CSC 226

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1 (a) "ALGORITHMS" 10 letter

There is no letter repeat

So $P(10,10) = \frac{10!}{(10-10)!} = 3628800$ arrangement

(b) "DATASTRUCTURES" 13 letter

There is T 3time, R 2time S2time U 2time A 2time D and C 1time

$\frac{14!}{3!\cdot 2!\cdot 2!\cdot 2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!} = 908107200$

(c) There are three vowel in "ALGORITHMS"

So $3!\cdot 8! = 241920$

(d) There are 4 vowel

$\frac{5!}{2!\cdot 2!\cdot 1!}\cdot\frac{9!}{3!\cdot 2!\cdot 2!\cdot 1!\cdot 1!} = 4536000$

2.(a) $\binom{13}{2}\cdot\binom{39}{3} = 712842$

(b) $712842 + \binom{13}{1}\cdot\binom{39}{4} + \binom{39}{5}\cdot\binom{13}{0} = 2357862$

(c) $\binom{13}{3}\cdot\binom{13}{2} = 22308$

(d) $\underbrace{\binom{12}{1}\cdot\binom{4}{2}\binom{24}{2}}_{19872} + \underbrace{\binom{13}{2}\cdot\binom{4}{2}\cdot\binom{24}{1}}_{11232} + \underbrace{\binom{13}{1}\binom{12}{1}\binom{4}{2}\binom{24}{1}}_{22464} + 1\cdot 14 = 54582$

3(a) $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

as we know: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$\therefore \binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!(n-k)!}$ $\quad \binom{n-1}{k} = \frac{(n-1)!}{k!(n-k-1)!}$

$$\frac{(n-1)!}{(k-1)!(n-k)!} + \frac{(n-1)!}{k!(n-k-1)!} = (n-1)!\left(\frac{k!(n-k-1)! + (k-1)!(n-k)!}{k!(n-k-1)!(k-1)!(n-k)!}\right)$$

$$= (n-1)!\left(\frac{\cancel{(k-1)!}\cancel{(n-k-1)!}(k+(n-k))}{k!\cancel{(n-k-1)!}\cancel{(k-1)!}(n-k)!}\right) = \frac{(n-1)!\cdot n}{k!\,(n-k)!} = \frac{n!}{k!(n-k)!}$$

(b) Binomial theorem is : $(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots \binom{n}{n}x^0 y^n$

$-2^n = x^n - n(2x^{n-1}+x^n) + \frac{n(n-1)}{2}(2+x)^2 x^{n-2} - \cdots + (-1)^n(2+x)^n$

Let $x = x$ $y = -(2+x)$

By the Binomial theorem :

$(x+(-(2+x)))^n = \binom{n}{0}x^n(-(2+x))^0 + \binom{n}{1}x^{n-1}(-(2+x))^1 + \cdots \binom{n}{n}x^0(-(2+x))^n$

$= 1 \cdot x^n (-1)^0 \cdot (2+x)^0 + \frac{n!}{1!(n-1)!} \cdot x^{n-1} \cdot (-1)^1 \cdot (2+x)^1 + \cdots \cdot 1 \cdot 1 \cdot (-1)^n (2+x)^n$

$= x^n - n \cdot x^{n-1} \cdot (2+x) + \cdots (-1)^n(2+n)^n$

$= x^n - n \cdot (2x^{n-1}+x^n) + \cdots + (-1)^n(2+n)^n$

So it is true.

4.(a) $\binom{32+4-1}{4-1} = \frac{35 \cdot 34 \cdot 33}{6} = 6545$

(b). $x_1 x_2 \geq 2$ $x_3 x_4 \geq 1$

we give 2 to X_1, 2 to X_2, 1 to X_3, 1 to X_4 26 left

$r=26$ $n=4$

$\binom{4+26-1}{26} = \binom{29}{26} = 3654$

5. $A \subseteq \{1,2,3,\dots 25\}$ $|A|=9$ $|C|=|D|=5$ B is subset of A

$1 \leq S_A \leq 21+22+23+24+25=115$

$\binom{9}{5}=126$ $|C|$ or $|D|=5$

For pigeonhole Primciple:

If m pigeons occupy n pigeonholes and $m>n$, then at least one pigeonhole has two or more pigeons roosting in it.

$126 > 115$

there are two different subset have same sum.