Each multiple choice question has **EXACTLY ONE** correct answer.

1. Which one of the following is an unambiguous context-free grammar for

$$\{w \in \{(,)\}^* \mid |w| \geq 0 \text{ and } w \text{ is balanced}\}?$$

(Note: the empty string is balanced)

(a) $S \to SS \mid (S)$
(b) $S \to S(S) \mid \epsilon$
(c) $S \to SS \mid ()$
(d) $S \to S(S) \mid ()$
(e) $S \to SS \mid (S) \mid \epsilon$

2. Which one of the following is a CNF grammar that is equivalent to $S \to 1S0S \mid 0S1S \mid \epsilon$

(a) $S_0 \to S \mid \epsilon$
$S \to UR \mid ZT \mid SS \mid ZU \mid UZ$
$R \to SZ$
$T \to SU$
$U \to 1$
$Z \to 0$

(b) $S_0 \to USZ \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon$
$S \to USZ \mid ZT \mid SS \mid ZU \mid UZ$
$T \to SU$
$U \to 1$
$Z \to 0$

(c) $S_0 \to UR \mid ZT \mid SS \mid 01 \mid 10 \mid \epsilon$
$S \to UR \mid ZT \mid SS \mid 01 \mid 10$
$R \to S0$
$T \to S1$

(d) $S_0 \to UR \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon$
$S \to UR \mid ZT \mid SS \mid ZU \mid UZ$
$R \to SZ$
$T \to SU$
$U \to 1$
$Z \to 0$

(e) None of the above.

3. Consider the following CNF grammar

$$\begin{aligned} S &\to SS \mid LX \mid LR \\ X &\to SR \\ L &\to (\\ R &\to) \end{aligned}$$

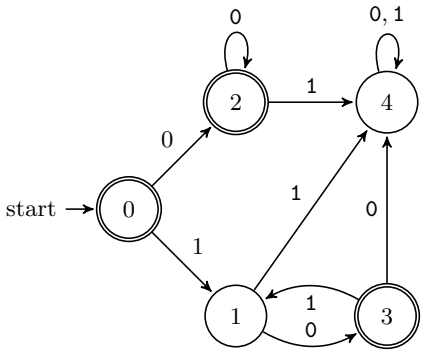
Suppose we run the CYK algorithm on this grammar, and the input ()(()), producing the following table (with missing entries):

1	2	3	4	5	6	
L	(i)	$\emptyset$	$\emptyset$	$\emptyset$	(vi)	1
	R	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	2
		L	$\emptyset$	$\emptyset$	S	3
			(ii)	(iii)	(v)	4
				(iv)	$\emptyset$	5
					R	6

What are the correct values for the missing entries?

(a) (i)=S, (ii)=R, (iii)=S, (iv)=L, (v)=X, (vi)=S
(b) (i)=S, (ii)=R, (iii)=S, (iv)=R, (v)=X, (vi)=S
(c) (i)=S, (ii)=L, (iii)=S, (iv)=S, (v)=X, (vi)=S
(d) (i)=S, (ii)=S, (iii)=S, (iv)=R, (v)=X, (vi)=S
(e) None of the above.

4. Which one of the following grammars (with start symbol S) generates the language recognized by the following DFA?



(a)

$$\begin{aligned} S &\rightarrow 0B \mid 1A \mid \epsilon \\ A &\rightarrow 0D \mid 1C \\ B &\rightarrow 0B \mid 1D \mid \epsilon \\ C &\rightarrow 0D \mid 1A \mid \epsilon \\ D &\rightarrow 0D \mid 1D \end{aligned}$$

(b)

$$\begin{aligned} S &\rightarrow 0B \mid 1A \mid \epsilon \\ A &\rightarrow 0C \mid 1D \mid \epsilon \\ B &\rightarrow 0B \mid 1D \\ C &\rightarrow 0D \mid 1A \mid \epsilon \\ D &\rightarrow 0D \mid 1D \end{aligned}$$

(c)

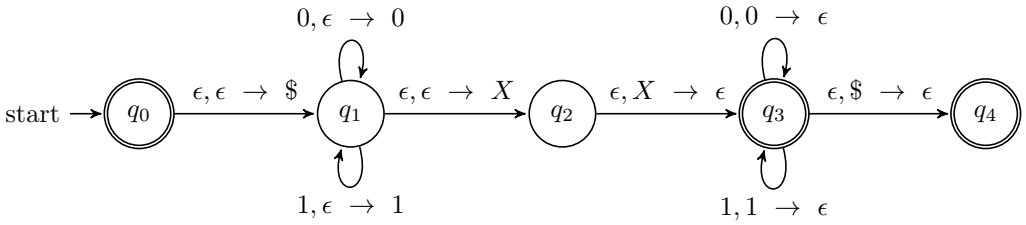
$$\begin{aligned} S &\rightarrow 0B \mid 1A \mid \epsilon \\ A &\rightarrow 0C \mid 1D \\ B &\rightarrow 0B \mid 1D \mid \epsilon \\ C &\rightarrow 0D \mid 1A \mid \epsilon \\ D &\rightarrow 0D \mid 1D \end{aligned}$$

(d)

$$\begin{aligned} S &\rightarrow 0B \mid 1A \mid \epsilon \\ A &\rightarrow 0D \mid 1C \mid \epsilon \\ B &\rightarrow 0B \mid 1D \\ C &\rightarrow 0D \mid 1A \mid \epsilon \\ D &\rightarrow 0D \mid 1D \end{aligned}$$

(e) None of the preceding.

5. Which one of the following grammars generates the language recognized by the following PDA?



(a) $S \rightarrow \epsilon \mid 1S1 \mid 0S0$
(b) $S \rightarrow \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0$
(c) $S \rightarrow \epsilon \mid 1S1 \mid 0S0 \mid SS$
(d) $S \rightarrow \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0 \mid SS$
(e) None of the preceding

6. Suppose that L is decided by a 3-tape TM than runs in time $O(n^2)$. Which one of the following statements could be false?

   (a) L is decided by a 2-tape TM that runs in time $O(n^4)$.
   (b) L is decided by a 1-tape TM that runs in time $O(n^4)$.
   (c) L is decided by a 1-tape TM that runs in time $O(n^3)$.
   (d) L is decided by a 1-tape TM that runs in time $O(n^6)$.
   (e) None of the above.

7. Which one of the following statements could be false?

   (a) If L is decided by a nondeterministic TM then it is decided by a TM that halts on every input.
   (b) Any language that can be enumerated by a Turing machine is recognizable.
   (c) There are countably many decidable languages.
   (d) There are countably many recognizable languages.
   (e) None of the above

8. Which of the following statements is false?

   (a) Every language decided by a nondeterministc TM is decided by a deterministic TM
   (b) Every language decided by a deterministic TM is recognized by a deterministic TM
   (c) Every langauge recognized by a deterministic TM is recognized by a nondeterministic TM
   (d) Every language recognized by a deterministic TM is decided by a deterministic TM
   (e) None of the above

Suppose we have the following TM for the language $\{a^n b^n \mid n \geq 0\}$: $M = (\{q_0, q_1, q_2, q_3, q_a, q_r\}, \{a, b\}, \{a, b, a', b', \sqcup$

	a	b	a'	b'	$\sqcup$
q_0	(q_1, a', R)	(q_r, a', R)	(q_r, a', R)	(q_3, b', R)	
q_1	(q_1, a, R)	(q_2, b', L)	(q_r, a', R)	(q_1, b', R)	
q_2	(q_2, a, L)	(q_r, a', R)	(q_0, a', R)	(q_2, b', L)	
q_3	(q_r, a', R)	(q_r, a', R)	(q_r, a', R)	(q_3, b', R)	$(q_a, \sqcup, R)$

9. What is the initial configuration of M when started on input $aaabb$?

   (a) $q_0 aaabb\sqcup$

   (b) $aq_0 aabb$

   (c) $a'a'a'b'b'q_r$

   (d) $q_0 aaabb$

   (e) None of the above

10. What state is M in when it halts on input $aaabb$?

    (a) M does not halt on this input

    (b) We don't know because it is undecidable whether M halts on this input

    (c) q_r

    (d) q_3

    (e) None of the above

11. Suppose that A is TM-decidable and $A \leq_m B$. Which one of the following statements could be false?

    (a) $\overline{A}$ is TM-decidable.

    (b) A is TM-recognizable.

    (c) $\overline{B}$ is TM-decidable.

    (d) $\overline{A}$ is TM-recognizable.

    (e) None of the above

12. Which one of the following statements could be false?

    (a) There is a languge L such that neither L nor $\overline{L}$ is decidable.

    (b) There is a languge L such that neither L nor $\overline{L}$ is recognizable.

    (c) If L is undecidable and $\overline{L}$ is not recognizable, then L must be recognizable.

    (d) If L is recognizable and $\overline{L}$ is recognizable, then L must be decidable.

    (e) None of the above.