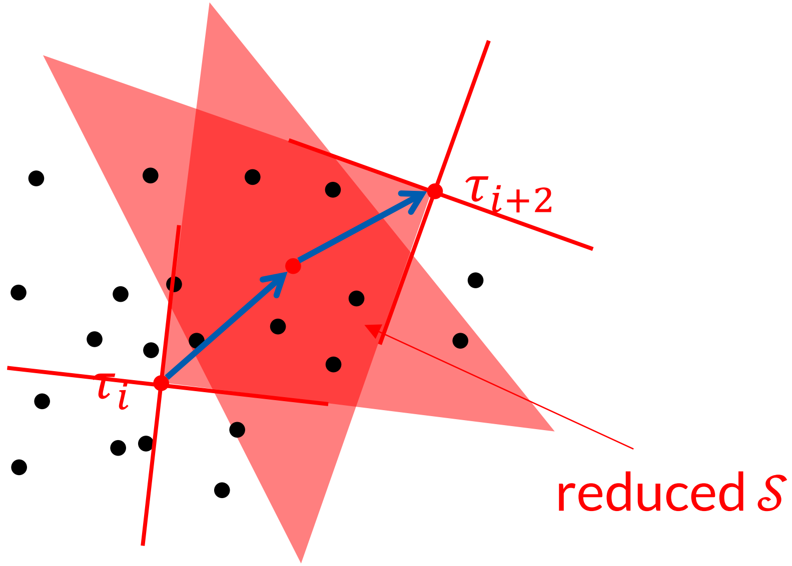
  - *ii.* perturb sector

  - *iii.* $\mathcal{M}_{\exp}(\tau)$ on reduced $\mathcal{S}$



Existing Methods: **Weaknesses**

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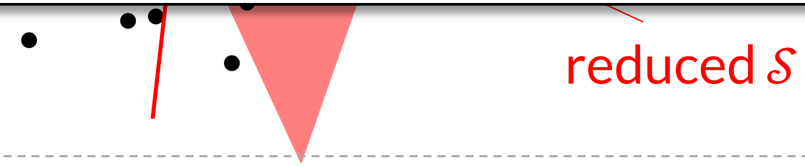
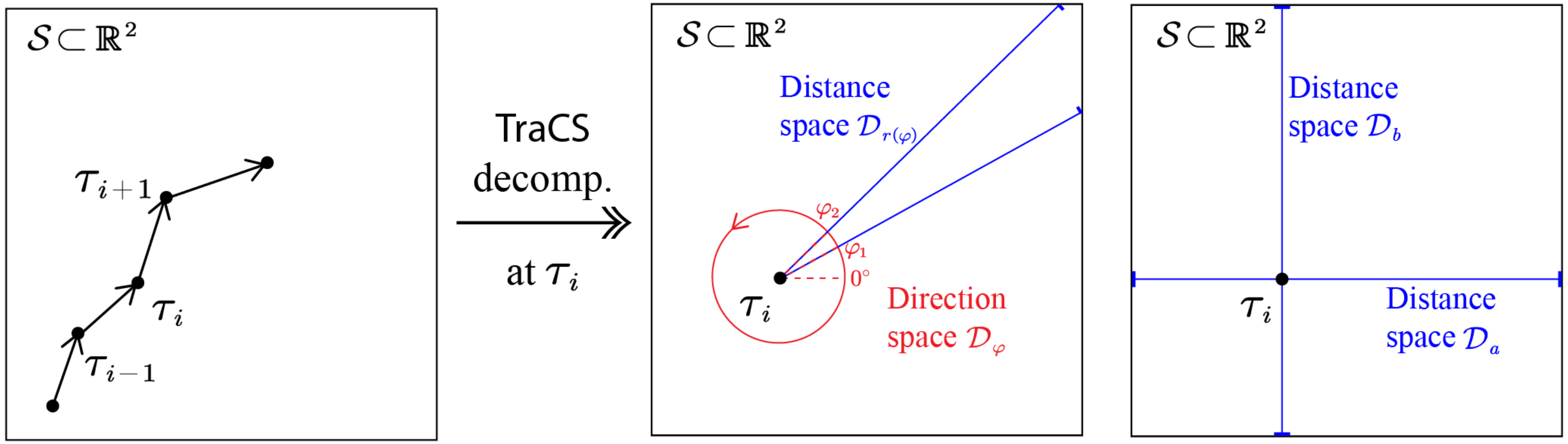


This Paper: Continuous Space

- TraCS-D: direction-distance perturbation
- TraCS-C: coordinates perturbation

- **Key idea:** decomposes $\mathcal{S}$ into two subspaces



TraCS-D: $\mathcal{D}_\varphi \otimes \mathcal{D}_{r(\varphi)}$ TraCS-C: $\mathcal{D}_a \times \mathcal{D}_b$

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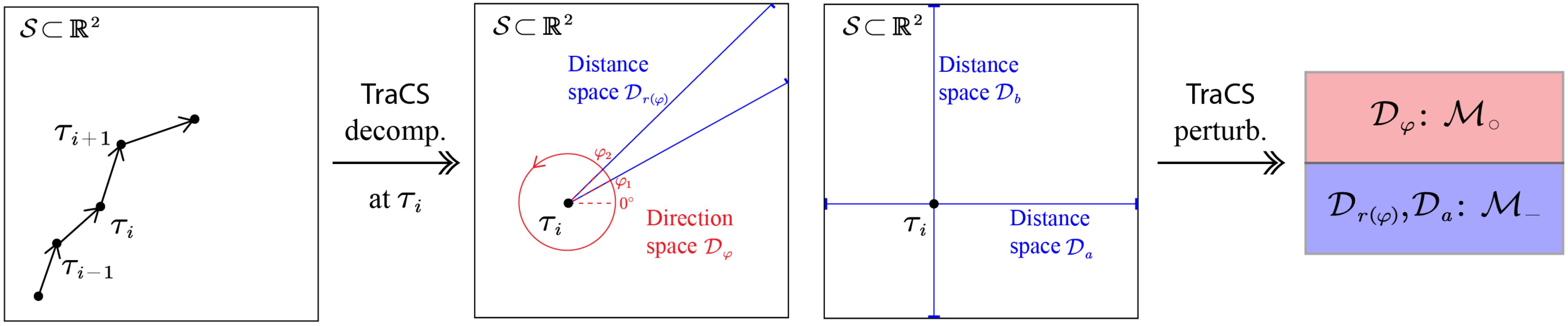

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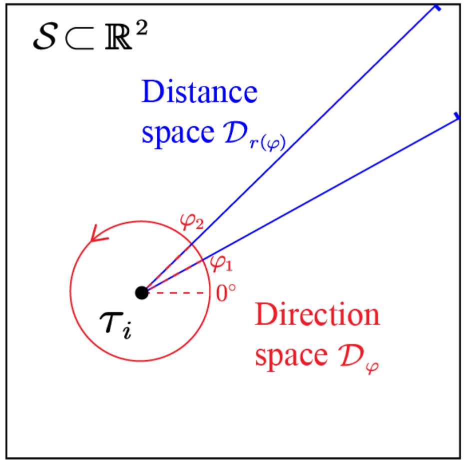
Decomposition of Continuous Space

- $\mathcal{S} = \mathcal{D}_{\varphi} \otimes \mathcal{D}_{r(\varphi)}$

2D space 1D subspaces

- Each location $\tau_{i+1} \in \mathcal{S}$ has a unique representation $(\varphi, r(\varphi))$

- Perturb φ and $r(\varphi)$ using 1D mechanisms



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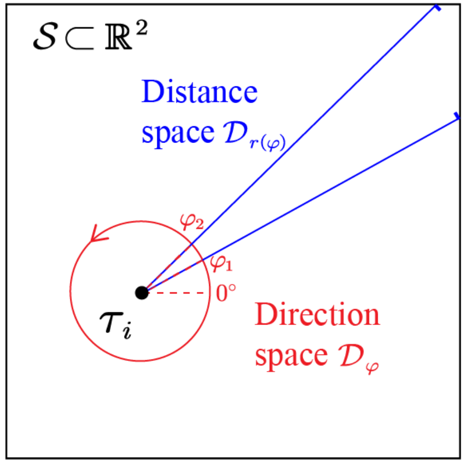

- $\mathcal{S} = \mathcal{D}_a \times \mathcal{D}_b$
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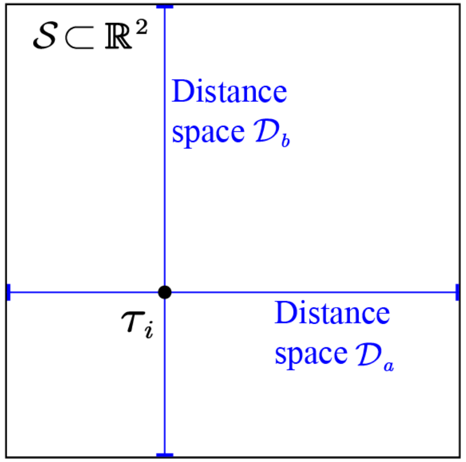
- Perturb a and b using 1D mechanisms



TraCS-C: $\mathcal{D}_a \times \mathcal{D}_b$

$\mathcal{M}$ for Continuous Space

- **Q:** How to design LDP mechanisms for:

  - circular space $[0,2\pi) \to [0,2\pi)$? linear space $[a_{\text{sta}}, a_{\text{end}}) \to [a_{\text{sta}}, a_{\text{end}})$?

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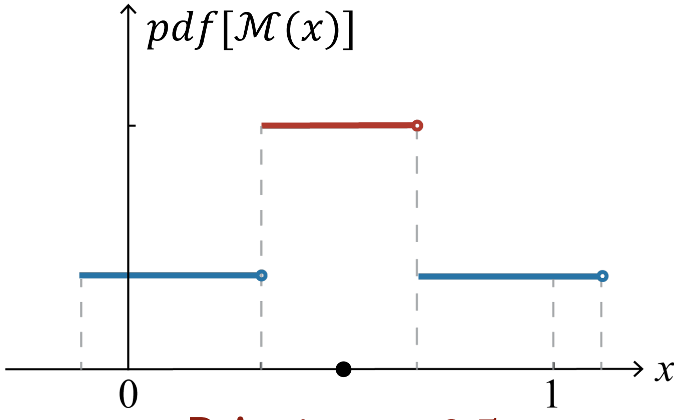
- **S:** Piecewise-based mechanism

  - originally designed for mean * /distribution estimation**

$$pdf[\mathcal{M}(x) = y] = \begin{cases} p_\varepsilon & \text{if } y \in [l_{x,\varepsilon}, r_{x,\varepsilon}), \\ p_\varepsilon / \exp(\varepsilon) & \text{otherwise,} \end{cases}$$

  - ensure LDP for $[0,1) \rightarrow [-C, C)$



Private $x = 0.5$

* Collecting and Analyzing Multidimensional Data with Local Differential Privacy, ICDE 2019

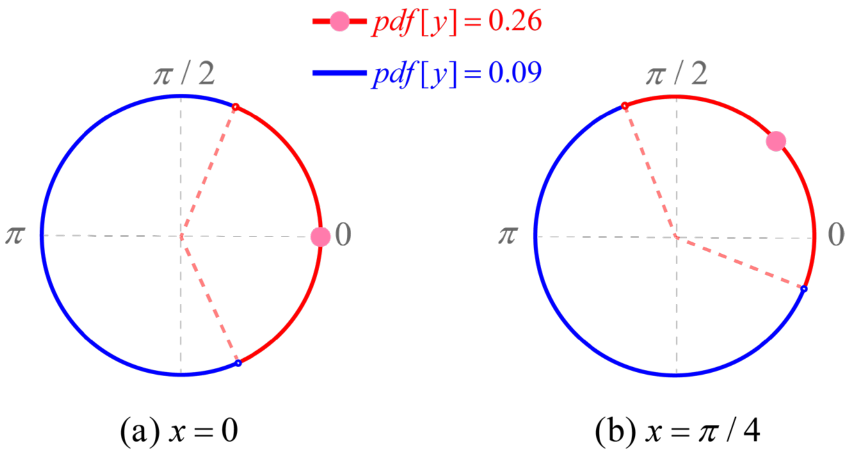
** Estimating Numerical Distributions under Local Differential Privacy, SIGMOD 2020

$\mathcal{M}_\circ$ and $\mathcal{M}_-$

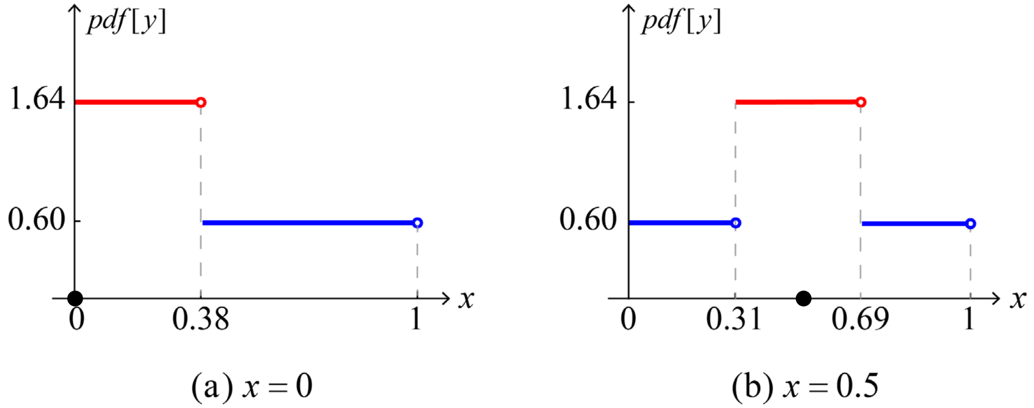
- Design piecewise-based mechanisms $\mathcal{M}_\circ$ and $\mathcal{M}_-$

$[0,2\pi) \rightarrow [0,2\pi)$ $[0,1) \rightarrow [0,1)$

- Examples of $\mathcal{M}_\circ$ and $\mathcal{M}_-$



$\mathcal{M}_\circ(x; \varepsilon = 1)$



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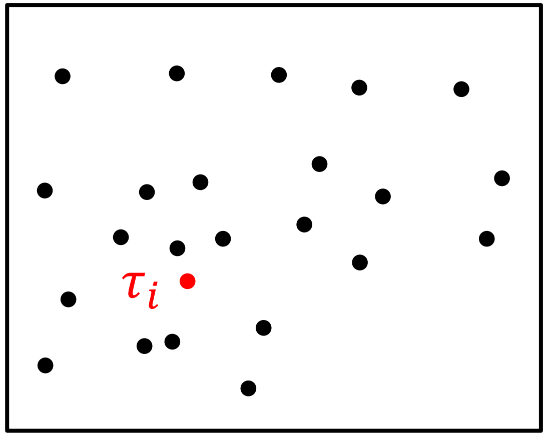
Rounding to Discrete Spaces

- $\mathcal{S}$ = [longtitude] × [latitude]



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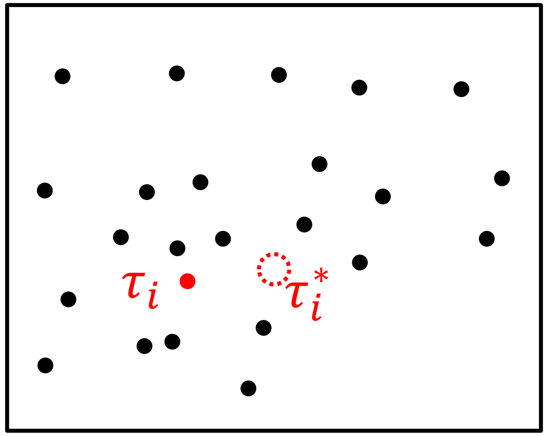

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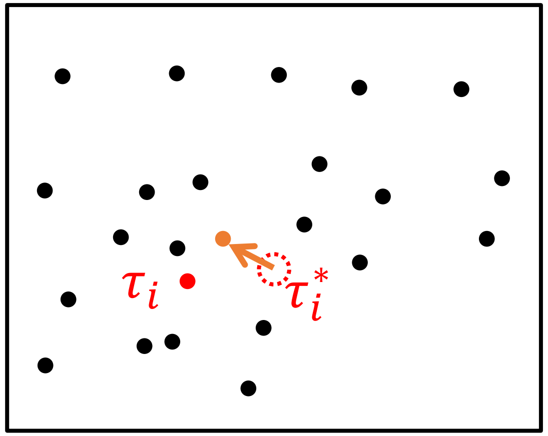

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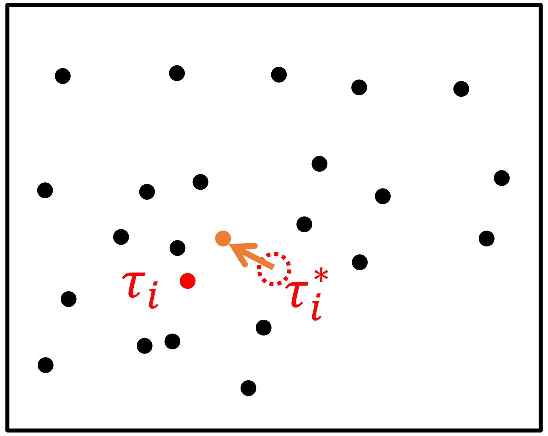

- Advantages:

  - not affected by # locations/cells

  - efficient sampling ($\Theta(1)$ time complexity; EM: $\Theta(m)$)

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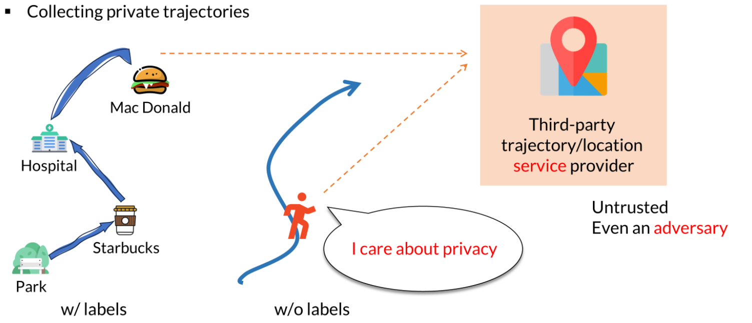

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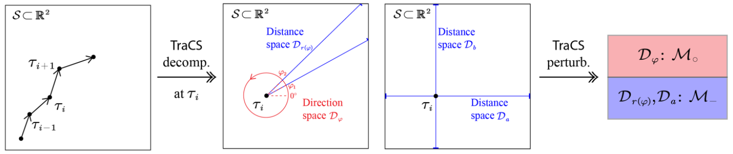

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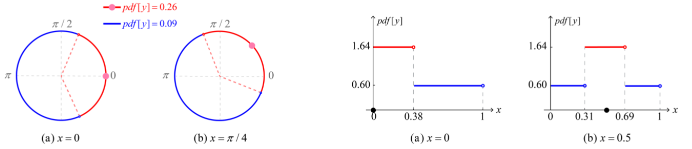

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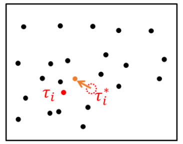

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