Question 1

Given Matrix:

$$\begin{bmatrix} 2 & -2 & 4 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & -3 & 2 & 1 \\ -4 & 0 & -4 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ -5 \\ 0 \end{bmatrix}$$

The forward Elimination

$$\begin{bmatrix} 2 & -2 & 4 & 0 & | & -6 \\ 1 & 2 & 3 & 0 & | & 2 \\ 0 & -3 & 2 & 1 & | & -5 \\ -4 & 0 & -4 & 0 & | & 0 \end{bmatrix} \xrightarrow[-\frac{1}{4}R_4]{\frac{1}{2}R_1} \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 1 & 2 & 3 & 0 & | & 2 \\ 0 & -3 & 2 & 1 & | & -5 \\ 1 & 0 & 1 & 0 & | & 0 \end{bmatrix} \xrightarrow[R_4-R_1]{R_2-R_1} \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 0 & 3 & 1 & 0 & | & 5 \\ 0 & -3 & 2 & 1 & | & -5 \\ 0 & 1 & -1 & 0 & | & 3 \end{bmatrix}$$

$$\xrightarrow[3R_4-R_2]{R_3+R_2} \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 0 & 3 & 1 & 0 & | & 5 \\ 0 & 0 & 3 & 1 & | & 0 \\ 0 & 0 & -4 & 0 & | & 4 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_4} \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 0 & 3 & 1 & 0 & | & 5 \\ 0 & 0 & 3 & 1 & | & 0 \\ 0 & 0 & 1 & 0 & | & -1 \end{bmatrix} \xrightarrow{R_3-R_4} \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 0 & 3 & 1 & 0 & | & 5 \\ 0 & 0 & 0 & 1 & | & 3 \\ 0 & 0 & 1 & 0 & | & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 & | & -3 \\ 0 & 3 & 1 & 0 & | & 5 \\ 0 & 0 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & 3 \end{bmatrix}$$

back - substitution

$X_2 = \frac{5-(-1)}{3} = 2$

$X_1 = -3 + 2 - 2 \cdot (-1) = -1 + 2 = 1$

$X_1 = 1 \quad X_2 = 2 \quad X_3 = -1 \quad X_4 = 3$

Solution $X = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$

2(a)

```
A=[0.866 0 -0.5 0 0 0;
      0.5 0 0.866 0 0 0;
      -0.866 -1 0 -1 0 0;
      -0.5 0 0 0 -1 0;
      0 1 0.5 0 0 0;
      0 0 -0.866 0 0 -1];
B = [0;-1000;0;0;0;0];
```

X = linsolve(A,B)

X1 =-500.0220

X2= 433.0191

X3=-866.0381

X4=0.0000

X5=250.0110

X6=749.9890

(b)y = inv(A)

y =

0.8660	0.5000	0	0	0	0
0.2500	-0.4330	0	0	1.0000	0
-0.5000	0.8660	0	0	0	0
-1.0000	0.0000	-1.0000	0	-1.0000	0
-0.4330	-0.2500	0	-1.0000	0	0
0.4330	-0.7500	0	0	0	-1.0000

© z=det(A)

z =1.0000

3(a)

```
A=[(-1.4)^3 (-1.4)^2 -1.4 1;
      (-1)^3 (-1)^2 -1 1;
      (0.1)^3 (0.1)^2 0.1 1;
      (1.1)^3 (1.1)^2 1.1 1;
      (1.4)^3 (1.4)^2 1.4 1];
B = [tan(-1.4); tan(-1); tan(0.1); tan(1.1); tan(1.4)];
```

(b)

```
p = A\B

p =  2.8344

      -0.0973

      -1.4308

      0.0901
```

Question 4

(a) Given table:

x_i	$x_0 = 2$	$x_1 = 2.1$	$x_2 = 2.2$	$x_3 = 2.7$	$x_4 = 3$
$f(x_i)$	$f(x_0) = 6$	$f(x_1) = 7.752$	$f(x_2) = 10.256$	$f(x_3) = 36.576$	$f(x_4) = 66$

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$= \frac{(x-2.1)(x-2.2)(x-2.7)(x-3)}{(-0.1)(-0.2)(-0.7)(-1)} \cdot 6 + \frac{(x-2)(x-2.2)(x-2.7)(x-3)}{(0.1)(-0.1)(-0.6)(-0.9)} \cdot 7.752$$
$$+ \frac{(x-2)(x-2.1)(x-2.7)(x-3)}{(0.2)(0.1)(-0.5)(-0.8)} \cdot 10.256 + \frac{(x-2)(x-2.1)(x-2.2)(x-3)}{(0.7)(0.6)(0.5)(-0.3)} \cdot 36.576$$
$$+ \frac{(x-2)(x-2.1)(x-2.2)(x-2.7)}{(1)(0.9)(0.8)(0.3)} \cdot 66$$

$$= \frac{(x-2.1)(x-2.2)(x-2.7)(x-3)}{0.014} \cdot 6 - \frac{(x-2)(x-2.2)(x-2.7)(x-3)}{0.0054} \cdot 7.752$$
$$+ \frac{(x-2)(x-2.1)(x-2.7)(x-3)}{0.008} \cdot 10.256 - \frac{(x-2)(x-2.1)(x-2.2)(x-3)}{0.063} 36.576$$
$$+ \frac{(x-2)(x-2.1)(x-2.2)(x-2.7)}{0.216} \cdot 66$$

(b) when $x = 2.5$

$$f(2.5) = \frac{(2.5-2.1)(2.5-2.2)(2.5-2.7)(2.5-3)}{0.014} \cdot 6 - \frac{(2.5-2)(2.5-2.2)(2.5-2.7)(2.5-3)}{0.0054} \cdot 7.752$$
$$+ \frac{(2.5-2)(2.5-2.1)(2.5-2.7)(2.5-3)}{0.008} \cdot 10.256 - \frac{(2.5-2)(2.5-2.1)(2.5-2.2)(2.5-3)}{0.063} \cdot 36.576$$
$$+ \frac{(2.5-2)(2.5-2.1)(2.5-2.2)(2.5-2.7)}{0.216} \cdot 66$$
$$= \frac{0.012}{0.014} \cdot 6 - \frac{0.015}{0.0054} 7.752 + \frac{0.02}{0.008} \cdot 10.256 + \frac{0.03}{0.063} \cdot 36.576 - \frac{0.012}{0.216} \cdot 66$$
$$= \frac{36}{7} - \frac{323}{15} + \frac{641}{25} + \frac{3048}{175} - \frac{11}{3}$$
$$= 23$$