$X^k \quad \left[\quad x_{ij} \quad\right]_{I \times J_k}$ $\qquad T_{I \times R} \quad \left[\quad t_{ir} \quad\right]_{I \times R}$

$P^{kT} \quad \left[\quad p_{rj} \quad\right]_{R \times J_k}$ （有转置！是 P^T）

$$\| Vec(X^k) - (I \otimes T) Vec(P^{kT}) \|_2^2$$

$$Vec(X^k) = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{I1} \\ \vdots \\ x_{IJ_k} \end{pmatrix} \qquad \underset{\displaystyle J_k \times J_k}{\underset{\downarrow}{I}} \otimes T = \begin{bmatrix} T & & & \\ & T & & \\ & & \ddots & \\ & & & T \end{bmatrix} ~~I_k~~ \; I \times J_k$$

$R \times J_k$

$$Vec(P^{k^T}) = \begin{bmatrix} p_{11} \\ p_{21} \\ \vdots \\ p_{R1} \\ \vdots \\ p_{RJ_k} \end{bmatrix}$$

$$I \otimes T \; Vec(P^{k T}) \Rightarrow \overset{T}{\begin{bmatrix} \quad \end{bmatrix}} \underset{\Downarrow}{\begin{bmatrix} \quad \end{bmatrix}} \Big| \begin{bmatrix} \quad \end{bmatrix} \nearrow$$

一这部分的 $Vec(P^T)$

$i=1 \; j=1 \qquad t_{11} P_{11} + t_{12} P_{21} + \cdots + t_{1R} P_{R1} \Rightarrow \sum_r^R t_{1r} P_{r1}$

$i=2 \; j=1 \qquad t_{21} P_{11} + t_{22} P_{21} + \cdots + t_{2R} P_{R1} \Rightarrow \sum_r^R t_{2r} P_{r1}$

$\vdots \qquad\qquad\qquad \vdots$

$i=I \; j=1 \qquad t_{I1} P_{11} + t_{I2} P_{21} + \cdots + t_{IR} P_{R1} \Rightarrow \sum_r^R t_{Ir} P_{r1}$

$i=1 \; j=2 \qquad t_{11} P_{12} + t_{12} P_{22} + \cdots + ~~t_{2R}~~ \, t_{1R} P_{R2} \Rightarrow \sum_r^R t_{1r} P_{r2}$

i

$$\therefore\ J \otimes T\ Vec(\overset{\to k\ \text{block}}{P^T})$$

$$= \begin{pmatrix} \sum_r^R t_{1r}p_{r1} \\ \sum_r^R t_{2r}p_{r1} \\ \vdots \\ \sum_r^R t_{Ir}p_{r1} \\ \vdots \\ \sum_r^R t_{ir}p_{rj} \\ \vdots \\ \sum_r^R t_{ir}p_{rJ_k} \\ \vdots \\ \sum_r^R t_{Ir}p_{rJ_k} \end{pmatrix} \begin{array}{l} i=1\ \ j=1 \\ \\ \\ i=I\ \ j=1 \\ \\ i=i\ \ j=j \\ \\ i=i\ \ j=J_k \\ \\ \cancel{p}\ i=I\ \ j=J_k \end{array}$$

$$Vec(X) = \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{I1} \\ \vdots \\ x_{IJ_k} \end{pmatrix}$$

$$\downarrow$$

$$Vec(X) - J \otimes T\,(P^T)$$

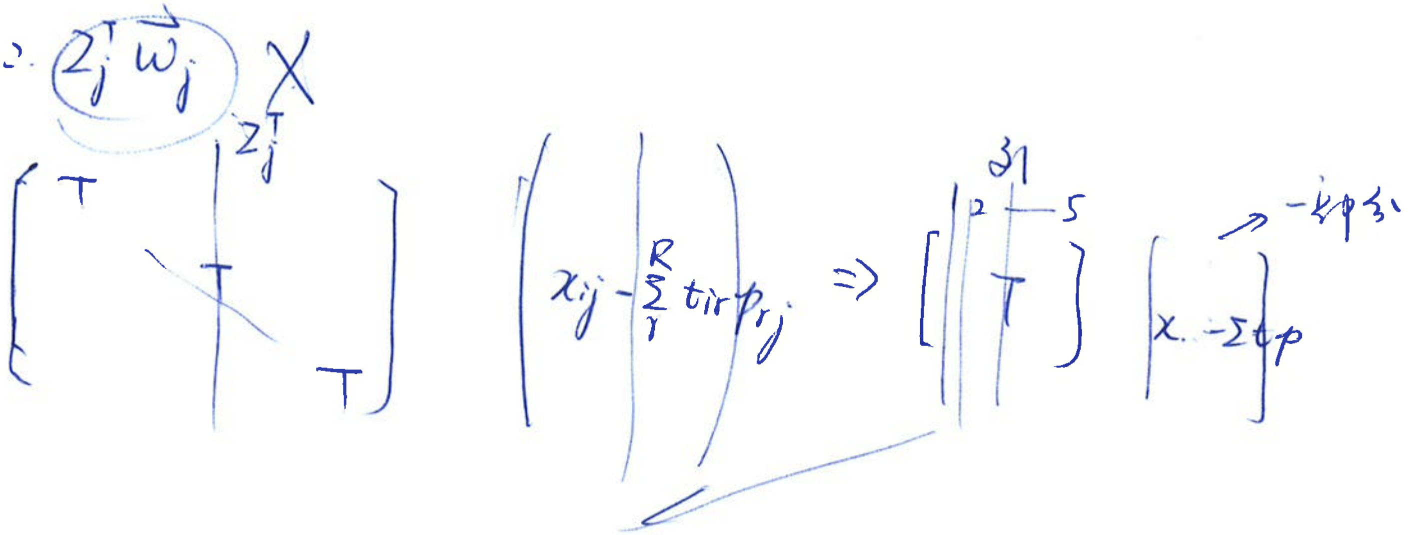
$Z_j^T \vec{w}_j$ | $Z = X^{(DaTA)} = J \otimes T = \begin{bmatrix} T & & \\ & \ddots & \\ & & T \end{bmatrix}$ $\to Z_j^{\cancel{\phi}}$

$$\vec{w}_j : Vec(X^k) - Vec(TP_k^T)$$

$\cancel{P}\,Vec(P^T)_{\cancel{p}\ jth} = 0 \Rightarrow$ to estimate

$$= \begin{pmatrix} \phi \\ \vdots \\ x_{ij} - \sum_r^R t_{ir}p_{rj} \\ \vdots \end{pmatrix}$$

ii



$$\underset{\uparrow\uparrow\ i\ r}{t_{11}}(\underset{\uparrow\uparrow\ i\ j}{x_{11}} - \Sigma_r^R \underset{\uparrow\ i}{t_{1r}} \underset{\uparrow\ j}{p_{r1}}) + t_{21}(x_{21} - \Sigma_r^R t_{2r} p_{r1}) + \cdots$$

$$+ t_{I1}(x_{I1} - \Sigma_r^R t_{Ir} p_{r1}) = \Sigma_i^I [t_{i1}(x_{i1} - \Sigma_r^R t_{ir} p_{r1})]$$

~~t_{i2}~~

$$\hat{p}_{11}$$
$$\hat{p}_{21}$$
$$\vdots$$
$$\hat{p}_{r1}$$
$$\vdots$$
$$\hat{p}_{R1}$$
$$\vdots$$
$$\hat{p}_{rj}$$
$$\vdots$$
$$\hat{p}_{RJ}.$$

~~$\Sigma_i^I [t_{i2}(x_{i2}$~~

$$\Sigma_i^I [t_{i2}(x_{i1} - \Sigma_r^R (t_{ir} p_{r1})]$$
$$\vdots$$
$$\Sigma_i^I [t_{iR}(x_{i1} - \Sigma_r^R (t_{ir} p_{r1})]$$
$$\Sigma_i^I [t_{i1}(x_{i2} - \Sigma_r^R (t_{ir} p_{r2})]$$
$$\vdots$$
$$\Sigma_i^I [t_{iR}(x_{i2} - \Sigma_r^R (t_{ir} p_{r2})]$$
$$\vdots$$
$$\Sigma_i^I [t_{iR}(x_{ij} - \Sigma_r^R (t_{ir} p_{rj})]$$
$$\vdots$$
$$\Sigma_i^I [t_{iR}(x_{iJ} - \Sigma_r^R (t_{ir} p_{rJ})]$$

iii（不需要）

Note that this page is ~~not~~ no longer needed

$$S.(X^{k'} r_{(-k)}, \lambda_2)$$

$$X^{k} = I \otimes T = \begin{bmatrix} T & & & \\ & T & & \\ & & \ddots & \\ & & & T \end{bmatrix}$$

[illegible] notation

[illegible]

$$r_{(-k)} \equiv \underline{DATA_{(-k)}} = \underset{\hookrightarrow \text{data}}{Vec(X)}$$

$$X^{k^T} r_{(-k)} = \begin{bmatrix} T & & \\ & \ddots & \\ & & T \end{bmatrix} \overset{Vec(X_k) \rightarrow DATA}{\begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{I1} \\ x_{12} \\ \vdots \end{bmatrix}}$$

$$t_{11}x_{11} + t_{21}x_{21} + \cdots + t_{I1}x_{I1} = \cancel{\sum_i^I x_i} \; \sum_i^I t_{i1} x_{i1}$$

$$t_{12}x_{11} + t_{22}x_{21} + \cdots + t_{I2}x_{I1} = \sum_i^I t_{i2} x_{i1}$$

$$\begin{array}{c|cc} \begin{bmatrix} \sum_i^I t_{i1} x_{i1} \\ \sum_i^I t_{i2} x_{i1} \\ \vdots \\ \sum_i^I t_{iR} x_{i1} \\ \sum_i^I t_{i1} x_{i2} \\ \vdots \\ \sum_i^I t_{iR} x_{i2} \\ \vdots \\ \sum_i^I t_{iR} x_{iJ} \end{bmatrix} & \begin{matrix} r & j \\ 1 & 1 \\ 2 & 1 \\ \vdots & \\ R & 1 \\ 1 & 2 \\ \vdots & \\ R & 2 \\ \vdots & \\ R & J. \end{matrix} \end{array}$$

```
j
for(j = 1 : J ) {
    for(r = 1 : R){
$\Delta_{rj} = \sum_i t_{ir} x_{ij}$
```