CSC 320 - ASSIGNMENT 2 - SOLUTIONS

1\. (a) $\{w \mid w \text{ begins and ends with a } 01\}$

$$01 \cup 01(0\cup 1)^*01$$

(b) $\{w \mid w \text{ has at most one pair of consecutive 1's}\}$

$$(0\cup 10)^*(\varepsilon \cup 11)^*(01\cup 0)^* \cup 1$$

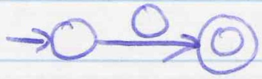
(c) $\{w \mid w \text{ contains } n \text{ 0's where } n \text{ is divisible by } 5\}$

$$(1^*01^*01^*01^*01^*01^*)^* \cup 1^*$$

<u>Note</u>: With all of these there may be multiple correct solutions

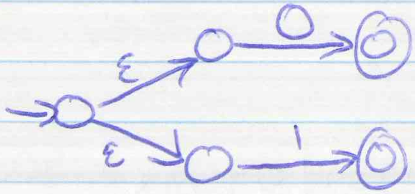
2\. (a) $R = (0\cup 1)^*1(0\cup 1)$

<u>0</u>:

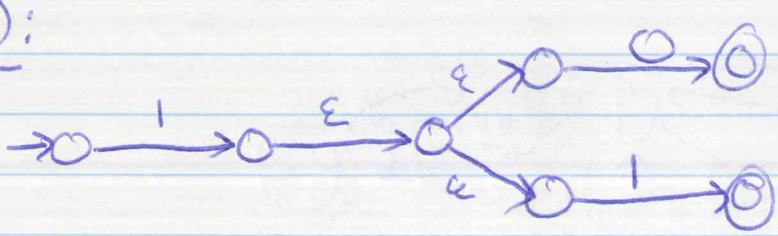


<u>1</u>:

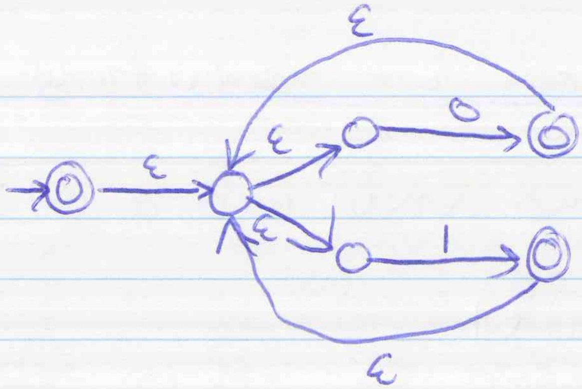
<u>0∪1</u>:



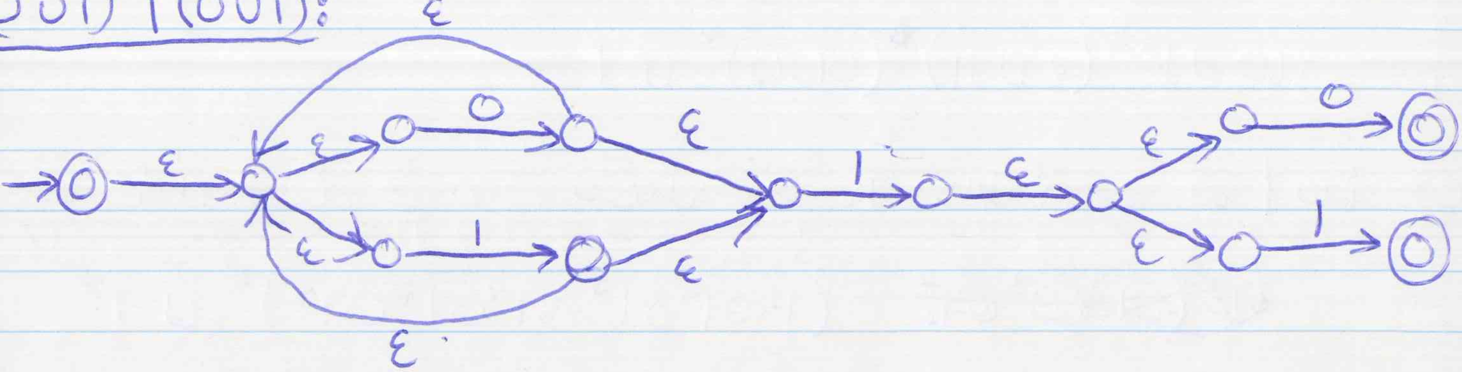
<u>1(0∪1)</u>:



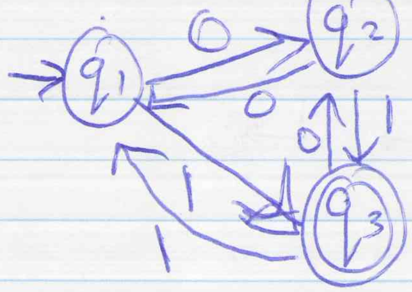
$(0 \cup 1)^*$:



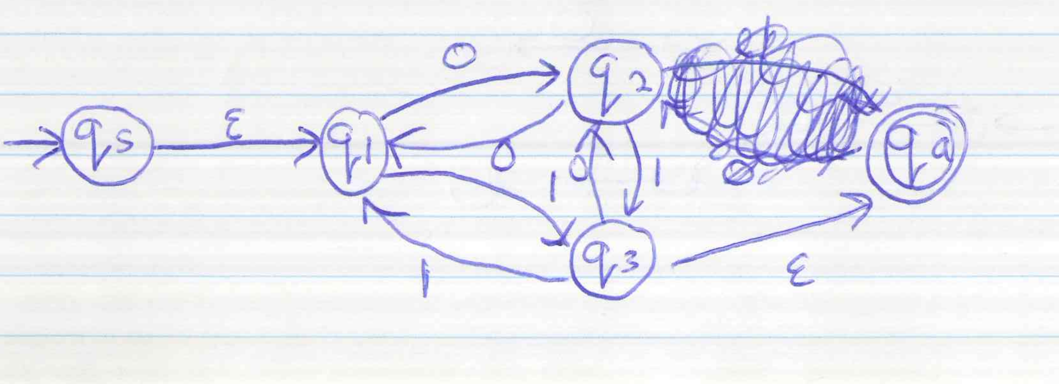
$(0 \cup 1)^* 1 (0 \cup 1)$:



(b) Let $M = (\{q_1, q_2, q_3\}, \{0,1\}, \delta, q_1, \{q_3\})$ where δ is

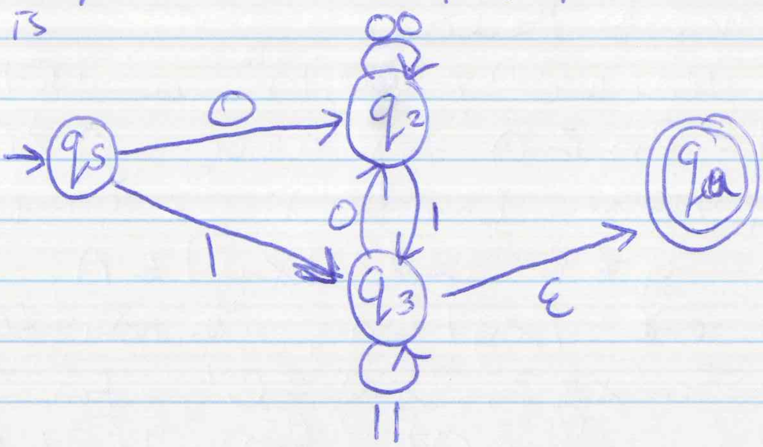


Construct GNFA, $G = (\{q_1, q_2, q_3, q_s, q_a\}, \{0,1\}, \delta_G, q_s, \{q_a\})$ where δ_G is given by

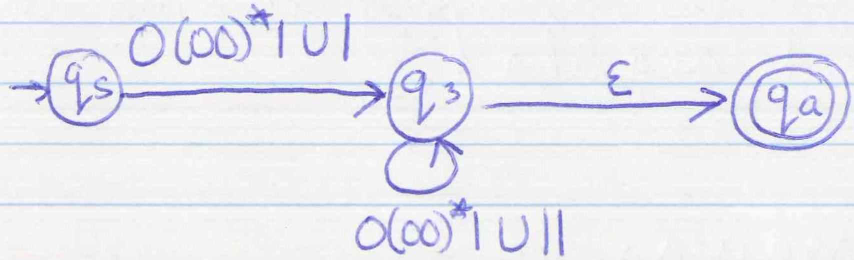


Now run CONVERT(G), [I will remove q_1, q_2, q_3 in that order. This is not a must.]

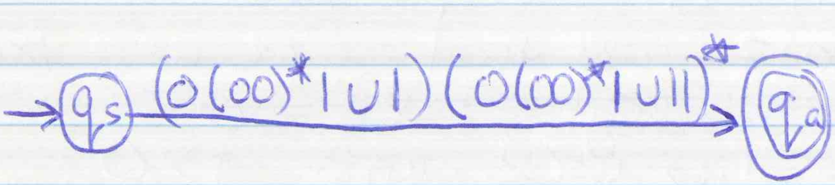
Remove q_1: $G_1 = (\{q_2, q_3, q_s, q_a\}, \{0,1\}, \delta_{G_1}, q_s, \{q_a\})$ where δ_{G_1} is



Remove q_2: $G_2 = (\{q_3, q_s, q_a\}, \{0,1\}, \delta_{G_2}, q_s, \{q_a\})$ where δ_{G_2} is



Remove q_3: $G_3 = (\{q_s, q_a\}, \{0,1\}, \delta_{G_3}, q_s, \{q_a\})$ where δ_{G_3} is



Therefore, the regular expression that describes M is

$$(0(00)^*1 \cup 1)(0(00)^*1 \cup 11)^*$$

3. Let $L = \{ w\bar{w} \mid w \in \{0,1\}^*$ and $\bar{w}$ is w with all the bits toggled$\}$.

Let L be regular, then there exists $n > 0$ such that for every $w \in L$ where $|w| \geq n$ the pumping lemma holds.

Let $w = 0^n 1^n$. Here $w = 0^n \in \{0,1\}^*$ and $\bar{w} = 1^n$, thus $w \in L$ and $|w| = 2n \geq n$ for any $n > 0$.

By the pumping, there exists x, y, z such that $w = xyz$ with $y \neq \varepsilon$ and $|xy| \leq n$.

Since $w = 0^n 1^n$ and $|xy| \leq n$, no matter how w is divided into xyz, $xy = 0^i$ for $i \leq n$.

Also, since $y \neq \varepsilon$, $y = 0^j$ for $1 \leq j \leq i$.

Consider $xy^0z = xz = 0^{n-j}1^n$. Since $j \geq 1$, $n - j < n$ and thus $xz \notin L$. This is a contradiction of the pumping lemma and thus L is not regular.

<u>In summary:</u>

- Let L be regular
- $\exists n > 0$ s.t. $\forall w \in L$ with $|w| \geq n$, p.l. holds
- Let $w = 0^n 1^n \in L$
- $|w| = 2n \geq n$ so p.l. holds
- Let $w = xyz$ such that $y \neq \varepsilon$ and $|xy| \leq n$
- ~~Let $k=0$ in $xy^kz = xz = 0^n$~~
- This implies $y = 0^j$ with $j > 0$
- Let $k = 0$ in $xy^kz = xz = 0^{n-j}1^n$
- $n - j \neq n$ thus $xz \notin L$
- Therefore L is <u>not</u> regular by contradiction

4. Let $L = \{1^p \mid p$ is a prime number$\}$. Assume L is regular, then $\exists n > 0$ such that $\forall w \in L$ with $|w| \geq n$, the pumping lemma holds.
Let $w = 1^p$ where p is a prime number larger than n. That is, $p \geq n$.
Since $|w| = p \geq n$, the pumping lemma states that there exists x, y, z such that $w = xyz$ where $|xy| \leq n$ and $y \neq \varepsilon$.
This implies that $y = 1^i$ where $i > 0$. Thus, for any $k \geq 0$, the pumping lemma says that xy^kz should be in L. But,

$$xy^kz = 1^{p+(k-1)i} \quad \text{for any } k \geq 0.$$

Let $k = p+1$, then
$$xy^kz = 1^{p+(p+1-1)i} = 1^{p+pi} = 1^{p(1+i)}$$

where p is a prime number and $i > 0$.
But, $p(1+i)$ is divisible by p a prime number and $(1+i) > 1$.
This implies that $xy^kz \notin L$, and thus a contradiction.

5. Let $L = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$, then $\bar{L} = \{w \in \{0,1\}^* \mid w$ is a palindrome$\}$. [Every string in $\{0,1\}^*$ is either a palindrome ~~or~~ it isn't.].
We know that regular languages are closed under complement, that is if L is regular then $\bar{L}$ is regular.

This implies the contrapositive is also true, that is if $\overline{L}$ is not regular, then L is not regular.
We will prove that $\overline{L} = \{w$ is a palindrome$\}$ is not regular.
Let $\overline{L}$ be regular for sake of contradiction.
Then, there exists an $n > 0$ such that for every $w \in \overline{L}$ where $|w| \geq n$, the pumping lemma holds.
Let $w = 0^n 1 0^n$, then $w^R = 0^n 1 0^n$ implying that $w \in \overline{L}$. Also, $|w| = 2n+1 > n$, and so the pumping lemma holds.
So, there exists x, y, z such that $w = xyz$ with $y \neq \varepsilon$ and $|xy| \leq n$.
This implies that $xy = 0^i$ for $i \leq n$ and $y = 0^j$ for $j > 0$.
Consider $xy^0z = xz = 0^{n-j} 1 0^n$ where $j > 0$ and $j \leq i$.
Note that $|0^{n-j}1| \leq n$ since $j \geq 1$.
This says that the first half of the string has a 1 but the second half is all 0's.
Thus $xz \neq (xz)^R$ which means $xz \notin \overline{L}$.
Therefore, $\overline{L}$ is not regular and thus L is not regular.