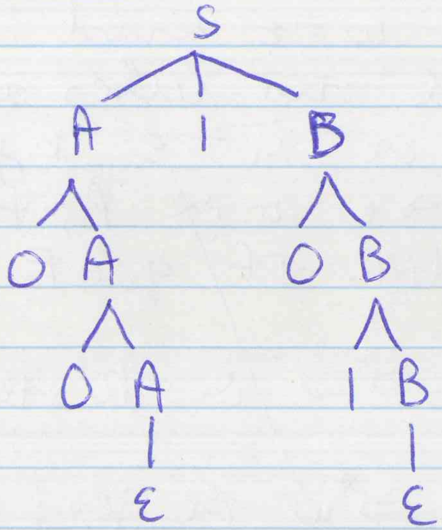
CSC 320 - ASSIGNMENT 3 - SOLUTIONS

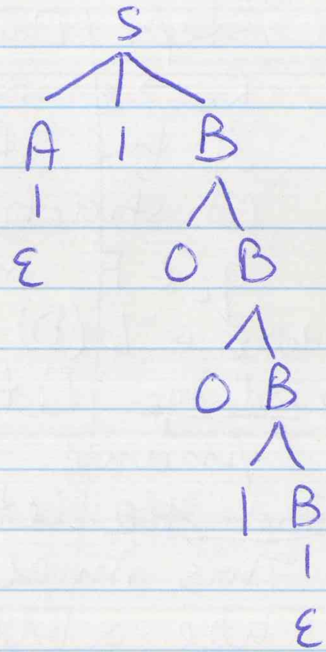
1. (a) Let $G = (\{S, A, B\}, \{0,1\}, P, S)$ be a CFG where P is given as follows:

$$S \rightarrow A1B$$
$$A \rightarrow 0A \mid \varepsilon$$
$$B \rightarrow 0B \mid 1B \mid \varepsilon$$

(b) 00101:



1001:



2. Let DFA $D = (Q, \Sigma, \delta, q_0, F)$ accept the regular language L, i.e. $L = L(D)$. We will construct the right-linear grammar $G = (V, \Sigma, P, R_0)$ as follows:
   1. $\forall q_i \in Q$, make variable $R_i \in V$
   2. $\forall q_i, q_j \in Q$, if $\delta(q_i, a) = q_j$ where $a \in \Sigma$, then let

$$R_i \rightarrow aR_j$$

be in P

3. $\forall q_i \in F$, let $R_i \to \varepsilon$ in P.
4. Let R_0 be start.

<u>Claim:</u> $w \in L(G)$ if and only if $w \in L(D)$

<u>Proof:</u> $\Rightarrow$ Let $w \in L(G)$, then $w = w_1 \cdots w_k$ such that each $w_i \in \Sigma$, or $w = \varepsilon$. If $w = \varepsilon$, then $R_0 \to \varepsilon$ in in P as all other R_0 rules are of the form $R_0 \to aR_i$ where $a \in \Sigma$.
If $w = w_1 \cdots w_k$, then $\exists$ a sequence of production rules $R_{i-1} \to w_i R_i$ for $i = 1$ to k in G and rule $R_k \to \varepsilon$.
This implies, by construction, that there exists rules $\delta(q_{i-1}, w_i) = q_i$ for $i = 1$ to k in D and that $q_k \in F$. By definition, $w \in L(D)$

$\Leftarrow$ Let $w \in L(D)$, then $w = w_1 \cdots w_k$ such that $w_i \in \Sigma$, $\exists q_0, \ldots, q_k$ such that q_0 is the start symbol and $q_k \in F$. Furthermore, $\delta(q_{i-1}, w_i) = q_i$ for $i = 1$ to k.
This implies that $R_{i-1} \to w_i R_i$ in G and that $R_k \to \varepsilon$ is in G.
Thus

$$R_0 \Rightarrow w_1 R_1 \Rightarrow w_1 w_2 R_2 \Rightarrow^* w_1 w_2 \cdots w_k R_k$$
$$\Rightarrow w_1 w_2 \cdots w_k = w$$

Therefore, $w \in L(G)$.

3. CFG $G = (V, \Sigma, P, S)$ where P is

$S \rightarrow 0A0 \mid 1B1 \mid BB$
$A \rightarrow C$
$B \rightarrow S \mid A$
$C \rightarrow S \mid \varepsilon$

Note: $S \Rightarrow BB \Rightarrow^* AA \Rightarrow^* CC \Rightarrow^* \varepsilon$
$\rightarrow$ so, $S_0 \rightarrow \varepsilon$ needs to be added to the grammar.

Add S_0:
$S_0 \rightarrow S \mid \varepsilon$
$S \rightarrow 0A0 \mid 1B1 \mid BB$
$A \rightarrow C$
$B \rightarrow S \mid A$
$C \rightarrow S \mid \varepsilon$

Remove $C \rightarrow \varepsilon$:
$S_0 \rightarrow S \mid \varepsilon$
$S \rightarrow 0A0 \mid 1B1 \mid BB$
$A \rightarrow C \mid \varepsilon$
$B \rightarrow S \mid A$
$C \rightarrow S$

Remove $A \rightarrow \varepsilon$:
$S_0 \rightarrow S \mid \varepsilon$
$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid BB$
$A \rightarrow C$
$B \rightarrow S \mid A \mid \varepsilon$
$C \rightarrow S$

Remove $B \rightarrow \varepsilon$:
$S_0 \rightarrow S \mid \varepsilon$
$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B$
$S \rightarrow \varepsilon$
$A \rightarrow C$
$B \rightarrow S \mid A \cancel{\varepsilon}$
$C \rightarrow S$

Remove $S \rightarrow \varepsilon$:
$S_0 \rightarrow S \mid \varepsilon$
$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B$
$A \rightarrow C$
$B \rightarrow S \mid A$
$C \rightarrow S$

Note that $B \rightarrow \varepsilon$, $C \rightarrow \varepsilon$ have been removed

→ Note: Removing $S \to B$, leads to $S \to S$ so we just remove $S \to B$ and add nothing.

→ Also ~~all~~ the other unit rules amounts to Variable → S, thus we have

$S_0 \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid \varepsilon$
$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$C \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

→ Now, S and C are unreachable from S_0, so are unnecessary

$S_0 \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid \varepsilon$
$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

→ Now let $C \to 0$, $D \to 1$:

$S_0 \to CAC \mid CC \mid DBD \mid DD \mid BB \mid \varepsilon$
$A \to CAC \mid CC \mid DBD \mid DD \mid BB$
$B \to CAC \mid CC \mid DBD \mid DD \mid BB$
$C \to 0$
$D \to 1$

→ Finally, let $E \to AC$, $F \to BD$:

$S_0 \to CE \mid CC \mid BF \mid DD \mid BB \mid \varepsilon$
$A \to CE \mid CC \mid BF \mid DD \mid BB$
$B \to CE \mid CC \mid BF \mid DD \mid BB$
$E \to AC$
$F \to BD$
$C \to 0$
$D \to 1$

4. Let $G = (\{A, B, C, S\}, \{a, b\}, P, S)$ where P is:

$$\begin{aligned} S &\to AB \mid BC \\ A &\to BA \mid a \\ B &\to CC \mid b \\ C &\to AB \mid a \end{aligned}$$

(a) aabab:

a	a	b	a	b
$\{A,C\}$	$\{B\}$	$\{B\}$	$\{S,C,A\}$	$\{S,C\}$
	$\{A,C\}$	$\{S,C\}$	$\{B\}$	$\{B\}$
		$\{B\}$	$\{A,S\}$	$\{S,C\}$
			$\{A,C\}$	$\{S,C\}$
				$\{B\}$

→ Since $T(1,5) = \{S, C\}$ which contains S, then string $aabab \in L(G)$

(b) bababb:

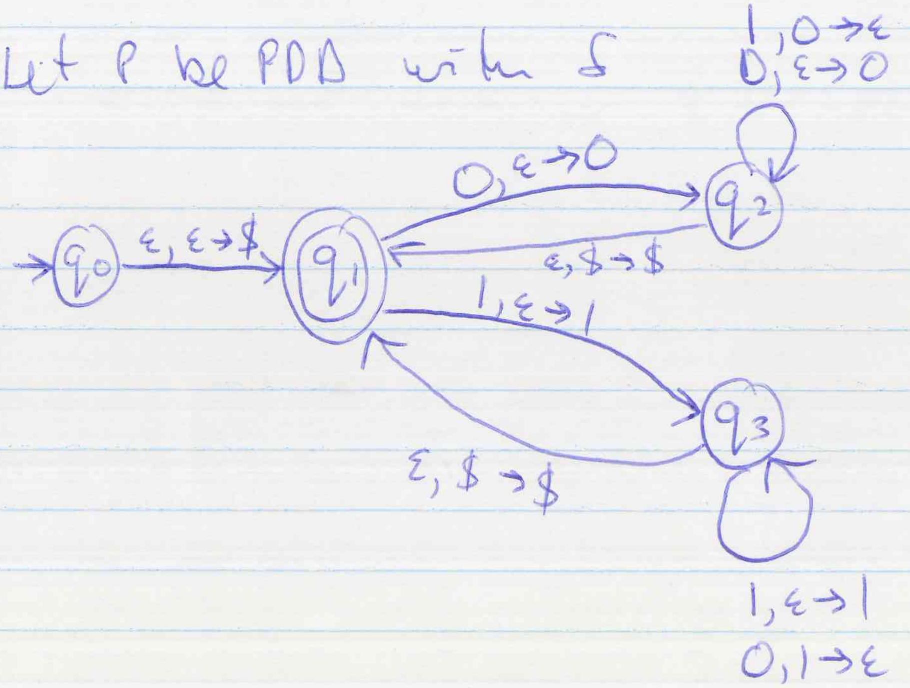
b	a	b	a	b	b
$\{B\}$	$\{A,S\}$	$\{S,C\}$	$\{B\}$	$\{B\}$	$\emptyset$
	$\{A,C\}$	$\{S,C\}$	$\{B\}$	$\{B\}$	$\emptyset$
		$\{B\}$	$\{A,S\}$	$\{S,C\}$	$\emptyset$
			$\{A,C\}$	$\{S,C\}$	$\emptyset$
				$\{B\}$	$\emptyset$
					$\{B\}$

→ Here $T(1,6) = \emptyset$ and thus $bababb \notin L(G)$

5. (a) $L = \{w \mid w$ has equal number 0's + 1's$\}$

→ Idea: If there are more 0's than 1's while processing, push 0's ~~&~~ when you see 0's & pop 0's when you see 1's
→ If more 1's then push + pop 1's.
→ when stack is empty go to accepting state

→ Let P be PDA with δ

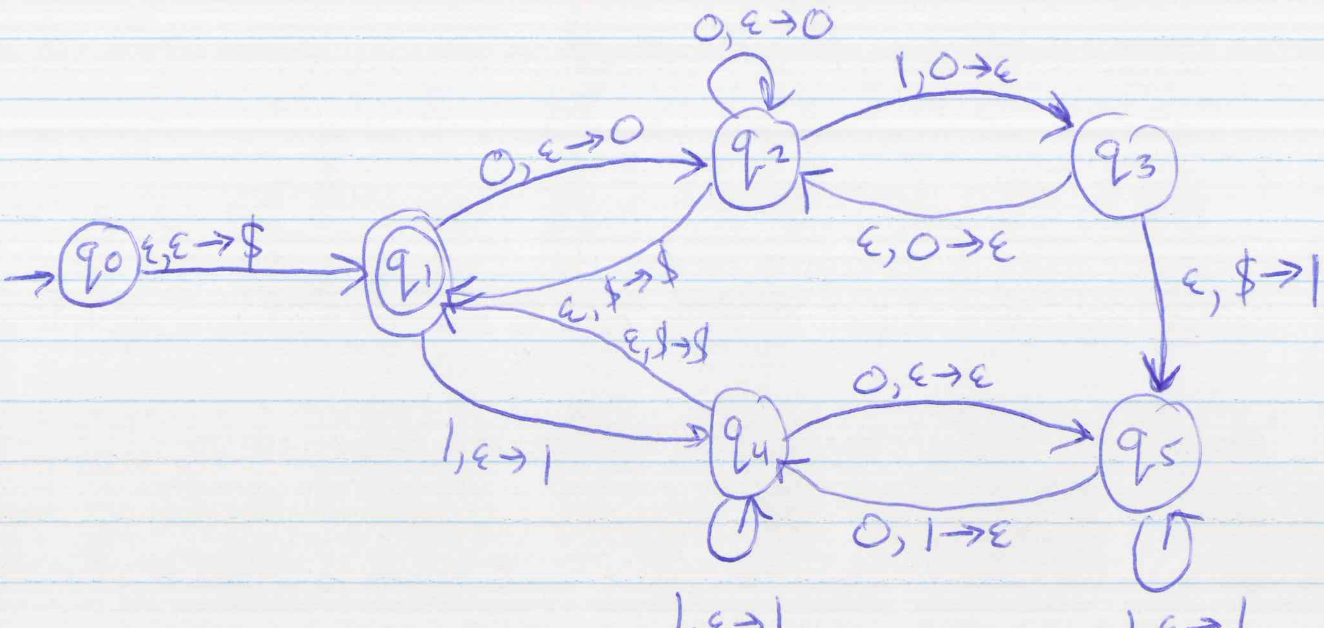


$P = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,\$\}, \delta, q_0, \{q_1\})$

(b) $L = \{w \mid w$ has twice as many 0's as 1's$\}$

→ Idea, similar to (a) but when there are more 0's we push one 0 ~~for~~ every 0 read and pop two 0's for every 1 read
→ when more ones we add one 1 for every 1 read and remove one 1 for every two 0's read.

→ There is one special case when removing 0's or seting 1's in the first part, where there becomes more 1's where we move to the other part



→ I have looked at this one a while and can't see that I have missed anything but I could be wrong so keep an eye out.

→ Their PDA's will probably be all different.

→ Here, $P = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \{0, 1, \$\}, \delta, q_0, \{q_1\})$