Theorem: Let $G = (Q, \Sigma, \delta, q_{start}, q_{accept})$ be a GNFA and let $G' = (Q', \Sigma, \delta', q_{start}, q_{accept})$ be the GNFA resulting from running CONVERT(G) for one iteration. Then, $L(G) = L(G')$.

⇒ Let $w \in L(G)$, then $w = w_1 \cdots w_k$ such that there exists a sequence of states $q_0, \dots, q_k$ where $q_0 = q_{start}$, $q_k = q_{accept}$, and $w_i \in L(R_i)$ where $R_i = \delta(q_{i-1}, q_i)$. Let q_t be the state removed from G to get G'. If none of the $q_0, \dots, q_k$ are q_t then $w \in L(G')$ since the rest of the states are untouched in G'.

If any of the $q_0, \dots, q_k$ are q_t, then the sequence will look like the following

$$q_0, \dots, q_i, q_t, q_t, \dots, q_t, q_j, \dots, q_k$$

where there are one or more occurrences of q_t in a row. That means the string w looks as follows

$$w_1 \cdots w_i w_{i+1} w_{i+2} \cdots w_{j-1} w_j \cdots w_k$$

where $w_i \in L(\delta(q_{i-1}, q_i))$, $w_{i+1} \in L(\delta(q_i, q_t))$, $w_{i+2} \in L(\delta(q_t, q_t))$, …, $w_{j-1} \in L(\delta(q_t, q_t))$, and $w_j \in L\left(\delta(q_t, q_j)\right)$.

In G', $\delta'(q_i, q_j) = (\delta(q_i, q_t))(\delta(q_t, q_t))^* \left(\delta(q_t, q_j)\right) \cup \delta(q_i, q_j)$, which implies that $w_{i+1} w_{i+2} \cdots w_{j-1} w_j \in L\left(\delta'(q_i, q_j)\right)$ which implies that $w \in L(G')$.

⇐ Let $w \in L(G')$, then there exists a sequence of states $q_0, \dots, q_k$ such that $q_0 = q_{start}$, $q_k = q_{accept}$, and $w_i \in L(R_i')$ where $R_i' = \delta'(q_{i-1}, q_i)$ for each w_i in $w = w_1 \cdots w_k$. For any $\delta'(q_{i-1}, q_i)$ in G' if it equals $\delta(q_{i-1}, q_i) = R_i$ in G, then $w_i \in L(R_i)$.

Otherwise, when processing w_i in G it must pass through the state that has been removed, q_t, one or more times when going from q_{i-1} to q_i. Thus, w_i can be subdivided in to substrings $w_{(i-1)t} w_{tt_1} w_{tt_2} \cdots w_{ti}$ such that $w_{(i-1)t} \in L(\delta(q_{i-1}, q_t))$, $w_{tt_j} \in L(\delta(q_t, q_t))$, and $w_{ti} \in L(\delta(q_t, q_i))$. This implies that $w \in L(G)$ by definition of acceptance in a GNFA.