CSC 226 SUMMER 2018
ALGORITHMS AND DATA STRUCTURES II
MIDTERM EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: _____

2. Name: _____

3. DATE: 20 JUNE 2018
   DURATION: 50 MINUTES
   INSTRUCTOR: RICH LITTLE

4. THIS QUESTION PAPER HAS SEVEN PAGES, DOUBLE-SIDED, INCLUDING THE COVER PAGE.

5. THIS QUESTION PAPER HAS FOUR QUESTIONS.

6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.

7. WRITE YOUR NAME ON THE BACK OF THE EXAM.

8. THIS IS A CLOSED BOOK EXAM. CALCULATORS ARE ALLOWED.

9. READ THROUGH ALL THE QUESTIONS AND ANSWER THE EASY QUESTIONS FIRST. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (10)	
Q2 (10)	
Q3 (10)	
Q4 (10)	
TOTAL (40) =	

1. (a). [2 mark] If a free tree has n nodes, then how many edges does it have?

(b). [2 mark] Given a graph $G = (V, E)$ where $|V| = n$ and $|E| = m$, what is the worst-case running time of Prim's algorithm for finding a minimum spanning tree?

(c). [2 mark] Suppose that $\log_{n\to\infty} f(n)/g(n) = \infty$. Is $f(n) \in o(g(n))$ or is $g(n) \in o(f(n))$?

(d). [2 Marks] Consider the following id[] array which is the result of a sequence of quick-unions. Draw the corresponding disjoint set trees.

id[i]	9	7	7	9	3	7	6	7	5	9
i	0	1	2	3	4	5	6	7	8	9

(e). [2 Marks] What is the least number of internal nodes in a left-leaning red-black tree with 2 *red* edges?

2. (a). [3 marks] In how many ways can the symbols a,b,c,d,e,e,e,e,e be arranged so that no e is adjacent to another e?

(b). [3 marks] With n a positive integer, evaluate the following sum. (**Hint:** Binomial theorem)

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^k\binom{n}{k} + \cdots + 2^n\binom{n}{n}$$

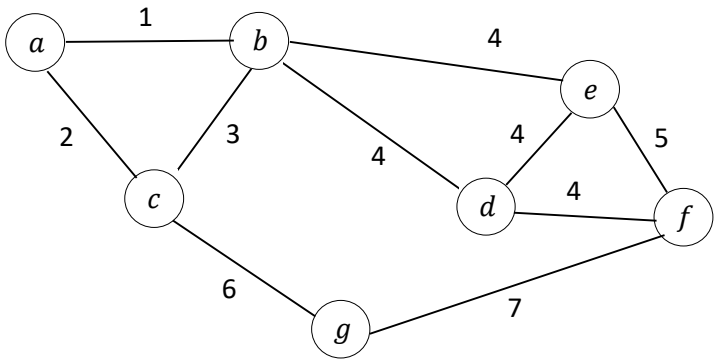
(c). [4 marks] Let S be a set of five distinct positive integers, the maximum of which is at most 9. Consider all the subsets A of S where $1 \leq |A| \leq 3$. Let s_A denote the sum of the elements in subset A. Prove that there must be at least two such subsets, say A and B, such that $s_A = s_B$.
(**Hint:** Compare the number of such subsets to the possible range of s_A.)

3. (a). [6 marks] Fill in the missing code below from the Sedgewick book's implementation of a right rotation for a Red-Black tree. The `Node` class is given.
Recall that you have access to global final variables `RED = true` and `BLACK = false` and the helper method `size(Node h)` which returns the number of internal nodes in the tree rooted at `h`.

```
private class Node
{
    Key key;              // key
    Value val;            // associated value
    Node left, right;     // subtrees
    int N;                // # of nodes in subtree
    boolean color;        // color of parent link
}

Node rotateRight(Node h) {

     Node x = h.left;
     // two assignments to adjust links

     h.left = _____________________________;

     x.right = _____________________________;

     // two assignments to adjust colour

     x.color = _____________________________;

     h.color = _____________________________;

     // two assignments to adjust N, the number of descendents

     x.N = _____________________________;

     h.N = _____________________________;

     return x
}
```

(b). [4 marks] Given a `Node h`, under what circumstances do we need to call `rotateRight(h)`?

4. (a). [5 marks] Consider graph G given below. Draw all the minimum spanning trees that exist for this graph.



(b). [5 marks] Let C be a cycle in a weighted graph G, where G has distinct edge weights (i.e. no two edges have the same weight.) Let edge e be the "heaviest" edge in the cycle. Prove that the minimum spanning tree for G does not contain edge e.

THE END