UNIVERSITY OF VICTORIA
MIDTERM EXAM 2 NOVEMBER 3 2020
COMPUTER SCIENCE 349A

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DURATION: 50 minutes

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THIS QUESTION PAPER HAS 4 PAGES INCLUDING THIS COVER PAGE.

NOTES: (0) OPEN BOOK EXAM; ONLY BASIC CALCULATORS ARE ALLOWED,
(1) THERE ARE A TOTAL OF 18 MARKS,
(2) YOU MAY NOT USE THE INTERNET FOR ANY PURPOSE OTHER THAN THE COURSE WEBSITE ON CONNEX WITH THE EXCEPTION OF AN ONLINE PDF TOOL IF NECESSARY

Question	Possible marks	Actual marks
1	6	
2	6	
3	6	
Total	18	

1. (a) [3 points] Determine the second order ($n = 2$) Taylor polynomial approximation for $f(x) = \sqrt{x-1}$ expanded about $a = 2$. Do not inculde the remainder term.

   (b) [1 point] Determine the remainder term for the polynomial approximation in (a).

   (c) [2 points] Determine a good upper bound for the truncation error of the Taylor polynomial approximation in (a) when $2 \le x \le 2.15$ by bounding the remainder term in (b).

(a) $f(x) = \sqrt{x-1}$

$f'(x) = \frac{1}{2\sqrt{x-1}}$

$f''(x) = \frac{1}{4(x-1)^{\frac{3}{2}}}$

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$

$= 1 + \frac{1}{2}(x-2) + \frac{\frac{1}{4}}{2}(x-2)^2$

$= 1 + \frac{1}{2}x - 1 + \frac{1}{8}(x^2 - 4x + 4)$

$= \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{2}x + \frac{1}{2}$

$= \frac{1}{8}x^2 + \frac{1}{2}$

(b) $R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$ where $n=2$ $a=2$ ξ is between x and a

$R_2 = \frac{f^3(\xi)}{6}(x-2)^3$ where ξ is between x and 2

$f^3(x) = -\frac{1}{4} \cdot (-\frac{3}{2})(x-1)^{-\frac{5}{2}} = -\frac{3}{8}(x-1)^{-\frac{5}{2}}$

$R_2 = \frac{\frac{3}{8}(\xi-1)^{-\frac{5}{2}}}{6}(x-2)^3 = \frac{1(\xi-1)^{-\frac{5}{2}}}{16}(x-2)^3$ where ξ is between x and 2

(c) $|R_2| < |\frac{1(\xi-1)^{-\frac{5}{2}}}{16}(x-2)^3|$ where take $x, \xi \in \{2, 2.15\}$ to maximum $|R_2|$

Take upper bound $x = 2.15$ $\xi = 2$ in R_2

$|R_2| = \frac{1(2-1)^{-\frac{5}{2}}}{16}(2.15-2)^3| \approx -0.0002109375$

2. (a) [2 points] Consider the following polynomial: $f(x) = x^4 + x^3 - 5x^2 - 6x + 6$. Use Horner's algorithm to compute $f(2.1)$ and $f'(2.1)$.

   (b) [1 point] Use Newton's method and the results of part (a) to compute x_1, an approximation to a zero of $f(x)$.

   (c) [1 point] If x_1 from (b) is taken to be the approximation to the zero, what is the approximate deflated polynomial?

   (d) [2 points] If the computations with Newton's method in (a) and (b) are continued, the method converges to a zero at approximately 2.0975. What will be the order of convergence? Justify.

(a) $f(x) = 6 + x\cdot(-6 + x\cdot(-5 + x(1 + x\cdot 1)))$

$b_4 = 1$ $\quad C_4 = 1$

$b_3 = 1 + (1\cdot 2.1) = 3.1$ $\quad C_3 = 3.1 + 2.1\cdot 1 = 5.2$

$b_2 = -5 + (3.1)(2.1) = 1.51$ $\quad C_2 = 1.51 + 5.2\cdot 2.1 = 12.43$

$b_1 = -6 + 1.51\cdot 2.1 = -2.829$ $\quad C_1 = -2.829 + 12.43\cdot 2.1 = 23.274$

$b_0 = 6 + 2.1\cdot(-2.829) = 0.0591$

$f(2.1) = b_0 = 0.0591$ $\quad f'(2.1) = C_1 = 23.274$

(b) $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = 2.1 - \dfrac{0.0591}{23.274} \approx 2.097460686$

(c) For $x_1 = 2.097460686$

$b_4 = 1$ $\quad C_4 = 1$

$b_3 = 1 + 2.097460686 = 3.097460686$ $\quad C_3 = b_3 + x_1\cdot C_4 = 5.194921372$

$b_2 = -5 + (b_3\cdot x_1) = 1.496802015$ $\quad C_2 = b_2 + C_3\cdot x_1 = 12.39294536$

$b_1 = -6 + (b_2\cdot x_1) = -2.860516619$ $\quad C_1 = b_1 + C_2\cdot x_1 = 23.13319906$

$b_0 = 6 + (b_1\cdot x_1) = 0.000178849979$

$Q(x_1) = -2.860516619 + 1.496802015x + 3.097460686x^2 + x^3$

(d) C_i for X_0 and X_1 are $\neq 0$ It is continued.

$X_2 = X_1 - \frac{f(x_0)}{f'(x_1)} = 2.097452955$

for $x_1 = 2.0975$

$b_4 = 1$ 		$C_4 = 1$

$b_3 = 1 + 1 \cdot 2.0975 = 3.0975$ 		$C_3 = 3.0975 + 2.0975 \cdot 1 = 5.195$

$b_2 = -5 + 3.0975 \cdot 2.0975 = 1.49700625$ 		$C_2 = 1.49700625 + 5.195 \times 2.0975 = 12.3935875$

$b_1 = -6 + 1.49700625 \cdot 2.0975 = -2.860029391$ 		$C_1 = -2.860029391 + C_2 \cdot X_1 = 23.13537619$

$b_0 = 0.001088352378$

From the question, the method converges to a zero at approximately 2.0975, and the computation is continued.

So, $f(x) = 0$ producing a sequence x_i which coverage to 2.0975, and $f'(x_t) = 23.13537619 > 0$

So the order of convergence is 2.

3. [6 points] Consider the following function,

$$g(x) = \frac{\ln(x+1) - x}{x^2}.$$

Using precision $k = 4$, $b = 10$, chopping, floating-point arithmetic and the Taylor polynomial approximation

$$\ln(x+1) \approx x - \frac{x^2}{2} + \frac{x^3}{3},$$

which is very accurate when x is close to 0, show that the computation of $fl(g(0.01234))$ is unstable.

$$\ln(x+1) \approx x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$g(x) = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} - x}{x^2}$$

$$= \frac{-\frac{x^2}{2} + \frac{x^3}{3}}{x^2} = -\frac{1}{2} + \frac{x}{3} = \frac{2x-3}{6}$$

$$fl(g(0.01234)) = \frac{2 \times (0.01234) - 3}{6} = -0.49588667$$

Since it should be accurate when x is close to 0. and from the definition of stability these float-number should be repending, So it is unstable.