Q1 $f(1.12) = 2 + \frac{(1.12-1)}{4} - \frac{(1.12-1)^2}{64} + \frac{(1.12-1)^3}{512}$

$= 2 + 0.03 - 0.000225 + 0.000003375$

$= 2.029778375 \approx 0.20298 \times 10^{-1}$

Thus the approxiuated relative error $|\epsilon_t| < 0.5 \times 10^{-6}$

a) $P_0 = 2$ $|\epsilon_t| = \left|\frac{f(1.12) - 2}{f(1.12)}\right| = 0.014670259 > 0.5 \times 10^{-6}$

b) $P_1 = 2 + \frac{(x-1)}{4} = 2.03$

$\therefore |\epsilon_t| = \left|\frac{f(1.12) - 2.03}{f(1.12)}\right| = 0.000109186 > 0.5 \times 10^{-6}$

c) $P_2 = P_1 - \frac{(x-1)^2}{64} = 2.03 - \frac{0.12^2}{64} = 2.029775$

$|\epsilon_t| = \left|\frac{f(1.12) - 2.029775}{f(1.12)}\right| = 0.000001662 < 5 \times 10^{-5}$

since $P_0, P_1, P_2 \cdots P_{n-1}, P_n$ is Converge. More P input creates more accurate result

$\Rightarrow$ True value is 2.029775

Q2(a) 02003004_5 to decimal

$+(0.2003)_5 \times 5_{10}$

$(0.2003)_5 = 2\times5^{-1} + (0\times5^{-2}) + (0\times5^{-3}) + (3\times5^{-4})$

$= \frac{2}{5} + \frac{3}{625} = (0.4048)_{10}$

$(04)_5 = (4)_{10}$

There fore $(0.2003)_5 \times (5)_{10}^4 = 0.4048 \times 5^4 = 253$

(b) Convert -25.8_{10} to base 5

25			0.8
25/5	0	5^0	$5\times0.8=4.0$
5/5	0	5^1	
1/5	1	2^2	

$-(25.8)_{10} = -(100.4)_5 = -0.1004 \times 5^{-3}$

1	0	0	4	1	0	3

(c) Smallest positive number

$+0.1000 \times (5_{10})^{(44)_5} = 0.1000 \times 5^{-24_{10}}$

$= 1\times5^{-1}\times5^{-24} = (5^{-25})_{10}$

(d) $(10-1)\ 10\ 10 \cdots 10 = 9\cdot10^{f_i-1}$

Size of interval : $125_{10} - 25_{10} = 100$

∴ Consecutive Number is

$\frac{100}{9\cdot10^{f_i-1}} = \frac{10^{3-f_i}}{9} = 9^{-1}\cdot10^{3-f_i} = \frac{1}{9}\cdot10^{3-f_i}$

Zheng Yin

3. (i) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (ii) $\frac{-2c}{b\pm\sqrt{b^2-4ac}}$

$fl\left(\frac{-2c}{b+\sqrt{b^2-4ac}}\right.$

(a) $a=1.2$ $b=-78.99$ $c=1.234$

$b^2=6239.4201$

$4ac = 4\times1.2\cdot1.234=5.9232$

$\sqrt{b^2-4ac}=\sqrt{6239.4201-5.9232}=\sqrt{6233.4969}\approx78.95244774$

$-2c=-2.468$ $2a=2.4$

$\frac{-2c}{b-\sqrt{b^2-4ac}}=\frac{-2.468}{-78.99-78.95244774}=\frac{2.468}{157.94244774}\approx0.1562\times10^{-1}$

$\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{78.99-78.95244774}{2.4}=\frac{0.03755226}{2.4}\approx0.1562\times10^{-1}$

(b) both function got 0.0156

If we use $V_A=0.01562594$ $V_E=0.01562$

$\left|\frac{V_A-V_E}{V_E}\right|\times100\%=\left|\frac{0.01562594-0.01562}{0.01562}\right|\times100\%=\frac{0.00000594}{0.01562}=0.000380281\approx0.0380281\%$

$\boxed{\text{The relative error is } 0.1662\%}$

If we use $V_A=0.0156$ $V_E=0.01562594$

$\left|\frac{V_A-V_E}{V_E}\right|\times100\%=\left|\frac{0.01562-0.01562594}{0.01562594}\right|=\frac{0.00000594}{0.01562594}=0.000380137\approx0.0380137\%$

$\boxed{\text{The relative error is about } 0.1660\%}$

(c)

polynomial	(i) is more accurate	(ii) is more accurate
$0.01x^2-125x+0.05$	X	
$-0.3x^2+125x+0.025$		X

(i) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ (ii) $\frac{-2c}{b+\sqrt{b^2-4ac}}$

Two function have exponent on b which is $\pm0.125\times10^3$ which exponent is 3, so b will not affect the accuracy of the function.

in First polynomial, the normalized floating-point number of a and C are 0.1×10^{-1} and 0.5×10^{-1}

in Second polynomial, the normalized floating-point number of a and C are -0.3×10^0 and 0.25×10^{-1}

For a, the first polynomial is more accurate on exponent which show smaller number

So within a is important on i. So i is more accurate for polynomial

Second polymoial C have more precision number. So it is accurate on C of (ii)

So ... (ii) is more accurate on polynomial 2.