COMPUTER SCIENCE 349A, FALL 2020
ASSIGNMENT #2 - 20 MARKS

DUE FRIDAY OCTOBER 2, 2020 (11:30 p.m. PST)

This is a really large class and the logistics of grading assignments are challenging. Me and the markers require your help in making this process go smoothly. Please ensure that your assignments conform to the following requirements - any violation will result in getting a zero for the particular assignment.

- All assignments should be submitted electronically through the ConneX course website and shoud be **SINGLE PDF FILES**. No other formats will be accepted.. Handwritten answers are ok but they will need to be scanned and merged into a single pdf file together with any code examples and associated plots.
- The assignment number, student name and student number should be clearly visible on the top of every page of your assignment submission.
- **PLEASE DO NOT COPY THE ASSIGNMENT DESCRIPTION IN YOUR SUBMISSION**
- The asnwers to the questions should be in the same order as in the assignment specification.
- Some of the questions of the assignments are recycled from previous years but typically with small changes in either the description or the numbers. Any submission that contains numbers from previous years in any questions will be immediately graded with zero.
- Any assignment related email questions should have a subject line of the form CSC349A Assignment X, where X is the number of the corresponding assignment.
- The total number of points for this assignment is 20.

Question #1 - 6 marks.

The following is a third degree Taylor polynomial approximation to some function $f(x)$.

$$f(x) \approx 2 + \frac{(x-1)}{4} - \frac{(x-1)^2}{64} + \frac{(x-1)^3}{512}$$

Determine the true value of the function at $x = 1.12$ to five significant digits. Justify your confidence in the number of correct significant digits in your approximation.

Question #2 - 8 marks.

Consider a base-5 normalized, floating-point number system. Assume that a hypothetical computer using this system has the following floating-point representation for a word,

s_m	f_1	f_2	f_3	f_4	s_e	e_1	e_2

where s_m is the sign of the mantissa, s_e is the sign of the exponent (1 for negative, 0 for positive), f_i are the digits of the mantissa, and e_j are the digits of the exponent.

(a) Consider the base-5 word, given using the above representation, 02003004_5. What exact decimal value does it represent?

(b) Consider the decimal value -25.8_{10}. Convert this value to its base-5 normalized floating-point representation. Place the appropriate 8 base-5 digits in the word correctly.

(c) What is the smallest positive, non-zero number that can be represented in this system? Give the answer in the above form (i.e. as 8 base-5 digits.) and in decimal.

(d) What is the distance between any two consecutive numbers in the interval 25_{10} and 125_{10} in this floating-point representation system? Your answer should be in decimal.

Question #3 - 6 Marks.

The roots of a quadratic polynomial $ax^2 + bx + c$ can be computed by

$$(i)\quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or equivalently} \quad (ii)\quad \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

Using floating-point arithmetic, one of these formulas is often much more accurate than the other. For example, if $(-b + \sqrt{b^2 - 4ac})/(2a)$ is used to compute one of the roots of $P(x) = x^2 - 83.12x + 3.123 = 0$ with base $b = 10$, precision $k = 4$, idealized chopping arithmetic, the relative error is about 0.00039 or 0.039%. On the other hand, it can be shown that

$$fl\left(\frac{-2c}{b+\sqrt{b^2-4ac}}\right) = 69.40 \text{ or } 0.6940 \times 10^2$$

which (using the exact value of $83.0824\cdots$) has a large relative error of 0.165 or 16.5%.

(a) Use base $b = 10$, precision $k = 4$, idealized <u>chopping</u> arithmetic and each of the mathematically equivalent formulas

$$\frac{-2c}{b-\sqrt{b^2-4ac}} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

to compute an approximation to one root of $P(x) = 1.2x^2 - 78.99x + 1.234 = 0$. Specify each step of the computation. Note that many of the computations for the two formulas are identical, and need only be done once. Use your calculator to do this, not MATLAB.

(b) Compute the relative errors of each of the approximations in (a) using the fact that the exact value of the root is $0.01562594\cdots$.

(c) One of the two zeros of a quadratic polynomial $ax^2 + bx + c$ can be computed using either the formula

$$(i)\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ or } (ii)\frac{-2c}{b+\sqrt{b^2-4ac}}$$

For each of the specified polynomials in the table below, place an X in the appropriate box to indicate which of these formulas is more accurate in precision $k = 4$ floating-point arithmetic. Put exactly one X in each row of the table. You do not need to do floating-point arithmetic to answer this question. You are going to predict the behaviour. Justify your answers.

polynomial	(i) is more accurate	(ii) is more accurate
$0.01x^2 - 125x + 0.05$		
$-0.3x^2 + 125x + 0.025$		