CSC 226

Algorithms and Data Structures: II
More Discrete Math

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The Pigeonhole Principle

If m pigeons occupy n pigeonholes and $m > n$, then at least one pigeonhole has two or more pigeons roosting in it.

Examples

1. If I draw 14 cards from a standard deck of 52, will there be a pair?
2. Given a text file with 500,000 words of size 4 or less, is it possible that the words are distinct?
3. While on a 4-week vacation I will drink at least 1 beer a day but no more than 40 total. Will there be a span of consecutive days in which I drink exactly 15 beers?

Posets

Cartesian Product

For sets A and B, the Cartesian product (or cross product) of A and B is

$$A \times B = \{(a, b) \colon a \in A, b \in B\}$$

Relation

For sets A and B, any subset of $A \times B$ is called a relation from A to B.

Posets

Partial Order

A relation $\Re$ on a set A is called a <u>partial order</u> if $\Re$ is

i. Reflexive: For all $x \in A, (x, x) \in \Re$

ii. Antisymmetric: For all $x, y \in A$, if $(x, y), (y, x) \in \Re$, then $x = y$.

iii. Transitive: For all $x, y, z \in A$, if $(x, y), (y, z) \in \Re$, then $(x, z) \in \Re$.

Posets

Partially Ordered Set

Let A be a set and $\Re$ a relation on A. The pair $(A, \Re)$ is called a <u>partially ordered set</u> (or <u>poset</u>) if $\Re$ on A is a partial order.

Hasse Diagram

If $\Re$ is a partial order on A, we construct a <u>Hasse diagram</u> for $\Re$ on A by connecting x "up" to y if and only if $(x, y) \in \Re$ and there are no other $z \in A$ such that $(x, z) \in \Re$ or $(z, y) \in \Re$.

Examples

4. Let $A = \{1,2,3,4\}$. Define $\Re$ on A by

$$\Re = \{(x, y) : x, y \in A \text{ and } x|y\}$$

Draw a relation diagram and a Hasse diagram for $\Re$.

5. Consider the power set, $\mathrm{P}(A)$, where $A = \{1,2,3\}$. Draw the Hasse diagram to illustrate the subset relation.

Posets

Total Order

If $(A, \Re)$ is a poset, it is a total order if for all $x, y \in A$, either $(x, y) \in \Re$ or $(y, x) \in \Re$.