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Each multiple choice question has **EXACTLY ONE** correct answer.

1. Which one of the following is an unambiguous context-free grammar for

$$\{w \in \{(,)\}^* \mid |w| \geq 0 \text{ and } w \text{ is balanced}\}?$$

(Note: the empty string is balanced)

(a) $S \rightarrow SS \mid (S)$
(b) $S \rightarrow S(S) \mid \epsilon$
(c) $S \rightarrow SS \mid ()$
(d) $S \rightarrow S(S) \mid ()$
(e) $S \rightarrow SS \mid (S) \mid \epsilon$

The answer is (b). Clearly (a), (c) and (d) are out because they do not generate ϵ. (e) is ambiguous because, e.g., ϵ has more than one deriviation (exercise: give two such deriviations.) That just leaves (b).

2. Which one of the following is a CNF grammar that is equivalent to $S \rightarrow 1S0S \mid 0S1S \mid \epsilon$

(a)
$$\begin{aligned} S_0 &\rightarrow S \mid \epsilon \\ S &\rightarrow UR \mid ZT \mid SS \mid ZU \mid UZ \\ R &\rightarrow SZ \\ T &\rightarrow SU \\ U &\rightarrow 1 \\ Z &\rightarrow 0 \end{aligned}$$

(b)
$$\begin{aligned} S_0 &\rightarrow USZ \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon \\ S &\rightarrow USZ \mid ZT \mid SS \mid ZU \mid UZ \\ T &\rightarrow SU \\ U &\rightarrow 1 \\ Z &\rightarrow 0 \end{aligned}$$

(c)
$$\begin{aligned} S_0 &\rightarrow UR \mid ZT \mid SS \mid 01 \mid 10 \mid \epsilon \\ S &\rightarrow UR \mid ZT \mid SS \mid 01 \mid 10 \\ R &\rightarrow S0 \\ T &\rightarrow S1 \end{aligned}$$

(d)
$$\begin{aligned} S_0 &\rightarrow UR \mid ZT \mid SS \mid ZU \mid UZ \mid \epsilon \\ S &\rightarrow UR \mid ZT \mid SS \mid ZU \mid UZ \\ R &\rightarrow SZ \\ T &\rightarrow SU \\ U &\rightarrow 1 \\ Z &\rightarrow 0 \end{aligned}$$

(e) None of the above.

The answer is (d). You can easily eliminate (a), (b) and (c) (why?) but you still need to run the CNF algorithm to make sure that (a) is correct.

3. Consider the following CNF grammar

$$\begin{aligned} S &\rightarrow SS \mid LX \mid LR \\ X &\rightarrow SR \\ L &\rightarrow (\\ R &\rightarrow) \end{aligned}$$

Suppose we run the CYK algorithm on this grammar, and the input ()(()), producing the following table (with missing entries):

1	2	3	4	5	6	
L	(i)	∅	∅	∅	(vi)	1
	R	∅	∅	∅	∅	2
		L	∅	∅	S	3
			(ii)	(iii)	(v)	4
				(iv)	∅	5
					R	6

What are the correct values for the missing entries?

(a) (i)=S, (ii)=R, (iii)=S, (iv)=L, (v)=X, (vi)=S

(b) (i)=S, (ii)=R, (iii)=S, (iv)=R, (v)=X, (vi)=S

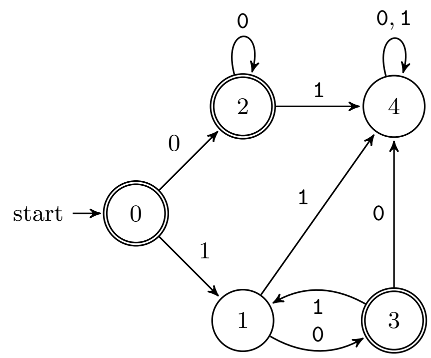
(c) (i)=S, (ii)=L, (iii)=S, (iv)=S, (v)=X, (vi)=S

(d) (i)=S, (ii)=S, (iii)=S, (iv)=R, (v)=X, (vi)=S

(e) None of the above.

Answer is (e). You can eliminate all of the other options just by looking at the diagonal entries.

4. Which one of the following grammars (with start symbol S) generates the language recognized by the following DFA?



(a)

$$\begin{aligned} S &\to 0B \mid 1A \mid \epsilon \\ A &\to 0D \mid 1C \\ B &\to 0B \mid 1D \mid \epsilon \\ C &\to 0D \mid 1A \mid \epsilon \\ D &\to 0D \mid 1D \end{aligned}$$

(b)

$$\begin{aligned} S &\to 0B \mid 1A \mid \epsilon \\ A &\to 0C \mid 1D \mid \epsilon \\ B &\to 0B \mid 1D \\ C &\to 0D \mid 1A \mid \epsilon \\ D &\to 0D \mid 1D \end{aligned}$$

(c)

$$\begin{aligned} S &\to 0B \mid 1A \mid \epsilon \\ A &\to 0C \mid 1D \\ B &\to 0B \mid 1D \mid \epsilon \\ C &\to 0D \mid 1A \mid \epsilon \\ D &\to 0D \mid 1D \end{aligned}$$

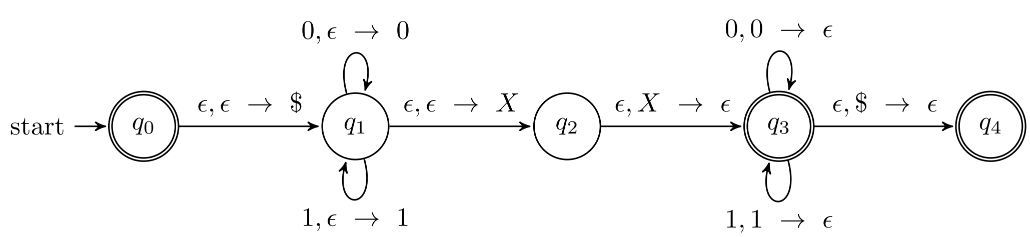
(d)

$$\begin{aligned} S &\to 0B \mid 1A \mid \epsilon \\ A &\to 0D \mid 1C \mid \epsilon \\ B &\to 0B \mid 1D \\ C &\to 0D \mid 1A \mid \epsilon \\ D &\to 0D \mid 1D \end{aligned}$$

(e) None of the preceding.

The answer is (c). Map 0 to S, 1 to A, 2 to B, 3 to C and 4 to D.

5. Which one of the following grammars generates the language recognized by the following PDA?



(a) $S \to \epsilon \mid 1S1 \mid 0S0$

(b) $S \to \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0$

(c) $S \to \epsilon \mid 1S1 \mid 0S0 \mid SS$

(d) $S \to \epsilon \mid 0 \mid 1 \mid 1S1 \mid 0S0 \mid SS$

(e) None of the preceding

The answer is (a). (b) and (d) are out because you can't have a single 0 or 1 (every push needs a matching pop of the same symbol.) (c) is out because it allows strings like 0011. You still need to run the transformation procedure to verify (a) (or just eyeball it.)

6. Suppose that L is decided by a 3-tape TM than runs in time $O(n^2)$. Which one of the following statements could be false?
   (a) L is decided by a 2-tape TM that runs in time $O(n^4)$.
   (b) L is decided by a 1-tape TM that runs in time $O(n^4)$.
   (c) L is decided by a 1-tape TM that runs in time $O(n^3)$.
   (d) L is decided by a 1-tape TM that runs in time $O(n^6)$.
   (e) None of the above.

   The answer is (c). This depends on 2 facts. (1) For any f, g, h, if $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ then $f(n)$ is $O(h(n))$ (this is just a general fact about big-Oh – CSC225) and (2) If L is decided by a multitape TM in time $O(f(n))$ then it is decided by a single tape TM in time $O((f(n)^2)$. So (a) is true by (1), (b) is true by (2), and (d) is true by (1) and (2). Or you could just note that (c) is possibly false.

7. Which one of the following statements could be false?
   (a) If L is decided by a nondeterministic TM then it is decided by a TM that halts on every input.
   (b) Any language that can be enumerated by a Turing machine is recognizable.
   (c) There are countably many decidable languages.
   (d) There are countably many recognizable languages.
   (e) None of the above

   The answer is (e) – all of the preceding are true

8. Which of the following statements is false?
   (a) Every language decided by a nondeterministc TM is decided by a deterministic TM
   (b) Every language decided by a deterministic TM is recognized by a deterministic TM
   (c) Every langauge recognized by a deterministic TM is recognized by a nondeterministic TM
   (d) Every language recognized by a deterministic TM is decided by a deterministic TM
   (e) None of the above

   The answer is (d). For example A_{TM} is recognized by a deterministic TM, but it is undecidable.

9. Suppose we have the following TM for the language $\{a^n b^n \mid n \geq 0\}$: $M = (\{q_0, q_1, q_2, q_3, q_a, q_r\}, \{a, b\}, \{a, b, a', b', \sqcup$

	a	b	a'	b'	$\sqcup$
q_0	(q_1, a', R)	(q_r, a', R)	(q_r, a', R)	(q_3, b', R)	
q_1	(q_1, a, R)	(q_2, b', L)	(q_r, a', R)	(q_1, b', R)	
q_2	(q_2, a, L)	(q_r, a', R)	(q_0, a', R)	(q_2, b', L)	
q_3	(q_r, a', R)	(q_r, a', R)	(q_r, a', R)	(q_3, b', R)	$(q_a, \sqcup, R)$

10. What is the initial configuration of M when started on input $aaabb$?
    (a) $q_0 aaabb\sqcup$
    (b) $aq_0 aabb$
    (c) $a'a'a'b'b'q_r$
    (d) $q_0 aaabb$
    (e) None of the above

    The answer is (d), just by the definition of initial configuration.

11. What state is M in when it halts on input $aaabb$?

    (a) M does not halt on this input
    (b) We don't know because it is undecidable whether M halts on this input
    (c) q_r
    (d) q_3
    (e) None of the above

    The answer is (e). We will get to state q_1, scanning $\sqcup$. There is no transition defined in this case.

12. Suppose that A is TM-decidable and $A \leq_m B$. Which one of the following statements could be false?

    (a) $\overline{A}$ is TM-decidable.
    (b) A is TM-recognizable.
    (c) $\overline{B}$ is TM-decidable.
    (d) $\overline{A}$ is TM-recognizable.
    (e) None of the above

    The answer if (c). Clearly we can eliminate (a), (b), (d) – any decidable language is recognizable and has a decidable complement. In general, the fact that $A \leq_m B$ and A is decidable does not tell us anything about the decidability of B. More specifically, here is an example that shows that (c) is false: let $A = \{\langle M \rangle \mid M \text{ has at most 10 states}\}$ and $B = A_{TM}$. Clearly A is decidable and $\overline{B}$ is not decidable. On the other hand, consider the following reduction. Letting M_r be the TM that rejects every input and M_a the TM that accepts every input, define

    $$f(\langle M \rangle) = \begin{cases} \langle M_r, \epsilon \rangle & \text{if } M \text{ has more than 10 states;} \\ \langle M_a, \epsilon \rangle & \text{otherwise.} \end{cases}$$

    Then f is a reduction from A to B. This shows that (c) could be false. Note that we are using the fact that A is decidable to get a "trivial" reduction.

13. Which one of the following statements could be false?

    (a) There is a languge L such that neither L nor $\overline{L}$ is decidable.
    (b) There is a languge L such that neither L nor $\overline{L}$ is recognizable.
    (c) If L is undecidable and $\overline{L}$ is not recognizable, then L must be recognizable.
    (d) If L is recognizable and $\overline{L}$ is recognizable, then L must be decidable.
    (e) None of the above.

    The answer is (c). We know that (b) is true, using the EQ_{TM} example from class. But this also shows that (c) is false: since EQ_{TM} is unrecognizable, it is also undecidable. But as we case the complement of EQ_{TM} is not recognizable. So $L = EQ_{TM}$ shows that (c) is false.