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Assignment 3
CSC 349A
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Question 1

(a) $f(x) = \ln(x+2)$

$P_n(x) = f(a) + f'(a)(x-a) + \cdots \frac{f^n(a)}{n!}(x-a)^n$

$a=-1 \quad n=3$

$P_3(x) = f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f'''(-1)}{3!}(x+1)^3$

$f(x) = \ln(x+2) =$

$f'(x) = \frac{d}{dx}(\ln(x+2)) = \frac{1}{x+2}$

$f''(x) = \frac{d}{dx}\left(\frac{1}{x+2}\right) = -\frac{1}{(x+2)^2}$

$f'''(x) = \frac{d}{dx}\left(-\frac{1}{(x+2)^2}\right) = \frac{2}{(x+2)^3}$

$\therefore P_3(x) = \ln(1) + \frac{1}{1}(x+1) + \frac{-\frac{1}{(1)^2}}{2}(x+1)^2 + \frac{\frac{2}{1^3}}{6}(x+1)^3$

$P_3(x) = 0 + x+1 + -\frac{1}{2}(x^2+2x+1) + \frac{1}{3}(x^3+3x^2+3x+1)$

$P_3(x) = x+1 - \frac{x^2}{2} - x - \frac{1}{2} + \frac{x^3}{3} + x^2 + x + \frac{1}{3}$

$P_3(x) = \frac{x^3}{3} + \frac{x^2}{2} + x + \frac{5}{6}$

(b) 6 significant digit

$fl(\ln(0.94)) = fl(-0.0618754403) = -0.618754 \times 10^{-1}$

when $x = -1.06$

$fl(P_3(x)) = \frac{(-1.06)^3}{3} + \frac{(-1.06)^2}{2} + (-1.06) + \frac{5}{6}$

$= fl(-0.397005) + 0.5618 - 1.06 + fl(0.83333333)$

$= -0.397005 + 0.5618 - 1.06 + 0.833333$

$\boxed{= -0.061872}$

(c) truncation error $\quad |E_t| = |-0.061872 + 0.0618754| = 0.0000034$

$P_3(x) = \frac{x^3}{3} + \frac{x^2}{2} + x + \frac{5}{6}$

$f^{(4)}(x) = \frac{d}{dx}\left(\frac{2}{(x+2)^3}\right) = -\frac{6}{(x+2)^4}$

$\ln(x+2) - P_3(x) = R_n = \frac{f^4(\varepsilon)}{4!}(x+1)^4 = \frac{-\frac{6}{(\varepsilon+2)^4}}{24}(x+1)^4 = -\frac{(x+1)^4}{4(\varepsilon+2)^4}$

$x = -1.06$

$R_3 = -\frac{(-0.06)^4}{4(\varepsilon+2)^4} = \frac{0.00001296}{4(\varepsilon+2)^4} = \frac{0.00000324}{(\varepsilon+2)^4} = \frac{324}{100000000(\varepsilon+2)^4}$

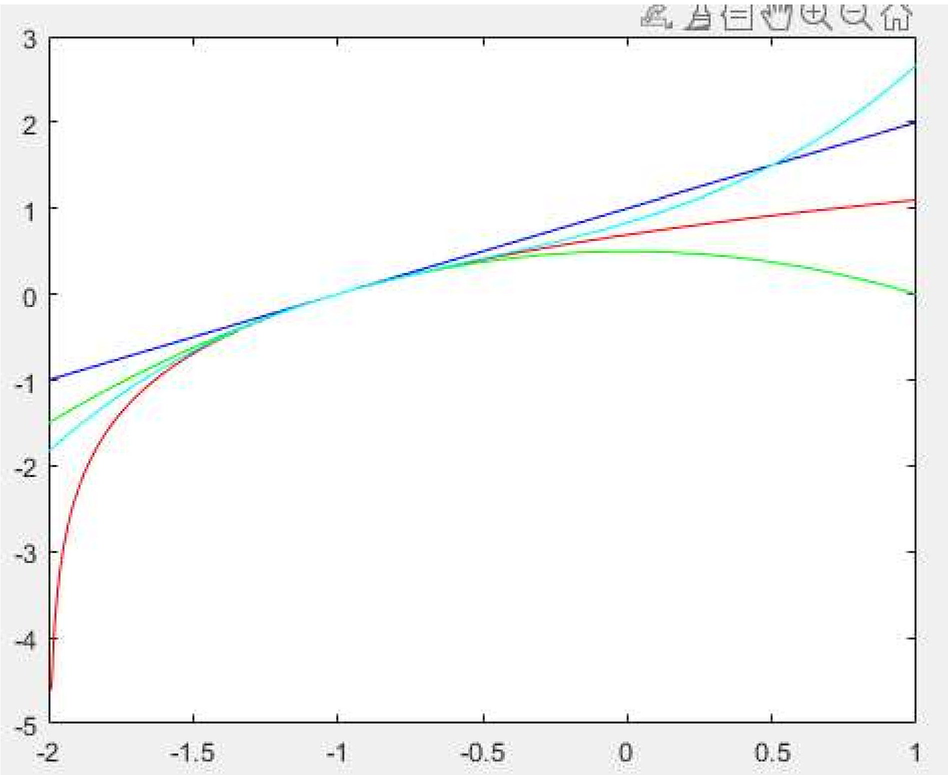
if upper bound $\varepsilon = 0$

$R_3 = \frac{324}{1\times10^8 \times 16} = 0.202 \times 10^{-6}$

$\therefore$ Upper bound of this truncation is 0.202×10^{-6}

(e)E

```
function polynomialApp()
x=[-2:0.01:1];
fx = log(x+2);
Px1 = x+1;
Px2 = Px1-(1/2)*(x.^2+2*x+1);
Px3 = Px2+1/3*(x.^3+3*x.^2+3*x+1);
plot(x,fx,'r',x,Px1,'b',x,Px2,'g', x,Px3,'c');
```



Question 2

$f(x) = 1 - \sin(x)$ $\quad 1 - \sin\frac{\pi}{2} = 0 \quad 1 - \sin 0.5 \approx 0.520574461$

(a) $\text{cond}_1 \approx \frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$

$f'(x) = -\cos(x)$

$\text{Cond}_1 = \frac{\tilde{x}(-\cos\tilde{x})}{1-\sin(\tilde{x})}$

when $\tilde{x} = 1.56$

$\text{Cond}_1 = \frac{1.56 \cdot (-0.010796117)}{0.000058279} = \frac{-0.016841942}{0.000058279} = -288.9843699$

Condition number of $\tilde{x} = 1.56$ is "large". $f(x)$ is ill-conditioned of x close to $\frac{\pi}{2}$

$\text{Cond}_2 = \frac{0.51(-\cos 0.51)}{1-\sin(0.51)} = -0.86936404$

Condition number of $\tilde{x} = 0.51$ is -0.86936404 is "small", $f(x)$ is well-condition of x close to 0.5

(b) For value close to $\frac{\pi}{2}$

Input $R_E = \left|\frac{x-\tilde{x}}{\tilde{x}}\right| = \left|\frac{\frac{\pi}{2}-1.56}{1.56}\right| \approx 0.00692$

Output $RE = \left|\frac{f(x)-f(\tilde{x})}{f(\tilde{x})}\right| = \left|\frac{0-288.9843699}{288.9843699}\right| = 1$

For value close to 0.5

Input $R_E = \left|\frac{0.5-0.51}{0.51}\right| \approx 0.019608$

Output $R_E = \left|\frac{(1-\sin 0.5)-(1-\sin 0.51)}{1-\sin 0.51}\right| = 0.017099$

Since Condition Number 1 $= -288.9843699$ Condition Number 2 is -0.86936404
Thus, the Condition number illustrusts the connection between two Relative Error
By two condition Number, small change error in input can cause great change in Output.

Question 3

(a) $\cos x \approx -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$

$$g(x) = \begin{cases} \frac{1+\cos x}{(x-\pi)^2} & x \neq \pi \\ 0.5 & x = \pi \end{cases} \longrightarrow \frac{1+\cos x}{(x-\pi)^2} = \frac{\frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}}{(x-\pi)^2} = \frac{1}{2} - \frac{(x-\pi)^2}{24}$$

$$\therefore g(x) = \begin{cases} \frac{1}{2} - \frac{(x-\pi)^2}{24} & x \neq \pi \\ 0.5 & x = \pi \end{cases}$$

(b) $g(3.16)$ is unstable.

$fl(g(3.16)) = 0.5908$

$g(3.16) \approx 0.499985$

$\Rightarrow g(3.16+\varepsilon) = \frac{1}{2} - \frac{(3.16+\varepsilon-\pi)^2}{24}$

$= \frac{1}{2} - \frac{1}{24}((\pi-3.16)^2 + \varepsilon^2 + 2(\pi-3.16)\varepsilon)$

$= -0.0416667\varepsilon^2 - 0.0015339\varepsilon + 0.4999859$

$3.16 - \pi = 0.018407$

since $\varepsilon = -0.018407$, $g(3.16+\varepsilon) \approx 0.499988$ has max value which is close to 0.4999859

instead of floating point value 0.5908

$\therefore g(3.16)$ is unstable.

(c) $fl(g(1.41)) = 0.3871$

solution $g(1.41) = \frac{1}{2} - \frac{1}{24}(1.41-\pi)^2 \approx 0.3750$

need to perturb $g(1.41+\varepsilon)$ s.t $|\frac{\varepsilon}{1.41}|$ small and $g(1.41+\varepsilon)$ is close to 0.3750

$g(1.41+\varepsilon) = \frac{1}{2} - \frac{1}{24}(1.41+\varepsilon-\pi)^2$

$= 0.375 - 0.1443\varepsilon - 0.0417\varepsilon^2$

$1.41 - \pi \approx -1.731$

when $\varepsilon = -1.731$ $g(1.41+\varepsilon)$ is maximum

$g(1.41+\varepsilon)_{max} = 0.4956$. but $|\frac{\varepsilon}{1.41}| > 1$!!

$0.3750 < 0.4956 \Rightarrow$ Computation is stable