COMPUTER SCIENCE 349A, FALL 2020
ASSIGNMENT #3 - 20 MARKS

DUE FRIDAY OCTOBER 16, 2020 (11:30 p.m. PST)

This is a really large class and the logistics of grading assignments are challenging. Me and the markers require your help in making this process go smoothly. Please ensure that your assignments conform to the following requirements - any violation will result in getting a zero for the particular assignment.

- All assignments should be submitted electronically through the ConneX course website and shoud be **SINGLE PDF FILES**. No other formats will be accepted.. Handwritten answers are ok but they will need to be scanned and merged into a single pdf file together with any code examples and associated plots.
- The assignment number, student name and student number should be clearly visible on the top of every page of your assignment submission.
- **PLEASE DO NOT COPY THE ASSIGNMENT DESCRIPTION IN YOUR SUBMISSION**
- The asnwers to the questions should be in the same order as in the assignment specification.
- Some of the questions of the assignments are recycled from previous years but typically with small changes in either the description or the numbers. Any submission that contains numbers from previous years in any questions will be immediately graded with zero.
- Any assignment related email questions should have a subject line of the form CSC349A Assignment X, where X is the number of the corresponding assignment.
- The total number of points for this assignment is 20.

Question #1 - 8 marks.

(a) **(3 points)** Determine the third order ($n = 3$) Taylor polynomial approximation for $f(x) = \ln(x+2)$ expanded about $a = -1$ without the remainder term. Leave your answer in terms of factors $(x+1)$ (that is, do not simplify). Show all your work.

(b) **(1 point)** Use the polynomial approximation in (a) (without the remainder term) to approximate $f(-1.06) = \ln(0.94)$. Use either hand computation, your calculator or MATLAB. Give an exact answer to 6 significant digits.

(c) **(1 point)** To 6 correct significant digits, the exact value of $f(-1.06) = \ln(0.94)$ is -0.0618754. Use this value to compute the absolute error of your computed approximation in (b).

(d) **(2 points)** Determine a good upper bound for the truncation error of the Taylor polynomial approximation in (b) by deriving the remainder term for the approximating polynomial in (a) and bounding it.

(e) **(1 point)** In MATLAB, plot a graph containing $f(x)$ and the polynomial approximation from (a) over the interval $[-2, 1]$ with a step size of 0.01. Include a legend in your plot, denoting the colour of each of the two curves. You need to include the commands and plot results in your submitted pdf.

Question #2 - 4 marks.
Consider the function

$$f(x) = 1 - \sin x$$

(a) **(2 points)** Is $f(x)$ well-conditioned or ill-conditioned for values of x close to $\frac{\pi}{2}$? Is $f(x)$ well-conditioned or ill-conditioned for values of x close to 0.5? To answer this, do the following. Consider the condition number $\tilde{x}f'(\tilde{x})/f(\tilde{x})$, and evaluate this for $\tilde{x} = 1.56$ and $\tilde{x} = 0.51$.

(b) **(2 points)** Now, calculate the relative error between the inputs and outputs in the two cases in (a)? Briefly explain how the condition number predicts these values.

Question #3 - 8 Marks
The evaluation of

$$g(x) = \begin{cases} \frac{(1+\cos x)}{(x-\pi)^2} & x \neq \pi \\ 0.5 & x = \pi \end{cases}$$

is inaccurate in floating-point arithmetic when x is approximately equal to π (radians). For example, if $x = 3.16$, then

$$fl(g(3.16)) = 0.5908$$

using $k = 4$, $b = 10$, idealized, chopping floating-point arithmetic. Note that the correct value of $g(3.16)$ is 0.499985..., so this computed approximation has a relative error of approximately 18%.

The fourth order ($n = 4$) Taylor polynomial approximation for $f(x) = \cos x$ expanded about $a = \pi$ is

$$\cos x \approx -1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$$

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(a) **(2 points)** Substitute the above Taylor polynomial approximation for $\cos x$ into the formula for $g(x)$, and simplify in order to obtain a polynomial approximation for $g(x)$ when $x \neq \pi$. (This polynomial approximates $g(x)$ very well when x is close to π since the Taylor polynomial approximation is very accurate when x is close to π.)

(b) **(3 points)** Show that the above floating-point computation of $g(3.16)$ is unstable. Use the notation and the definition of stability given in Handout 7 to show this.

Hint: consider a perturbation of the data $\hat{x} = 3.16 + \varepsilon$, where $\left|\frac{\varepsilon}{3.16}\right|$ is small. Use the polynomial approximation to $g(x)$ in (a) to determine a very accurate approximation to the exact value of $\hat{r} = \dfrac{1+\cos\hat{x}}{(\hat{x}-\pi)^2}$, and show that for all small values of ε, the exact value of $\hat{r}$ is not close to the computed floating-point approximation of 0.5908.

(c) **(3 points)** If $x = 1.41$ (radians), then

$$fl(g(1.41)) = 0.3871$$

$k = 4$, $b = 10$, idealized, chopping floating-point arithmetic. Show that this floating-point computation is stable (using the notation and the definition of stability given in Handout 7).