

CSC 226

Algorithms and Data Structures: II

All Pairs Shortest Paths

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ECS 466

All-Pairs Shortest Paths

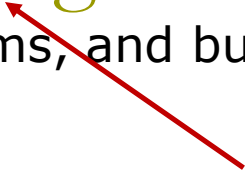


A dynamic programming
approach

All-Pairs shortest paths in weighted digraphs without negative-weight cycles

- Variation of Floyd-Warshall algorithm
- Algorithm-design technique: dynamic programming

Algorithmic Paradigms

- ✦ **Greedy** – Build up a solution incrementally, myopically optimizing some local criterion.
- ✦ **Divide-and-Conquer** – Break up a problem into **independent** subproblems, solve each subproblem, and combine solutions to subproblems to form solution to original problem.
- ✦ **Dynamic Programming** – Break up a problem into a series of **overlapping** subproblems, and build up solutions to larger and larger subproblems.


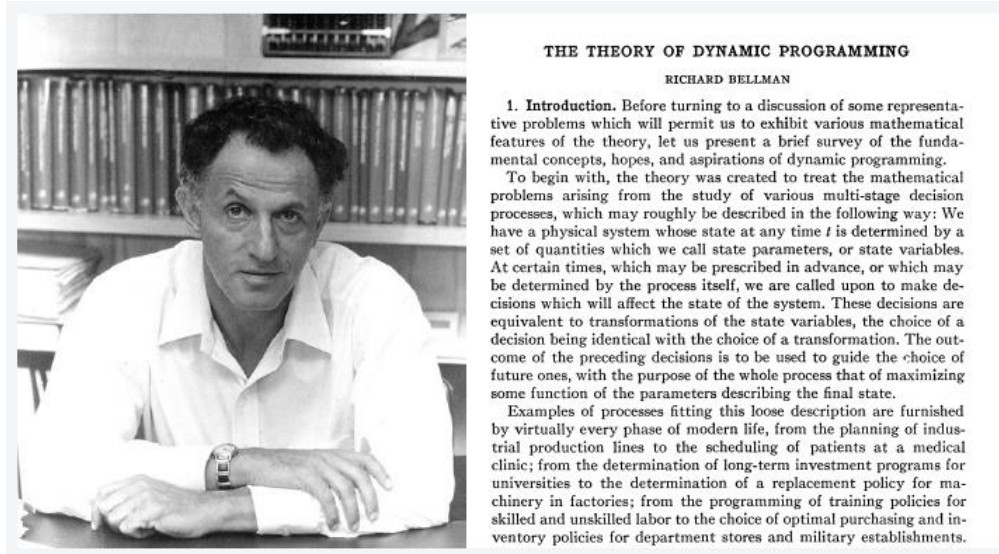
fancy name for caching
intermediate results in a table

Dynamic Programming History

✦ **Bellman** – Pioneered the systematic study in dynamic programming in the 1950s.

✦ **Etymology**

- Dynamic programming = planning over time.
- Secretary of Defense was hostile to mathematical research.
- Bellman sought an impressive name to avoid confrontation.



Algorithm Design Technique

Dynamic Programming

✦ **Simple subproblems**

- There is a way to break down the problem into simple subproblems of similar structure
- The subproblems can easily be defined with a few indices (i.e., i, j, k)

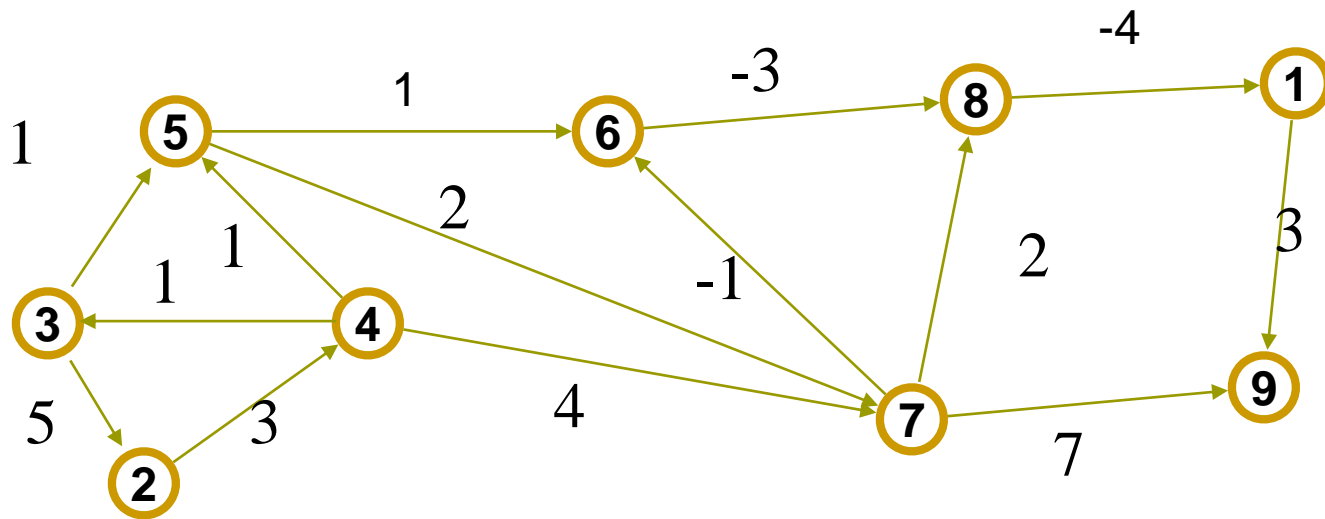
✦ **Subproblem optimality**

- An optimal solution to the global problem is a composition of optimal subproblem solutions using a simple combining operation

✦ **Subproblem overlap**

- Optimal solutions to unrelated subproblems can have subproblems in common. Such overlap is recognized and improves the efficiency significantly

Algorithm AllPairsShortestPaths(G)



Algorithm AllPairsShortestPaths(G)

Input: An edge-weighted digraph $G=(V,E)$ without negative-weight cycles. Further $V=\{1,2,\dots,n\}$

Output: Matrix D s.t. for all $i,j \in V$, $D[i,j]$ denotes the length of a shortest path from i to j

Terminology and Notation

- ✦ Let v_1, v_2, \dots, v_l be the vertices of a path p in directed graph G .
- ✦ Then the vertices v_2, v_3, \dots, v_{l-1} are called *intermediate vertices* of p .
- ✦ Let $d_{ij}^{(k)}$ be the length of a shortest path from i to j such that all intermediate vertices that are on the path are members of the set $\{1, 2, \dots, k\}$.
- ✦ Then $d_{ij}^{(0)}$ is the weight of the edge from i to j if such an edge exists, $+\infty$ otherwise
- ✦ $d_{ij}^{(n)}$ is the shortest path from i to j

Observations

1. A shortest path does not contain the same vertex twice.

Proof. Otherwise the path would contain a cycle, and removing the cycle would shorten the path.

Observations

2. If vertex k is *not* on a shortest path from i to j with intermediate vertices from $\{1, 2, \dots, k\}$ then $d_{ij}^{(k)} = d_{ij}^{(k-1)}$.

Proof. Since k is *not* on a shortest path from i to j with intermediate vertices from $\{1, 2, \dots, k\}$, adding k to the set $\{1, 2, \dots, k-1\}$ of intermediate vertices will not improve the length of the shortest path.

Observations

3. If vertex k is on a shortest path from i to j with intermediate vertices from $\{1, 2, \dots, k\}$ then

$$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

Proof. k is somewhere on the shortest path from i to j . Let us consider the shortest paths from (1) i to k and (2) k to j . The internal vertex set for these is $\{1, 2, \dots, k-1\}$ since k is an external vertex for both paths. Thus,

$$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

We conclude for a shortest path from i to j with intermediate vertices from $\{1, 2, \dots, k\}$:

$$d_{ij}^{(k)} = \min\{d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\}$$

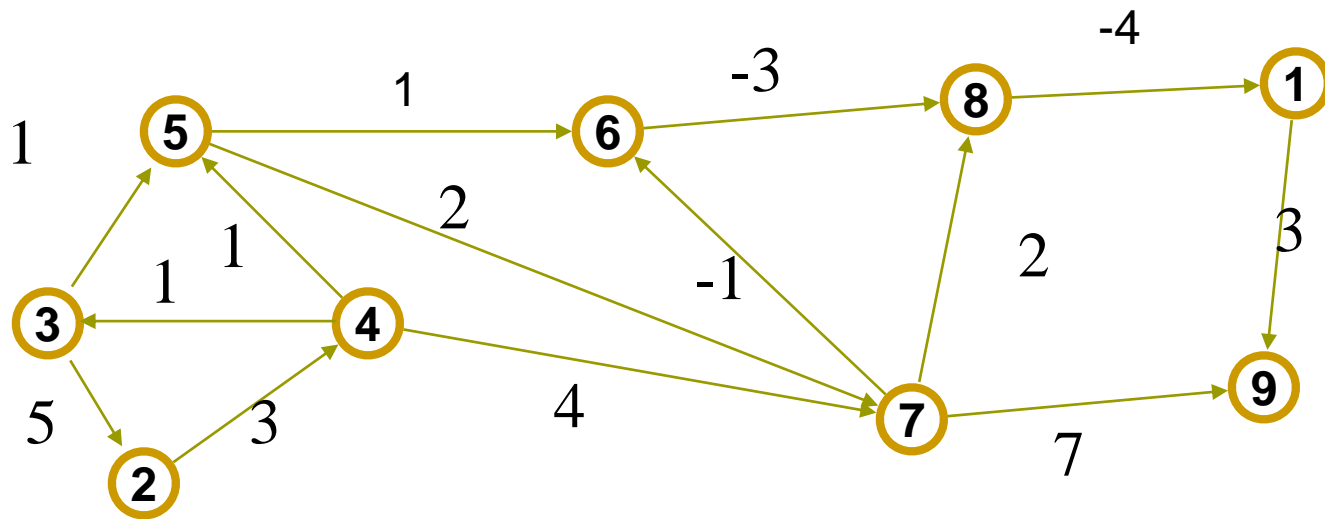
Developing the Dynamic Programming Algorithm

- ✦ Let D be an $n \times n$ matrix for each pair of vertices in the graph
- ✦ Initially, each cell $D[i, j]$ contains the weight of edge (i, j) , if existent, 0 if $i = j$, and $+\infty$ otherwise.
- ✦ We then iterate over the included intermediate vertices and update each cell for the (possibly) improved paths.
- ✦ Once all vertices are included in the set of intermediate vertices, $D[i, j]$ contains the weight of a shortest path from i to j if existent, 0 if $i = j$, and $+\infty$ otherwise.

Algorithm AllPairsShortestPaths(G)

```
// Initializing
for i  $\leftarrow$  1 to n do
    for j  $\leftarrow$  1 to n do
        if i=j then
            D[i,j]  $\leftarrow$  0
        if (i,j) $\in$ G then
            D[i,j]  $\leftarrow$  w((i,j))
        else
            D[i,j]  $\leftarrow$   $+\infty$ 
```

Algorithm AllPairsShortestPaths(G)



Algorithm AllPairsShortestPaths(G)

	1	2	3	4	5	6	7	8	9
1	0								
2		0							
3			0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

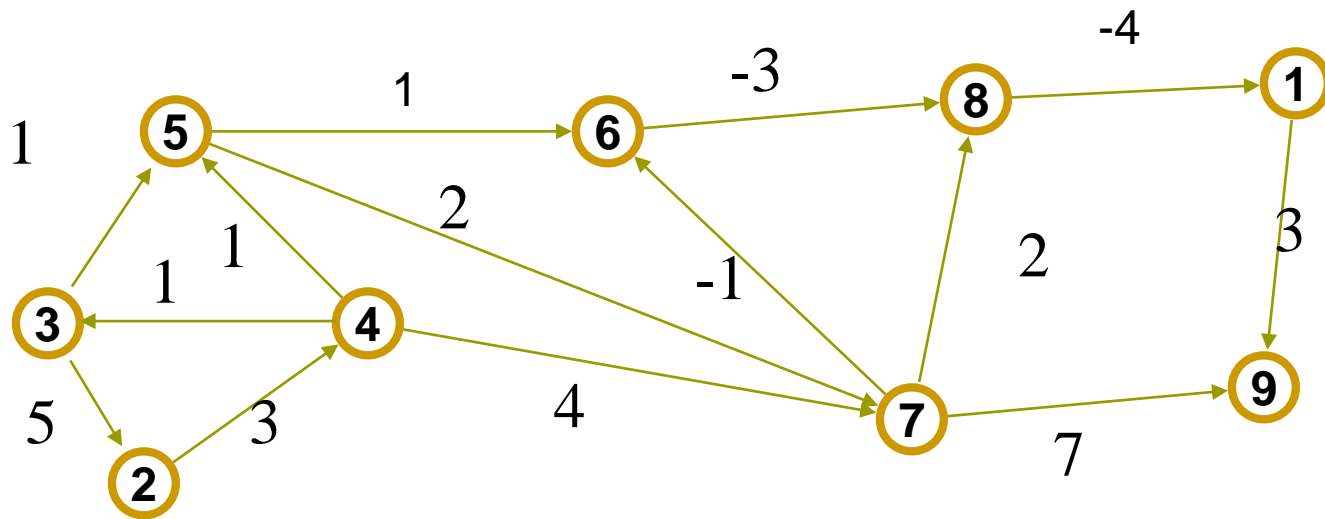
Algorithm AllPairsShortestPaths(G)

	1	2	3	4	5	6	7	8	9
1	0								3
2		0		3					
3		5	0		1				
4			1	0	1		4		
5					0	1	2		
6						0		-3	
7						-1	0	2	7
8	-4							0	
9									0

Algorithm AllPairsShortestPaths(G)

```
//Compute shortest paths
for k  $\leftarrow$  1 to n do //k is the latest included vertex
    for i  $\leftarrow$  1 to n do //path starts at i
        for j  $\leftarrow$  1 to n do //path ends at j
            if  $i \neq k$  and  $j \neq k$  then
                 $D[i,j] \leftarrow \min\{D[i,j], D[i,k] + D[k,j]\}$ 
return D
```

Algorithm AllPairsShortestPaths(G)



[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

j

i

[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

$K = 1$	j 1	2	3	4	5	6	7	8	9
i 1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

[illegible]

Algorithm AllPairsShortestPaths(G)

j

i

[illegible]

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

i

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

j

$K = 1$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

j-loop not
necessary

Algorithm AllPairsShortestPaths(G)

```
//Compute shortest paths
for k  $\leftarrow$  1 to n do //k is the latest included vertex
  for i  $\leftarrow$  1 to n do //path starts at i
    if  $i \neq k$  then
      if  $D[i,k] < +\infty$  then
        for j  $\leftarrow$  1 to n do //path ends at j
          if  $k \neq j$  then
            if  $D[k,j] < +\infty$  then
               $D[i,j] \leftarrow \min\{D[i,j], D[i,k] + D[k,j]\}$ 
return D
```

[illegible]

Algorithm AllPairsShortestPaths(G)

	j	1	2	3	4	5	6	7	8	9
K = 1	i	1	0	+∞	+∞	+∞	+∞	+∞	+∞	3
		2	+∞	0	+∞	3	+∞	+∞	+∞	+∞
		3	+∞	5	0	+∞	1	+∞	+∞	+∞
		4	+∞	+∞	1	0	1	+∞	4	+∞
		5	+∞	+∞	+∞	+∞	0	1	2	+∞
		6	+∞	+∞	+∞	+∞	+∞	0	+∞	-3
		7	+∞	+∞	+∞	+∞	+∞	-1	0	2
		8	-4	+∞	+∞	+∞	+∞	+∞	+∞	0
		9	+∞	+∞	+∞	+∞	+∞	+∞	+∞	+∞

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

j

$K = 1$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

i

Algorithm AllPairsShortestPaths(G)

j

[illegible]

i

Algorithm AllPairsShortestPaths(G)

j

$K = 1$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

j

$K = 1$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty/-1$
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

[illegible]

i

j

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

[illegible]

[illegible]

i

Algorithm AllPairsShortestPaths(G)

j

$K = 2$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$ /8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

Algorithm AllPairsShortestPaths(G)

j

$K = 2$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$ /8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

i

Algorithm AllPairsShortestPaths(G)

j

$K = 2$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	$+\infty$ /8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

[illegible]

Algorithm AllPairsShortestPaths(G)

j

$K = 3$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
i 4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

Algorithm AllPairsShortestPaths(G)

j

$K = 3$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
i 4	$+\infty$	$+\infty$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

Algorithm AllPairsShortestPaths(G)

j

[illegible]

Algorithm AllPairsShortestPaths(G)

j

$K = 3$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty$	0	$+\infty$	3	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
3	$+\infty$	5	0	8	1	$+\infty$	$+\infty$	$+\infty$	$+\infty$
i 4	$+\infty$	$+\infty/6$	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

i

j

[illegible]

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

[illegible]

Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

		j								
	$K = 4$	1	2	3	4	5	6	7	8	9
1		0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2		$+\infty$	0	$+\infty/4$	3	$+\infty/4$	$+\infty$	$+\infty/7$	$+\infty$	$+\infty$
3		$+\infty$	5	0	8	1	$+\infty$	$+\infty/12$	$+\infty$	$+\infty$
4		$+\infty$	6	1	0	1	$+\infty$	4	$+\infty$	$+\infty$
5		$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8		-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0
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Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

		j								
	$K = 5$	1	2	3	4	5	6	7	8	9
1		0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2		$+\infty$	0	4	3	4	$+\infty/5$	7/6	$+\infty$	$+\infty$
3		$+\infty$	5	0	8	1	$+\infty/2$	12/3	$+\infty$	$+\infty$
4		$+\infty$	6	1	0	1	$+\infty/2$	4/3	$+\infty$	$+\infty$
5		$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	1	2	$+\infty$	$+\infty$
6		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty$
7		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	2	7
8		-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9		$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0
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Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

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Algorithm AllPairsShortestPaths(G)

		j								
K = 8		1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty/-2$	0	4	3	4	4	5	6	2	13/1
3	$+\infty/-5$	5	0	8	1	2	2	3	-1	10/-2
4	$+\infty/-5$	6	1	0	1	2	2	3	-1	10/-2
5	$+\infty/-6$	$+\infty$	$+\infty$	$+\infty$	0	1	2	2	-2	9/-3
6	$+\infty/-7$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty/-4$
7	$+\infty/-8$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	-4	7
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

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Algorithm AllPairsShortestPaths(G)

		j							
$K = 8$	1	2	3	4	5	6	7	8	9
1	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	3
2	$+\infty/-2$	0	4	3	4	5	6	2	13/1
3	$+\infty/-5$	5	0	8	1	2	3	-1	10/-2
4	$+\infty/-5$	6	1	0	1	2	3	-1	10/-2
5	$+\infty/-6$	$+\infty$	$+\infty$	$+\infty$	0	1	2	-2	9/-3
6	$+\infty/-7$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	$+\infty$	-3	$+\infty/-4$
7	$+\infty/-8$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	-1	0	-4	7/-5
8	-4	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0	-1
9	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	0

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Algorithm AllPairsShortestPaths(G)

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