Name:
Student #: SOLUTION

Question [2 marks]

Consider any sequence of three distinct values, that we denote $S = [x_1, x_2, x_3]$. Using a **decision tree**, illustrate all the possible comparisons made by the **merge-sort algorithm**. Each comparison must be of the form $x_i < x_j$? and is a node in the tree whose left branch is the direction for "yes" and the right branch is for "no". Assuming that each $x_i \in \{1,2,3\}$, label each of the external nodes in the decision tree with the particular permutation of sequence S associated with the path taken by the algorithm from the root to that node. (That is, label each leaf with the permutation associated with that leaf like we did in class). Assume that the first split is $S_1 = [x_1, x_2]$ and $S_2 = [x_3]$ and that in the merge step we always make the comparison $x_i < x_j$? where $x_i \in S_1$ and $x_j \in S_2$.

- $x_i < x_j$? where i...
- label each leaf with S.

Decision tree:

Root: $X_1 < X_2$?

- yes → $X_1 < X_3$?
  - yes → $X_2 < X_3$?
    - yes → $X_1 < X_2 < X_3$, $S = [1,2,3]$
    - no → $X_1 < X_3 < X_2$, $S = [1,3,2]$
  - no → $X_3 < X_1 < X_2$, $S = [2,3,1]$
- NO → $X_2 < X_3$?
  - yes → $X_1 < X_3$?
    - yes → $X_2 < X_1 < X_3$, $S = [2,1,3]$
    - no → $X_2 < X_3 < X_1$, $S = [3,1,2]$
  - no → $X_3 < X_2 < X_1$, $S = [3,2,1]$