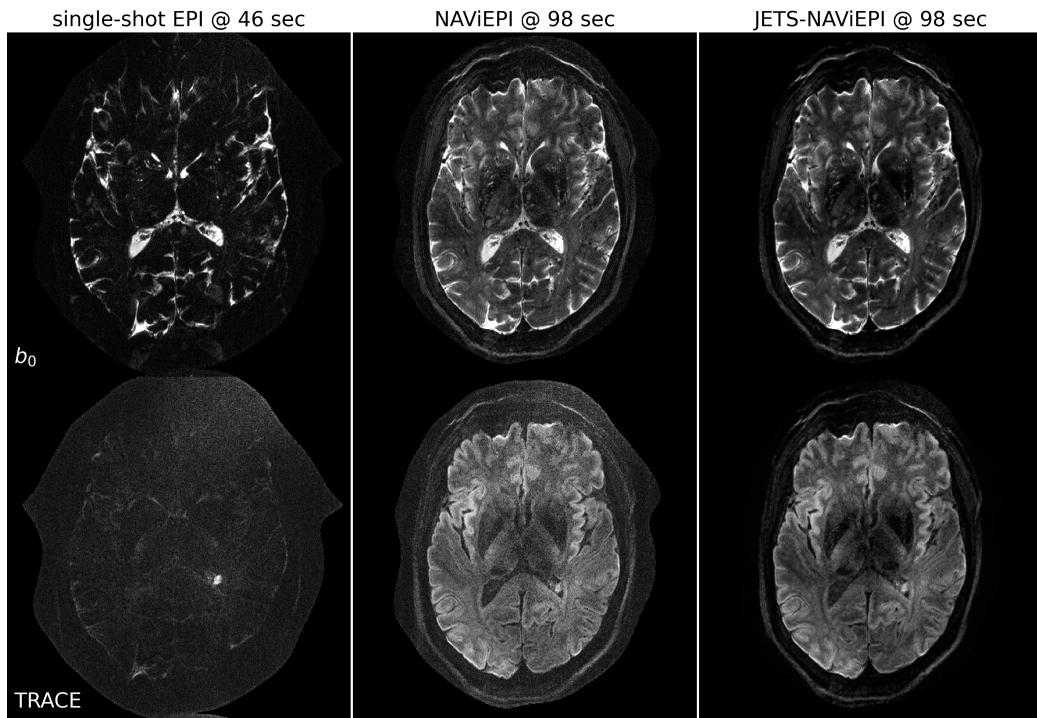


Graphical Abstract

Accelerated Diffusion Weighted Magnetic Resonance Imaging at 7 T: Joint Reconstruction for Shift-Encoded Navigator-based Interleaved Echo Planar Imaging (JETS-NAViEPI)

Zhengguo Tan, Patrick Alexander Liebig, Robin Martin Heidemann, Frederik Bernd Laun, Florian Knoll

3-scan trace acquisition with voxel size 0.5 X 0.5 X 2.0 mm³



Highlights

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- Navigator-based interleaved EPI acquisition with minimal distortion mismatch between echoes
- Novel accelerated diffusion acquisition with shifted phase encoding among diffusion directions for complementary k - q -space sampling at 7 T
- Generalized joint k - q -slice diffusion-weighted image reconstruction with overlapping locally low-rank regularization
- Efficient simultaneous multi-slice (SMS) image reconstruction
- 3-scan trace acquisition with the voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ and 60 slices at 1.5 min

Accelerated Diffusion Weighted Magnetic Resonance Imaging at 7 T: Joint Reconstruction for Shift-Encoded Navigator-based Interleaved Echo Planar Imaging (JETS-NAViEPI)

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Abstract

The pursuit of high spatial-angular-temporal resolution for in vivo diffusion-weighted magnetic resonance imaging (DW-MRI) at ultra-high field strength (7 T and above) is important in understanding brain microstructure and function. Such pursuit, however, faces several technical challenges. First, increased off-resonance and shorter T_2 relaxation require faster echo train readouts. Second, existing high-resolution DW-MRI techniques usually employ in-plane fully-sampled multi-shot EPI, which not only prolongs the scan time but also induces a high specific absorption rate (SAR) at 7 T. To address these challenges, we develop in this work navigator-based interleaved EPI (NAViEPI) which enforces the same effective echo spacing (ESP) between the imaging and the navigator echo. First, NAViEPI renders no distortion mismatch between the two echoes, and thus simplifies shot-to-shot phase variation correction. Second, NAViEPI allows for a large number of shots

(e.g. > 4) with undersampled iEPI acquisition, thereby rendering clinically-feasible high-resolution sub-millimeter protocols. To retain signal-to-noise ratio (SNR) and to reduce undersampling artifacts, we developed a k_y -shift encoding among diffusion encodings to explore complementary k - q -space sampling. Moreover, we developed a novel joint reconstruction with overlapping locally low-rank regularization generalized to the multi-band multi-shot acquisition at 7 T (dubbed JETS-NAViEPI). Our method was demonstrated with experimental results covering 1 mm isotropic resolution multi b -value DWI and sub-millimeter in-plane resolution fast TRACE acquisition.

Keywords: Diffusion-weighted magnetic resonance imaging, Echo planar imaging, Navigator, Ultra-high field, Joint reconstruction, Low rank, Simultaneous multi slice

¹ **1. Introduction**

² Diffusion-weighted magnetic resonance imaging (DW-MRI) ([Le Bihan et al., 1986; Merboldt et al., 1985](#)) is a non-invasive modality that is sensi-
³ tive to the intravoxel Brownian motion of water molecules. DW-MRI forms
⁴ the basis for diffusion tensor imaging (DTI) ([Basser et al., 1994; Mori et al., 1999](#)) and high angular resolution diffusion imaging (HARDI) ([Tuch et al., 2002](#)), and has been widely used in acute brain ischemia diagnosis, in tumor
⁵ detection and staging, and in neuroscience ([Jones, 2010](#)).

⁶ For DW-MRI acquisition, the commonly used pulse sequence is single-
⁷ shot echo-planar imaging (SS-EPI) ([Mansfield, 1977](#)). SS-EPI is capable of
⁸ rapidly acquiring one DW image per radio-frequency excitation at the order
⁹ of 100 ms, and is thus motion robust. However, conventional SS-EPI, even
¹⁰ with three-fold accelerated acquisition ([Bammer et al., 2001](#)) using parallel
¹¹ imaging ([Roemer et al., 1990; Ra and Rim, 1993; Pruessmann et al., 1999](#);
¹² [Griswold et al., 2002](#)), still suffers from low spatial resolution and geometric
¹³ distortions.

¹⁴ In the quest for high spatial-angular-temporal-resolution and minimal-
¹⁵ geometry-distortion DW-MRI, tremendous efforts have been made. Tech-
¹⁶ niques [for](#) the correction of image distortions induced by off-resonances and R249.Minor.5
¹⁷ eddy currents have been developed ([Andersson et al., 2003](#)). Furthermore,
¹⁸ gSlider ([Setsompop et al., 2018](#)) with blipped-CAIPI ([Setsompop et al., 2012](#))
¹⁹ for simultaneous multi-slice (SMS) ([Maudsley, 1980; Breuer et al., 2005](#))
²⁰ was proposed to achieve high-resolution DW-MRI. Advanced pulse sequences
²¹ based on multi-shot EPI have also been developed, including but not limited
²² to interleaved EPI (iEPI) ([Butts et al., 1993](#)), PROPELLER ([Pipe et al., 2004](#)),

26 2002), and readout-segmented EPI (rsEPI) (Porter and Heidemann, 2009;
27 Heidemann et al., 2010).

28 Based on four-shot iEPI, multiplexed sensitivity encoding (MUSE) image
29 reconstruction achieved DW-MRI with a sub-millimeter in-plane resolution
30 and maximal b -value 800 s/mm^2 at 3 T (Chen et al., 2013). The four-shot
31 iEPI employed in MUSE acquired an in-plane fully-sampled k -space, except
32 partial Fourier. Every shot (segment), corresponding to four-fold under-
33 sampling, was then reconstructed via parallel imaging to obtain shot-to-shot
34 phase variation. This indicates that increasing the number of shots in MUSE
35 will result in higher undersampling per shot, and consequently, degrade shot
36 phase estimation (Wu and Miller, 2017).

37 Alternatively, navigator-based iEPI acquisition has been proposed (Jeong
38 et al., 2013; Dai et al., 2017, 2018). These proposals allow for a larger num-
39 ber of shots, and hence higher spatial resolution. However, due to the use of
40 different ESP between the imaging echo and the navigator echo, these pro-
41 posals suffered from geometric distortion mismatch between the two echoes
42 and thus required specific compensation methods. In contrast, rsEPI (Porter
43 and Heidemann, 2009; Heidemann et al., 2010) used the same readout seg-
44 ment for both echoes, and thus required no distortion correction of navigator
45 echoes.

46 Beyond the MUSE-type parallel imaging reconstruction, compressed sens-
47 ing (Lustig et al., 2007; Block et al., 2007) has been explored. For instance,
48 multi-shot reconstruction techniques based on structured low-rank matrix
49 completion (MUSSELS) (Mani et al., 2017; Bilgic et al., 2019) achieved 5-
50 shot DW-MRI with 9-fold undersampling per shot. Recently, JULEP (Dai

51 et al., 2023) incorporated explicit phase mapping into MUSSELS. These re-
52 construction techniques, i.e., MUSE, MUSSELS and JULEP, targeted the
53 reconstruction of one DW image from interleaved EPI acquisition, and did
54 not explore joint- k - q -space undersampling or reconstruction.

55 Joint- k - q -space undersampling can be achieved via proper regularization
56 along the diffusion encoding direction. Relevant examples are diffusion un-
57 dersampling with Gaussian process estimated reconstruction (DAGER) (Wu
58 et al., 2019) and magnitude-based spatial-angular locally low-rank regular-
59 ization (SPA-LLR) (Hu et al., 2020). However, DAGER addressed the re-
60 construction problem of single-shot EPI acquisition and SPA-LLR focused
61 on the reconstruction of single-band and fully-sampled iEPI acquisition. R248.Major.2

62 In this work, we propose a Joint k - q -slice rEconsTruction framework
63 for Shift-encoded NAVigator-based interleaved EPI at 7 T (dubbed JETS-
64 NAViEPI). Our pulse sequence, NAViEPI, differs from most existing tech-
65 niques. First, NAViEPI builds upon interleaved EPI, thereby allowing for
66 fast and efficient k -space coverage. Second, inspired by rsEPI, NAViEPI en-
67 sures the same effective ESP between the imaging and the navigator echo,
68 thereby minimizing geometric distortion and allowing for the use of a larger
69 number of shots. NAViEPI essentially integrates the advantages of both iEPI
70 and rsEPI. Third, NAViEPI utilizes undersampled multi-shot iEPI, thereby
71 alleviating the SAR problem at 7 T. Fourth, NAViEPI shifts the k -space in-
72 plane sampling pattern along the phase encoding (k_y) direction. This shifting
73 creates complementary k - q -space sampling, which leads to the possibility of
74 our joint k - q -slice reconstruction. Specifically, we employ spatial-diffusion
75 overlapping LLR regularization to jointly reconstruct all diffusion encodings

⁷⁶ and multi-band slices. In vivo experiments at 7 T and comparisons with other
⁷⁷ techniques demonstrate the efficiency of our proposed method in achieving
⁷⁸ high spatial resolution DW-MRI at ultra-high field.

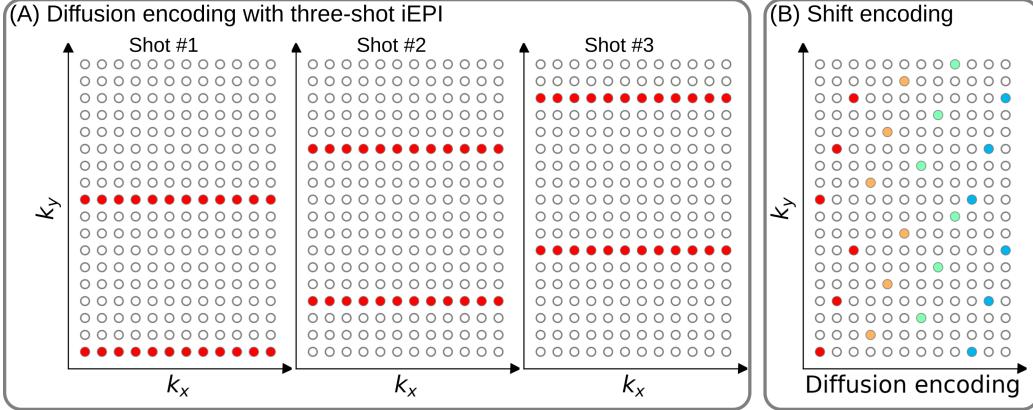


Figure 1: (A) An example DW-MRI acquisition with three-shot interleaved EPI acquisition. (B) The proposed k_y shifted diffusion encoding scheme. This example employs three shots per DW image. Therefore, every three columns have the same color.

79 2. Materials and methods

80 2.1. Multi-band shift-encoded iEPI acquisition

81 Fig. 1 (A) displays the diffusion-weighted image acquisition based on
 82 three-shot interleaved EPI with three-fold in-plane undersampling. Conven-
 83 tionally, such a sampling pattern is repeated for all diffusion directions. In
 84 contrast, we propose the k_y -shifted diffusion encoding, as shown in Fig. 1 (B).
 85 The interleaved EPI sampling pattern is shifted by one k_y line per diffusion
 86 direction, with the cycling period being the in-plane undersampling factor.

87 It is worth noting that, as shown in Fig. 1 (A), the undersampling factor
 88 of one segment is $R_{\text{in-plane}} \times N_{\text{shot}}$ (ignore multi-band undersampling here),
 89 yielding nine-fold in-plane undersampling in this example. In other words,
 90 the undersampling factor per segment linearly scales up with the number
 91 of shots. Consequently, conventional self-gating reconstruction techniques,
 92 e.g. MUSE, suffer from degraded shot-to-shot phase estimation, which in

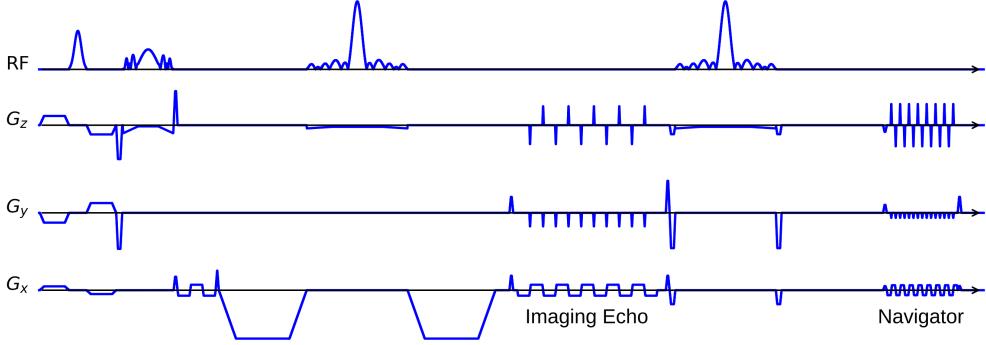


Figure 2: The NAViEPI sequence diagram. SMS is utilized for the acquisition of both imaging and navigator echoes. While the acceleration factor per navigator is the same as listed in Table 1, the acceleration factor per imaging echo is in addition linearly scaled by the number of shots.

93 turn limits the number of shots and spatial resolution.

94 *2.2. NAViEPI: Navigator-based iEPI with consistent effective ESP between
95 the imaging and the navigator echo - where iEPI meets rsEPI*

96 Instead of the self-gated MUSE with in-plane fully-sampled iEPI and
97 a limited number of shots, We propose NAVigator-based interleaved EPI
98 (NAViEPI), as illustrated in Fig. 2. Inspired by rsEPI (Porter and Hei-
99 demann, 2009), NAViEPI enforces a consistent effective ESP between the
100 imaging and the navigator echo, thereby minimizing distortion mismatch
101 between the two echoes.

102 Since one imaging echo presents one segment in multi-shot EPI acquisi-
103 tion, its effective ESP is defined as

$$\text{ESP}_{\text{eff}} = \frac{\text{ESP}}{R_{\text{in-plane}} \times N_{\text{shot}}} \quad (1)$$

104 Here, a larger number of shots (segments) increases the undersampling factor
105 per segment (see Fig. 1), but decreases the effective ESP. Since the navigator

106 echo is acquired for each segment, its in-plane undersampling factor equals
107 $R_{\text{in-plane}}$. Therefore, the effective ESP of the navigator echo must match that
108 of the imaging echo, as given in Eq. (1). With a matching effective ESP, the
109 base resolution of the navigator echo can then be determined.

110 *2.3. In vivo acquisition protocols*

111 We implemented multiple in-vivo acquisition protocols at a clinical 7 T
112 MR system (MAGNETOM Terra, Siemens Healthineers, Erlangen, Ger-
113 many) equipped with a 32-channel head coil (Nova Medical, Wilmington,
114 MA, USA) and the XR-gradient system (maximum gradient strength 80 mT/m
115 with a peak slew rate of 200 T/m/s). To calibrate coil sensitivity maps, refer-
116 ence scans employed a gradient-echo (GRE) sequence. Spectral fat saturation
117 and mono-polar diffusion-encoding gradients were used. The phase-encoding
118 direction was selected as anterior-to-posterior.

Table 1: NAViEPI acquisition protocols

Protocol	1.0 mm isotropic		sub-millimeter	
	#1	#2	#3	#4
Diffusion mode	MDDW ⁽¹⁾		3-scan trace	
Diffusion scheme	monopolar			
Diffusion direction	20	114	3	
<i>b</i> -value (s/mm ²)	1000	3-shell ⁽²⁾	1000	
<i>b</i> ₀	0	12	1	
FOV (mm ²)	200	214	220	
In-plane resolution (mm ²)	1.0		0.5	
Slice thickness (mm)	1.0		2.0	
Slices	141	114	60	
Navigator	No	No	Yes	No
Shots	4	2	5	1
TR (ms)	7700	5200	4400	8000
TEs (ms)	67	66	58/95.1	143
ESP (ms)	1.02	0.81	1.52	1.48
Bandwidth (Hz/Pixel)	1086	1460	758	
Partial Fourier			6/8	
Acceleration ⁽³⁾	1 × 3	3 × 3	3 × 2	
TA (min) ⁽⁴⁾	10 : 42	22 : 25	1 : 38	0 : 46

⁽¹⁾ MDDW: Multi-direction diffusion weighting;

⁽²⁾ 3-shell: 20, 30, and 64 directions with *b*-values of 1000, 2000, and 3000 s/mm², respectively;

⁽³⁾ Acceleration: Both in-plane and slice undersampling can be employed, denoted as (*R*_{in-plane} × *R*_{slice});

⁽⁴⁾ TA: Total acquisition time.

120 This study was approved by the local ethics committee. Three volunteers
121 with informed consent obtained before scanning participated in this study.
122 Detailed acquisition protocols are listed in Table 1.

123 *2.3.1. 20-diffusion-direction acquisition at 1 mm isotropic resolution*

124 As listed in Table 1, Protocol #1 with four-shot iEPI and without in- R248.Major.1a
125 plane undersampling was implemented. This protocol represents the acquisi-
126 tion scheme employed in many existing multi-shot reconstruction techniques,
127 (e.g., MUSE, SPA-LLR, and JULEP). The acquired data from this protocol
128 served as ground truth. Different reconstruction methods, specifically JETS,
129 MUSE, and JULEP were compared. We compared with JULEP instead of R249.Minor.6
130 MUSSELS, because JULEP uses not only structured low-rank constraints R248.Major.3a
131 but also explicit phase mapping.

132 We then retrospectively reduced the four-shot data to only one shot per R248.Major.1b
133 diffusion encoding without and with the proposed k_y shifting to simulate
134 four-fold in-plane undersampling. JETS reconstruction was performed on R248.Major.1b
135 the fully-sampled data and the retrospectively undersampled data to validate
136 the proposed k_y -shifted acquisition. R248.Major.1c

137 *2.3.2. Three-shell acquisition at 1 mm isotropic resolution*

138 Protocol #2 in Table 1 was implemented for multi-shell diffusion tensor R248.Major.3c
139 imaging (DTI) (Basser et al., 1994). We acquired a total of 114 diffusion
140 directions, whereas b_0 measurements were interspersed every ten diffusion
141 directions. This protocol was used to demonstrate the capability of of JETS
142 in achieving high spatial-angular-temporal resolution.

143 2.3.3. 3-scan trace acquisition at $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ voxel size

144 As listed in Table 1, Protocol #3 was implemented based on NAViEPI
145 with five shots per diffusion encoding. This protocol was compared against
146 single-shot EPI (Protocol #4) with the same spatial resolution and acceler-
147 ation, such as to demonstrate the sampling efficiency of NAViEPI.

148 2.4. Forward modeling

149 Our proposed acquisition method yields multi-dimensional multi-band
150 k -space data $\mathbf{y}_{c,q,s}$, where c, q, s denotes the index of the coil sensitivity
151 map, the diffusion encoding, and the shot, respectively. Acquisition modeling
152 needs to consider several aspects.

153 First, the acquired k -space data \mathbf{y} is mapped from individual shot images
154 $\mathbf{x}_{q,s,z}$ via the forward model,

$$\begin{aligned}\mathbf{y}_{c,q,s} &= \mathbf{P}_{q,s} \boldsymbol{\Sigma} \boldsymbol{\Theta}_z \mathbf{F} \mathbf{S}_c \mathbf{x}_{q,s,z} \\ \mathbf{y} &:= \mathbf{E}_1 \mathbf{x}\end{aligned}\tag{2}$$

155 Here, the encoding matrix \mathbf{E}_1 comprises a chain of linear operators. Every
156 shot image \mathbf{x} is point-wise multiplied by a set of coil sensitivity maps (\mathbf{S}) and
157 Fourier transformed (\mathbf{F}). The output is then point-wise multiplied by the
158 multi-slice phase map ($\boldsymbol{\Theta}$) with z the slice index in simultaneously excited
159 slices. This operator shifts individual slice along the phase-encoding direction
160 via varying phase modulation (Breuer et al., 2005). The SMS k -space data
161 is then summed (collapsed, $\boldsymbol{\Sigma}$) along the slice dimension and masked (point-
162 wise multiplied, \mathbf{P}) by the sampling pattern of each diffusion encoding and
163 shot.

164 Second, for diffusion MRI based on multi-shot EPI, multiple shots ac-
 165 quired for a given diffusion encoding need to be combined as one DW image
 166 ($\tilde{\mathbf{x}}$). One possibility is to perform magnitude average (Chen et al., 2013)
 167 or root-sum-squares (RSS) (Mani et al., 2017) of shot images. This method R249.Minor.7
 168 is robust to in-plane motion, but sub-optimal concerning SNR (Guhaniyogi
 169 et al., 2016). Alternatively, shot combination can be done via shot-to-shot
 170 phase variation correction (Liu et al., 2005; Chen et al., 2013). This can be
 171 incorporated into our formulation as point-wise multiplication between the R249.Minor.8
 172 shot-to-shot phase variation (Φ) and the DW image ($\tilde{\mathbf{x}}$),

$$\mathbf{x}_{q,s,z} = \Phi_{q,s,z} \tilde{\mathbf{x}}_{q,z} \quad (3)$$

173 Note that $\tilde{\mathbf{x}}$ can be obtained by applying the adjoint of Φ to \mathbf{x} . In MUSE,
 174 Φ is obtained by parallel imaging reconstruction of all shots with subsequent
 175 phase smoothing of every shot image. Based on this phase correction, the
 176 complete forward model follows

$$\mathbf{y} := \mathbf{E}_2 \tilde{\mathbf{x}} = \mathbf{E}_1 \Phi \tilde{\mathbf{x}} \quad (4)$$

177 where the encoding matrix \mathbf{E}_2 comprises the chain of the shot-to-shot phase
 178 variation Φ and the encoding matrix \mathbf{E}_1 . We implemented these two encoding
 179 operators in SigPy (Ong and Lustig, 2019).

180 *2.5. Joint k - q -slice reconstruction*

181 Based on the generalized forward models in Eqs. (2) and (4), our proposed
 182 joint k - q -slice reconstruction can be formulated as a three-step approach.

183 **I. Navigator echo reconstruction.** The acquisition of navigator echoes
 184 follows the forward model in Eq. (2), so the reconstruction of navigator

185 echoes can be formulated as:

$$\operatorname{argmin}_{\mathbf{x}} \|\mathbf{y} - \mathbf{E}_1 \mathbf{x}\|_2^2 + \lambda \mathbf{R}(\mathbf{x}) \quad (5)$$

186 where $\mathbf{R}(\mathbf{x})$ denotes the regularization functional with the regularization
187 strength λ . In this work, ℓ^2 regularization was used, i.e., $\mathbf{R}(\mathbf{x}) =$
188 $\|\mathbf{x}\|_2^2$. In the case of self-navigating (i.e., no navigator acquired) as
189 Protocol #2, the central k -space region (i.e., 1/4 of the full image matrix)
190 of each segment is used as \mathbf{y} in Eq. (5).

191 **II. Iterative phase smoothing.** Shot-to-shot phase variation was ex-
192 tracted from the reconstructed navigator echo phases. Assuming that
193 phase images are spatially smooth (Chen et al., 2013; Dai et al., 2023),
194 we employed the iterative approach to smooth phase,

$$\mathbf{x}^{(k+1)} = \mathbf{F}^{-1} \mathcal{H} \mathbf{F} \mathbf{x}^{(k)} \quad (6)$$

195 where the index k denotes the phase smoothing iteration step, and $x^{(0)}$
196 is then the reconstructed navigator image from Step I. \mathcal{H} is the Hanning R248.Minor.12
197 window.

198 **III. Shot-combined reconstruction.** Joint reconstruction of all DW im-
199 ages using the shot-combined forward model \mathbf{E}_2 with shot-to-shot phase
200 variation from Step II reads:

$$\operatorname{argmin}_{\tilde{\mathbf{x}}} \|\mathbf{y} - \mathbf{E}_2 \tilde{\mathbf{x}}\|_2^2 + \lambda \|\mathbf{T}(\tilde{\mathbf{x}})\|_* \quad (7)$$

201 Here, LLR regularization was employed in the local spatial-diffusion ma-
202 trices, based on the theory of partially separable functions (Liang, 2007;
203 Trzasko and Manduca, 2011; Zhang et al., 2015). \mathbf{T} represents a linear

operator that firstly slides a local patch window through all DW images
and then flattens every set of local patches to construct two-dimensional
(2D) spatial-diffusion matrices. The spatial dimension equals the block
size, and the diffusion dimension is the number of diffusion encodings.
 $\|\mathbf{T}(\tilde{\mathbf{x}})\|_*$ is the nuclear norm, i.e. the sum of singular values of a spatial-
diffusion matrix. This nuclear norm regularization was accomplished
via singular value thresholding (SVT) of each spatial-diffusion matrix R248.Minor.6
(Cai et al., 2010). After SVT, the adjoint of \mathbf{T} , \mathbf{T}^H , was needed to
reorder pixel values from the spatial-diffusion matrices back to DW im-
ages. To alleviate checkerboard artifacts induced by LLR regularization
with non-overlapping blocks (Hu et al., 2020), we employed overlapping
blocks. In this case, values from overlapping positions are summed up
to the output of \mathbf{T}^H . To enable the correct use of \mathbf{T}^H , we element-wise
divided the output of \mathbf{T}^H by a scaling matrix. This matrix was obtained
via $\mathbf{T}^H(\mathbf{T}(\mathbf{1}))$, where $\mathbf{1}$ denotes the matrix of all ones with the same
shape as the input \mathbf{x} . R249.Minor.9

220 2.6. Reconstruction

221 The acquired raw data was read in by twixtools (<https://github.com/pehses/twixtools>). Ramp-sampling regridding and FOV/2-ghost correc-
222 tion were also performed in twixtools. Subsequently, coil sensitivity maps
223 were computed from reference scans using ESPIRiT (Uecker et al., 2014) in
224 SigPy (Ong and Lustig, 2019).

225 With this pre-processing as well as the implemented forward models and
226 proximal operator, the inverse problem in Eq. (7) was solved by the alter-
227 nating direction method of multipliers (ADMM) (Boyd et al., 2010).
228

229 ADMM solves the minimization problems in an alternating update scheme,

$$\begin{cases} \mathbf{x}^{(k+1)} := \underset{\mathbf{x}}{\operatorname{argmin}} \| \mathbf{y} - \mathbf{E}(\mathbf{x}) \|^2 + \rho/2 \| \mathbf{T}\mathbf{x} - \mathbf{z}^{(k)} + \mathbf{u}^{(k)} \|_2^2 \\ \mathbf{z}^{(k+1)} := \mathcal{T}_{\lambda/\rho}(\mathbf{T}\mathbf{x}^{(k+1)} + \mathbf{u}^{(k)}) \\ \mathbf{u}^{(k+1)} := \mathbf{u}^{(k)} + \mathbf{T}\mathbf{x}^{(k+1)} - \mathbf{z}^{(k+1)} \end{cases} \quad (8)$$

230 where k denotes the ADMM iteration. \mathbf{z} is the auxiliary variable ($\mathbf{z} = \mathbf{T}\mathbf{x}$),
231 and \mathbf{u} is the Lagrangian multipliers. Importantly, when solving Eq. (2), \mathbf{x}
232 denotes shot images and \mathbf{E} denotes \mathbf{E}_1 in Eq. (8). In contrast, \mathbf{x} denotes shot-
233 combined images and \mathbf{E} denotes \mathbf{E}_2 when solving Eq. (4). \mathbf{x} can be solved
234 using linear least square algorithms, e.g. conjugate gradients (Hestenes and
235 Stiefel, 1952), while \mathbf{z} is updated via singular value thresholding (\mathcal{T}) with
236 the thresholding parameter λ/ρ . The coupling parameter ρ is effective in
237 both the update of \mathbf{x} and \mathbf{z} . It acts as Tikhonov regularization strength
238 when updating \mathbf{x} , but also inversely scales the thresholding strength when
239 updating \mathbf{z} .

240 In this work, 15 ADMM iterations with $\rho = 0.05$ and $\lambda = 0.08$ were used.

241 All reconstructions were done on a single A100 SXM4/NVLink GPU with
242 40 GB memory (NVIDIA, Santa Clara, CA, USA).

243 We compared our proposed joint reconstruction with established multi-
244 shot reconstruction techniques, specifically, MUSE (Chen et al., 2013) and
245 JULEP (Dai et al., 2023), hosted on GitHub by Dr. Dai (Dai et al., 2023).
246 Further, we performed the local-PCA denoising (Cordero-Grande et al., 2019)
247 as implemented in MRtrix (Tournier et al., 2019) on the MUSE reconstructed
248 complex DW images.

249 The in vivo data acquired from Protocol #2 in Table 1 consisted of 126
250 diffusion directions, which exceeds the available GPU memory. Therefore,

251 the 126 data volumes were uniformly split into three parts for our JETS
252 reconstruction with a LLR block width of 6 and the LLR regularization in
253 both Steps I and III in Section 2.5. In addition, MUSE reconstruction was
254 also performed, followed by the local-PCA denoising. Reconstructed DWIs
255 were then processed by DiPy ([Garyfallidis et al., 2014](#)) to obtain color-coded
256 fractional anisotropy (cFA) maps.

257 **3. Results**

258 *3.1. Iterative smoothing of shot-to-shot phase variation*

259 Navigators were acquired with the acceleration rate as listed in Table 1.
260 Besides, the base resolution of navigators (e.g. 32 in Protocol #3 in Table 1)
261 was smaller than imaging echoes. As a result, reconstructed navigator phases
262 (refer to the first column in Fig. 3) from Step I in Section 2.5 are not spatially
263 smooth. Such phases, when used in the shot-combined reconstruction, result
264 in signal void artifacts in DW images. To address this problem, we utilized
265 the iterative smoothing procedure. As shown in Fig. 3, the ripple-like phase
266 artifact disappears after five iterations. It can also be seen that such an R249.Minor.11
267 iterative procedure retains the shot-to-shot phase variation.

– 12

268 *3.2. Comparison to MUSE and JULEP with four-shot iEPI acquisition*

269 The iterative phase smoothing was also applicable to MUSE-type self-
270 navigating reconstruction, where shot phases were reconstructed from imag-
271 ing echoes. Fig. 4 compares our proposed JETS with MUSE (Chen et al.,
272 2013), MUSE with complex-valued local-PCA denoiser (Cordero-Grande et al.,
273 2019), and JULEP (Dai et al., 2023). The residual noise from MUSE can be
274 largely removed by the denoiser. However, when compared to JETS, the de-
275 noiser shows residual noise patterns within the globus pallidus (indicated by
276 the red arrow). JETS also shows better denoising than JULEP. The reason
277 is that JETS enforces spatial-diffusion regularization, whereas JULEP for-
278 mulates structured low-rank regularization of the four shots for one diffusion
279 encoding.

Iterative smoothing of shot-to-shot phase variation

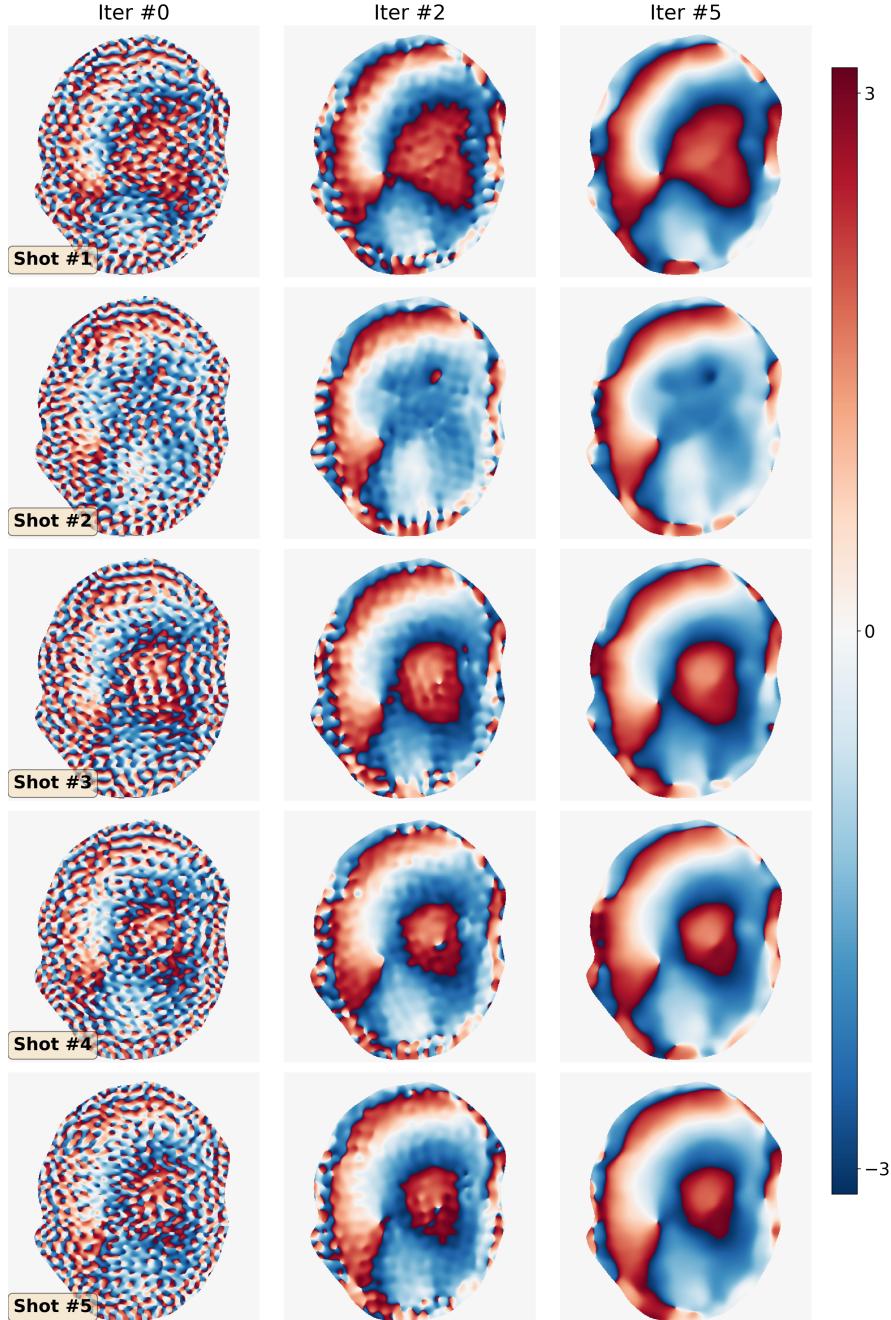


Figure 3: Iterative smoothing of shot-to-shot phase variation according to Eq. (6). Navigators from Protocol #3 were reconstructed based on Step I in Section 2.5 and then used as the input (iter #0, left column).

8th DW image from 4-shot iEPI @ 1 mm ISO

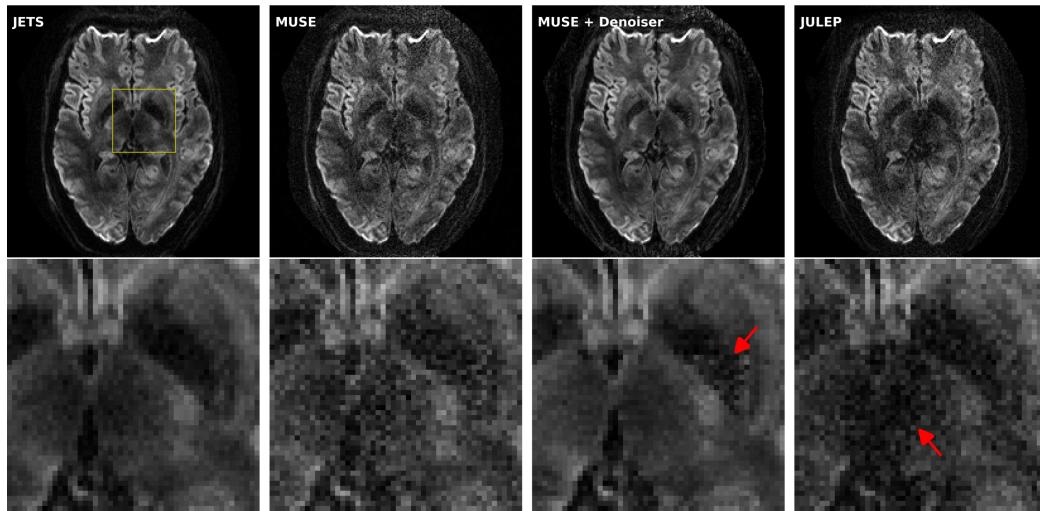


Figure 4: Reconstructed DW images (the 8th diffusion encoding) based on 4-shot iEPI acquisition with 1 mm isotropic resolution (Protocol #1 in Table 1). Four reconstruction methods are compared (from left to right): JETS, MUSE, MUSE with denoiser, and JULEP. The 2nd row displays the magnified views of the yellow square. The image from the denoiser (3rd column) shows residual noise patterns within the globus pallidus (indicated by the red arrow). The JULEP reconstruction (4th column) shows signal dropout in the central region (indicated by the red arrow).

R249.Minor.13

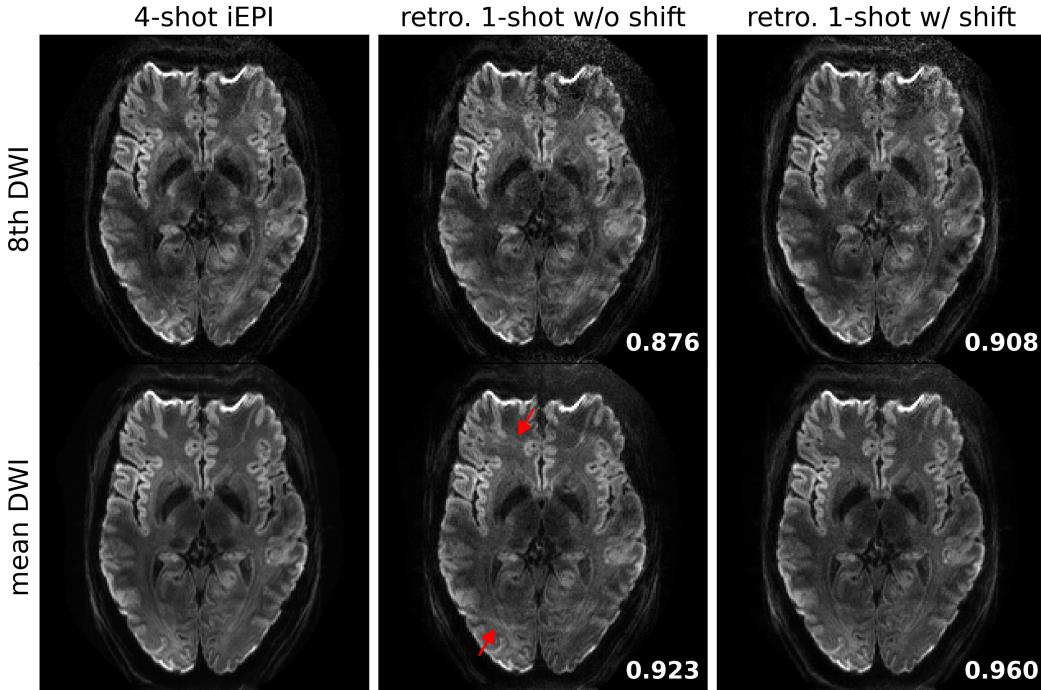


Figure 5: Quantitative validation of the proposed k_y -shift encoding sampling pattern based on 4-shot iEPI acquisition with 1 mm isotropic resolution (Protocol #1 in Table 1). (Top) the 8th diffusion encoding and (bottom) mean DWI over 20 diffusion encodings. (1st column) JETS reconstruction of 4-shot iEPI acquisition is used as the ground truth. The 2nd and the 3rd column displays JETS reconstruction of retrospectively undersampled 1-shot acquisition without and with k_y shifting, respectively. Residual aliasing artifacts are visible in the reconstruction without k_y shifting, as indicated by the red arrows. Structural similarity (SSIM) values are computed and displayed in the bottom right corners.

R249.Minor.14

- 16

280 3.3. Retrospectively undersampling from the four-shot iEPI acquisition

281 JETS reconstruction results on the four-shot prospectively fully-sampled
282 data from Protocol #1 in Table 1, as well as on the retrospectively under-
283 sampled one-shot data without and with the proposed k_y shift are displayed
284 in Fig. 5. Residual aliasing artifacts are visible in the reconstruction without
285 k_y shifting, as indicated by the red arrows. In contrast, the k_y shifting scheme
286 supplies a complementary k - q -space sampling pattern, which is beneficial for
287 joint reconstructions such as JETS. As shown in Fig. 5, JETS results in im-
288 proved SSIM values and reduced aliasing artifacts, when compared to the
289 reconstruction without k_y shifting.

R249.Minor.17,
18, 20

290 3.4. Analysis of reconstruction parameters

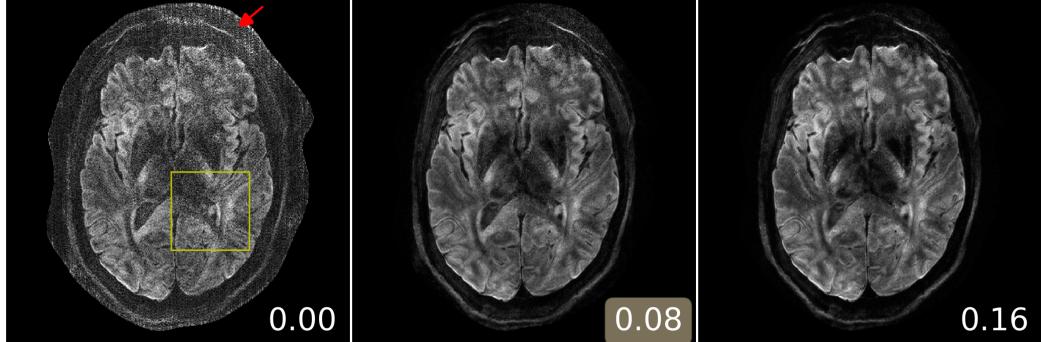
291 Here we provide a systematic analysis of the proposed JETS reconstruc-
292 tion with LLR regularization applied to the spatial-diffusion dimension, as
293 shown in Fig. 6.

294 First, we varied the regularization strength λ . We tested values of 0, 0.08,
295 and 0.16. The reconstruction with $\lambda = 0$ in Eq. (7) corresponds to parallel
296 imaging reconstruction without LLR regularization. It is worth noting that
297 the proposed NAViEPI sequence demonstrates high-quality sub-millimeter
298 DW images ($0.5 \times 0.5 \times 2.0$ mm³ in this example). The DW images can be
299 further improved with the use of LLR regularization, i.e., reduced noise, as
300 seen in the reconstruction with $\lambda = 0.08$. Increasing λ (e.g. 0.16) further
301 reduces noise, but at the cost of increased blurring. Therefore, $\lambda = 0.08$ was
302 selected in this work.

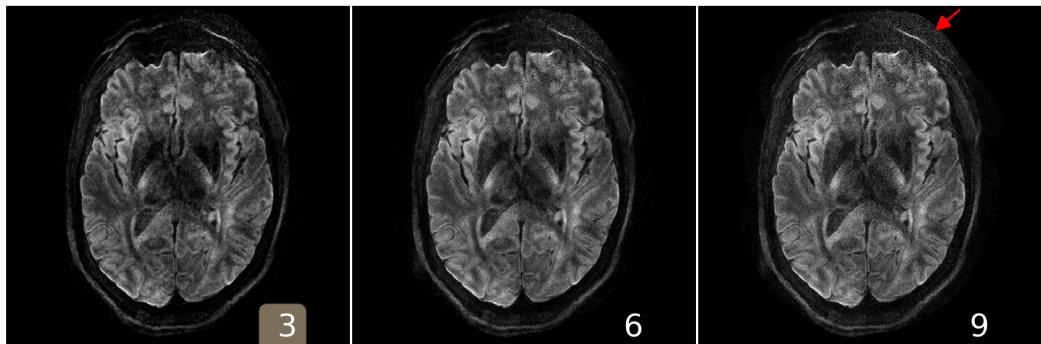
R249.Minor.19

303 Second, besides the regularization strength, the block size (i.e., the area
304 of 2D patches) also plays a role in denoising. We employed square blocks in

(A) varying λ , keeping block as 6 and stride as 1



(B) varying block size, keeping λ as 0.08 and stride as 1



(C) varying stride, keeping λ as 0.08 and block as 6

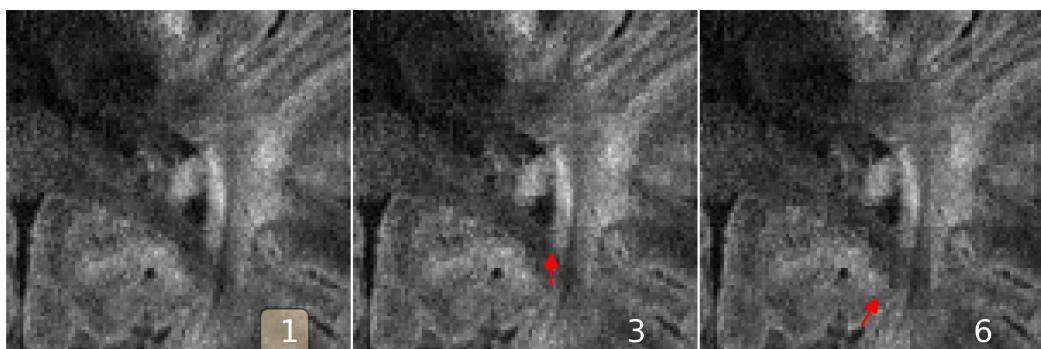


Figure 6: Analysis of reconstruction parameters based on the 3-scan trace acquisition with $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ (Protocol #3 in Table 1). Displayed are JETS reconstructed single-direction DW images. **(A)** Varying the regularization strength λ from 0 to 0.08 and 0.16. **(B)** Varying the block size from 3 to 6 and 9. **(C)** Varying the stride size from 1 to 3 and 6 (non-overlapping).

305 this work. Here, the block width of 2 shows the best denoising as compared
306 to 1 and 3, especially in the peripheral brain region. Among the three tested
307 block widths, the block size of 4 (with the block width 2) is the smallest one
308 which is no smaller than the diffusion directions in this 3-scan trace example
309 ($1 b_0$ plus 3 orthogonal diffusion directions). This observation agrees with
310 the suggestion that the patch size should be no smaller than and close to the
311 diffusion directions (Cordero-Grande et al., 2019).

312 Third, we varied the stride, i.e., the step from one local patch to the
313 next. The use of overlapping LLR (Fig. 6 (C) left) better suppresses blocky
314 artifacts, compared to the partially overlapping LLR (Fig. 6 (C) middle) and
315 the non-overlapping LLR (Fig. 6 (C) right).

R248.Major.4b

316 3.5. Sampling efficiency of NAViEPI

317 As shown in Fig. 7, NAViEPI achieves sub-millimeter resolution (voxel
318 size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$) with the use of a 5-shot acquisition. When compared
319 to a single-shot acquisition with the same voxel size, the acquisition time of
320 NAViEPI is about two times longer, but the image quality of NAViEPI is
321 remarkably improved.

R249.Minor.2¶

322 In the sub-millimeter imaging scenario, the increased base resolution re-
323 quires longer TE (143 ms) in the single-shot acquisition, which results in
324 significant signal loss due to T_2 relaxation. Therefore, sub-millimeter DWI
325 necessitates multi-shot acquisition, which is subject to shot-to-shot phase
326 variation and long scan time. However, NAViEPI solves both challenges. The
327 5-shot acquisition reduces TE to 58 ms, and thus retains SNR significantly
328 compared to the single-shot acquisition. Moreover, the JETS reconstruction
329 can help to reduce noise and improve structural visibility.

3-scan trace acquisition with voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$

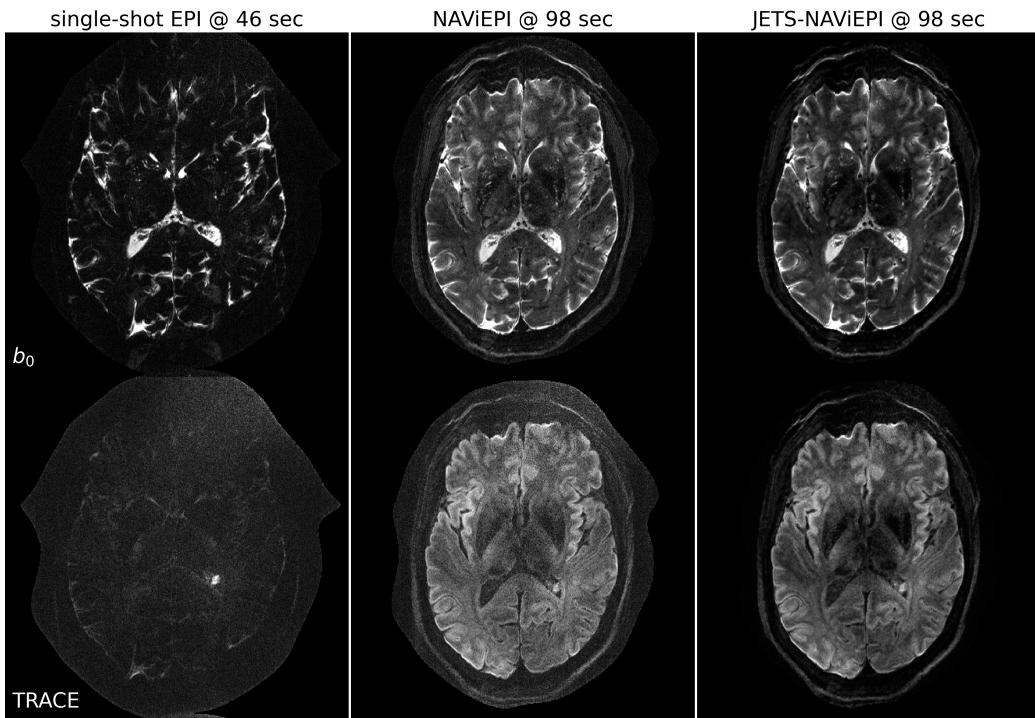


Figure 7: Sampling efficiency of the proposed NAViEPI sequence. 5-shot NAViEPI acquisition with the voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ (Protocol #3) was compared with single-shot EPI acquisition (Protocol #4). Both the 1st and the 2nd columns were reconstructed via parallel imaging without LLR regularization, whereas the 3rd column was reconstructed via JETS.

3-scan trace acquisition with voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$

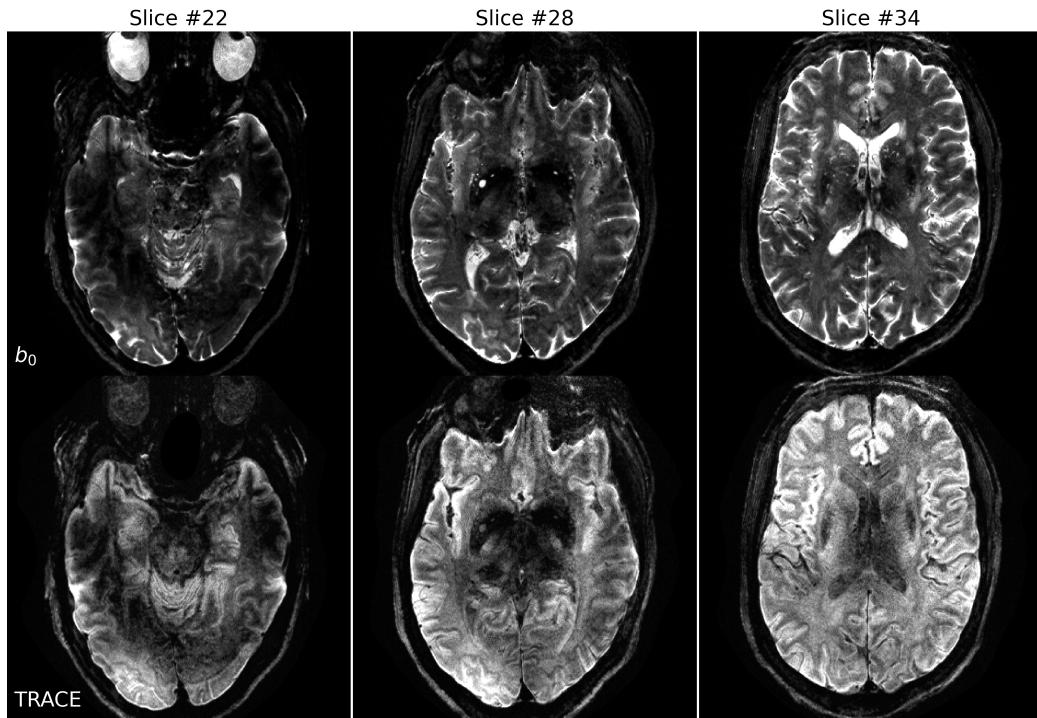


Figure 8: Reconstruction of the 3-scan trace acquisition with the voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ (Protocol #3) at different slices.

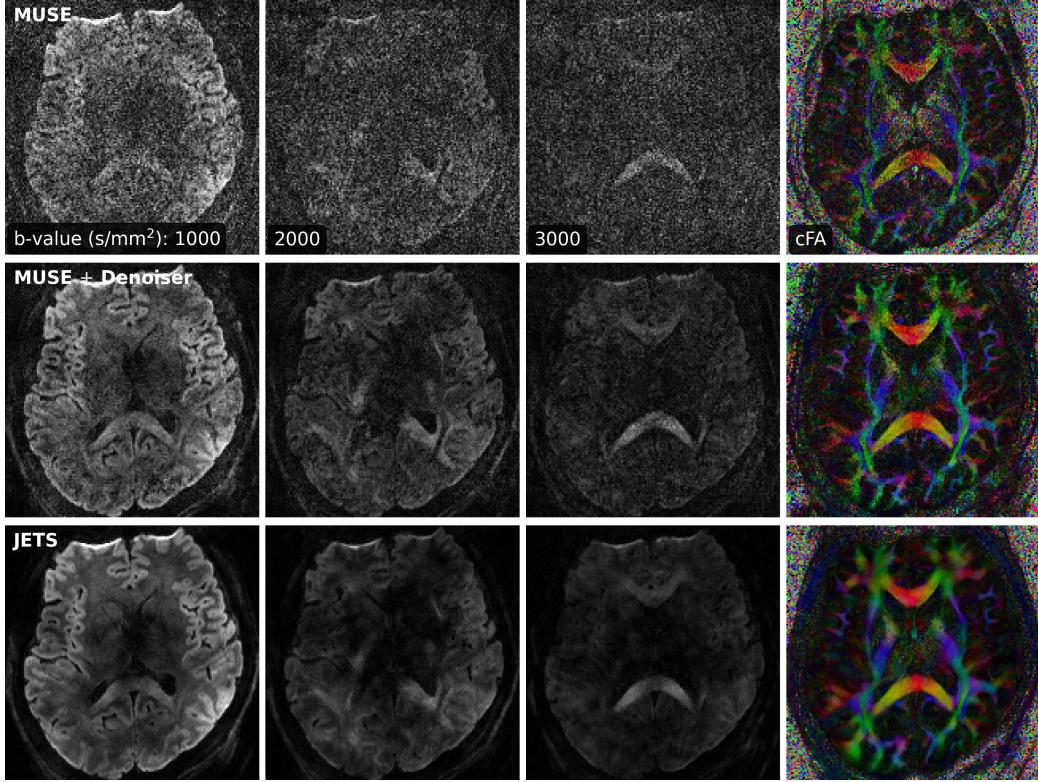


Figure 9: Comparison of three-shell DWIs and cFA maps with data acquired by Protocol #2 in Table 1. Reconstruction methods from top to bottom were MUSE, MUSE with the local-PCA denoiser, and the proposed JETS method.

Fig. 8 shows the JETS reconstructed b_0 and TRACE images in different slice locations. Admittedly, the lower brain region (e.g. slice #22) exhibits inhomogeneous and lower signal intensity than the upper slices. Such inhomogeneity can be alleviated with the use of multi-channel parallel transmission (Katscher et al., 2003; Grissom et al., 2010).

335 *3.6. Diffusion tensor imaging*

336 Protocol #2 in Table 1 yields an acceleration factor of 6×3 per shot, re-
337 sulting in severe noise amplification in MUSE reconstructed DWIs, as shown
338 in Fig. 9. The local-PCA denoiser substantially removes noise, but the DWI
339 at high b -values still illustrates more noise, compared to the proposed JETS
340 reconstruction.

341 4. Discussion

342 This work reports a novel DW-MRI technique, JETS-NAViEPI. NAViEPI
343 (1) achieves the fast and efficient acquisition of both imaging and navigator
344 echoes, (2) enforces consistent effective ESP between the two echoes, and (3)
345 allows for undersampled iEPI as well as a large number of shots. Moreover,
346 compared to the single-shot acquisition, joint k - q -slice reconstruction with k_y -
347 shift encoding on NAViEPI retains SNR and reduces aliasing artifacts in DW
348 images. As a result, JETS-NAViEPI renders high spatiotemporal resolution
349 diffusion MRI protocols in 7 T, e.g., a 3-scan trace acquisition with the voxel
350 size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$ at 1.5 min.

R249.Minor.24

351 One limitation of JETS-NAViEPI is the long reconstruction time due to
352 the simultaneous reconstruction of all DW images and the use of overlapping
353 locally low-rank regularization. The reconstruction for the Protocol #3 in
354 Table 1 on an A100 GPU takes about 2 min per multi-band slice. To reduce
355 the computation time, coil compression algorithms (Buehrer et al., 2007;
356 Huang et al., 2008) can be employed to reduce the number of coils for image
357 reconstruction. Moreover, one can deploy multi-GPU distributed computing
358 or modern optimization algorithms (e.g. stochastic gradient descent) (Ong
359 et al., 2020) to speed up the reconstruction.

360 Neither the signal modeling in Eqs. (2) and (4) nor the LLR regularization
361 considers the subject motion. In the presence of motion, the regularized
362 reconstruction can degrade. To overcome this problem, scout-informed motion
363 estimation and reconstruction (Polak et al., 2022) could be integrated
364 into the framework.

365 Another potential extension of this work is to incorporate distortion cor-

366 rection. The standard distortion correction method is known as TOPUP
367 (Andersson et al., 2003), which acquires two scans with opposing phase-
368 encoding directions to obtain the field inhomogeneity map and then performs
369 conjugate phase reconstruction to correct for distortion. Alternatively, the
370 multi-echo acquisition could be used for the coil sensitivity reference scan,
371 such that both coil sensitivity and B_0 field inhomogeneity maps could be
372 reconstructed from the data.

R249.Minor.25

373 This work employed a single regularization weight λ to enforce low rank-
374 ness along the spatial-diffusion direction. However, SNR may be heteroge-
375 neous within the FOV. Therefore, one single regularization scalar may be
376 inadequate to cover the whole FOV. Beyond this SVT-based reconstruction,
377 one can seek to use machine learning to learn a q -space prior as the regularizer
378 (Hammernik et al., 2018; Lam et al., 2019; Mani et al., 2021).

R249.Minor.26

379 Although NAViEPI employs navigators for the acquisition of shot-to-
380 shot phase variation, it is worth noting that phase behavior depends on
381 several hard-to-control factors such as pulsatile motion, bulk motion, loca-
382 tions within the brain, and diffusion sensitization strength. Therefore, more
383 comprehensive modeling or post-processing such as image registration can
384 be considered in future work.

385 This work compared LLR regularized JETS to MUSE post-processed by
386 the local PCA denoiser (Cordero-Grande et al., 2019). Technically, the LLR
387 regularization is realized by soft thresholding of the singular values of the
388 spatial-diffusion matrices, whereas the denoiser performs hard thresholding.
389 Both approaches demonstrate effective noise removal. In the scenario of ac-
390 celerated acquisitions, one can employ both approaches to maximally boost

R248.3b

391 SNR, i.e., the use of LLR regularization for image reconstruction followed by
392 the denoiser as a post-processing step.

393 While this work reconstructs all DW images and then performs model
394 fitting, an alternative approach is to directly estimate b_0 and diffusion ten-
395 sors from measured k - q -space data using model-based reconstruction (Knoll
396 et al., 2015; Dong et al., 2018; Shafieizargar et al., 2023). Compared to DW
397 image reconstruction, model-based reconstruction solves for a fewer number
398 of unknowns, but requires strict diffusion tensor modeling and the use of
399 nonlinear least square solvers.

400 **5. Conclusions**

401 We demonstrated the JETS-NAViEPI technique, which integrates a k_y -
402 shifted encoding interleaved EPI sequence and a joint reconstruction with
403 overlapping locally low-rank regularization for high spatial-angular-temporal
404 resolution DW-MRI at 7 T. This technique allows for high-quality DW image
405 reconstruction with accelerated acquisitions.

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412 **Data and code available statement**

413 In the spirit of reproducible and open science, we publish our source
414 code (<https://github.com/ZhengguoTan/sigpy>) as well as the raw k -space
415 data (<https://doi.org/10.5281/zenodo.7548595>). We also provide inter- R248.8
416 active demonstrations of the reconstruction procedure (https://github.com/ZhengguoTan/demo_jets_diffusion_mri_7t).
417

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431
432

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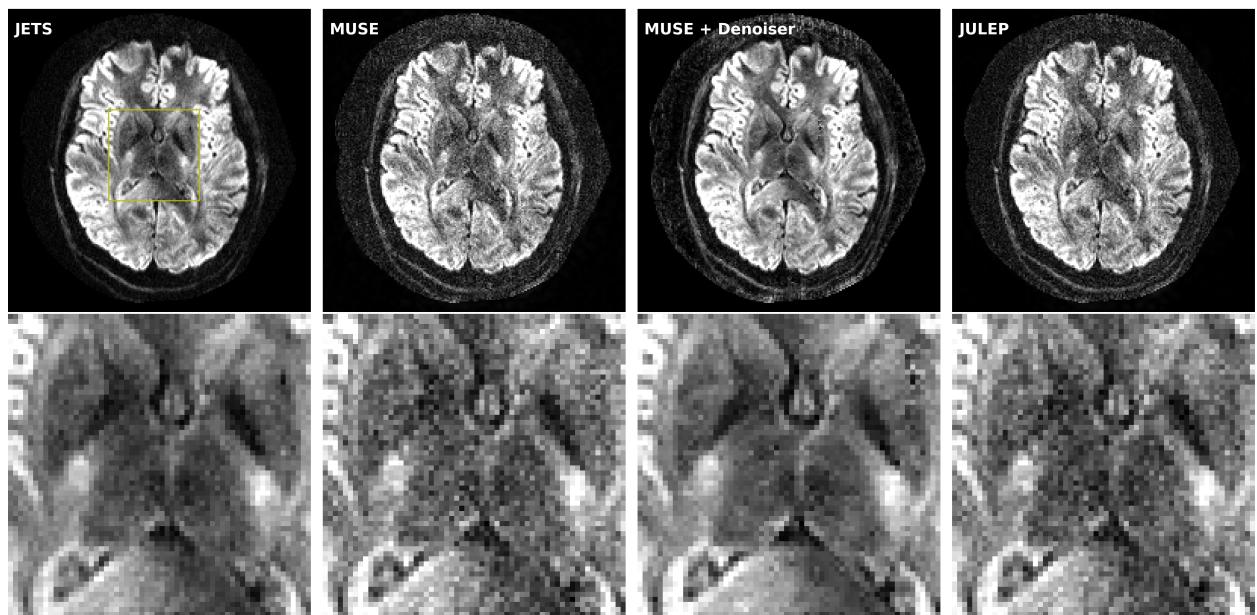
Supplementary Information

Accelerated Diffusion Weighted Magnetic Resonance Imaging at 7 T: Joint Reconstruction for Shift-Encoded Navigator-based Interleaved Echo Planar Imaging (JETS-NAViEPI)

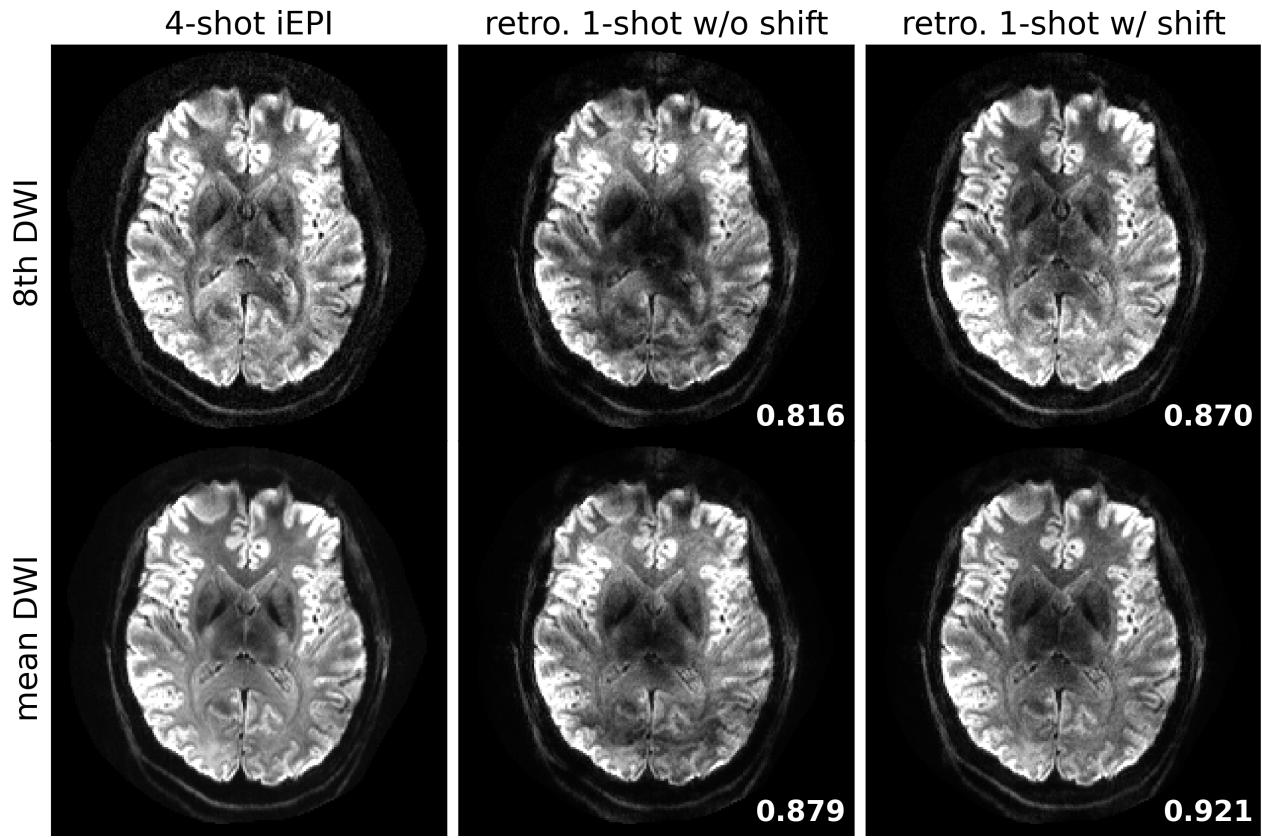
Zhengguo Tan, Patrick A. Liebig, Robin M. Heidemann, Frederik B. Laun, Florian Knoll

Here we aim to reproduce the results. Another subject was recruited and measured by all protocols listed in Table 1 in the main manuscript.

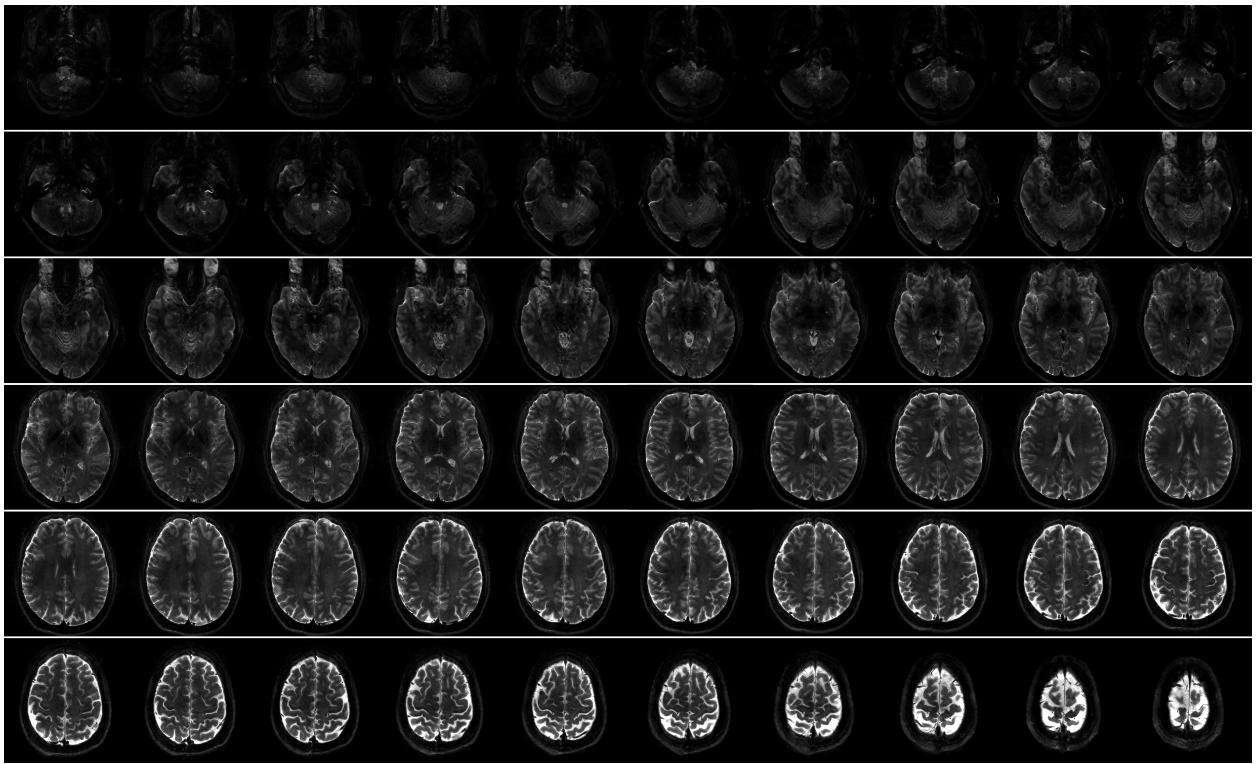
8th DW image from 4-shot iEPI @ 1 mm ISO



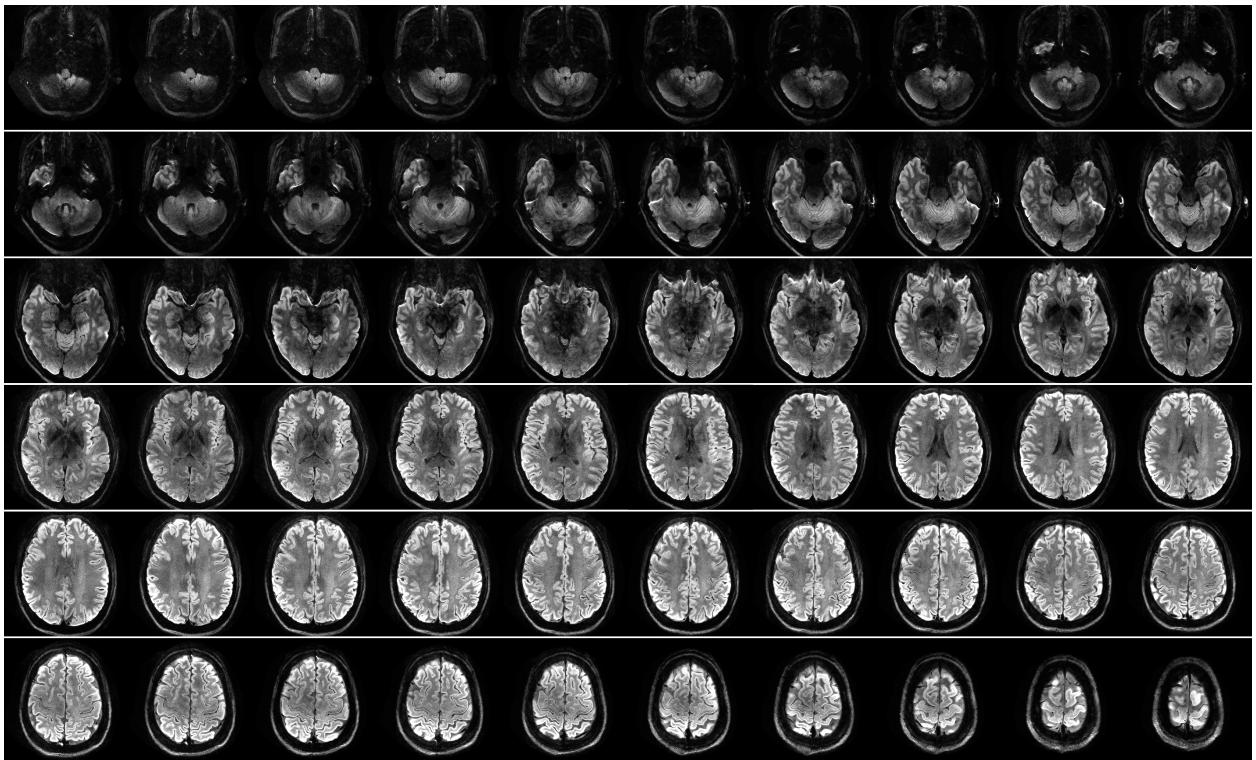
SI Figure S1: Reproducing Protocol #1. Reconstructed DW images (the 8th diffusion encoding) based on 4-shot iEPI acquisition with 1 mm isotropic resolution. Four reconstruction methods are compared (from left to right): JETS, MUSE, MUSE with denoiser, and JULEP. The 2nd row displays the magnified views of the yellow square.



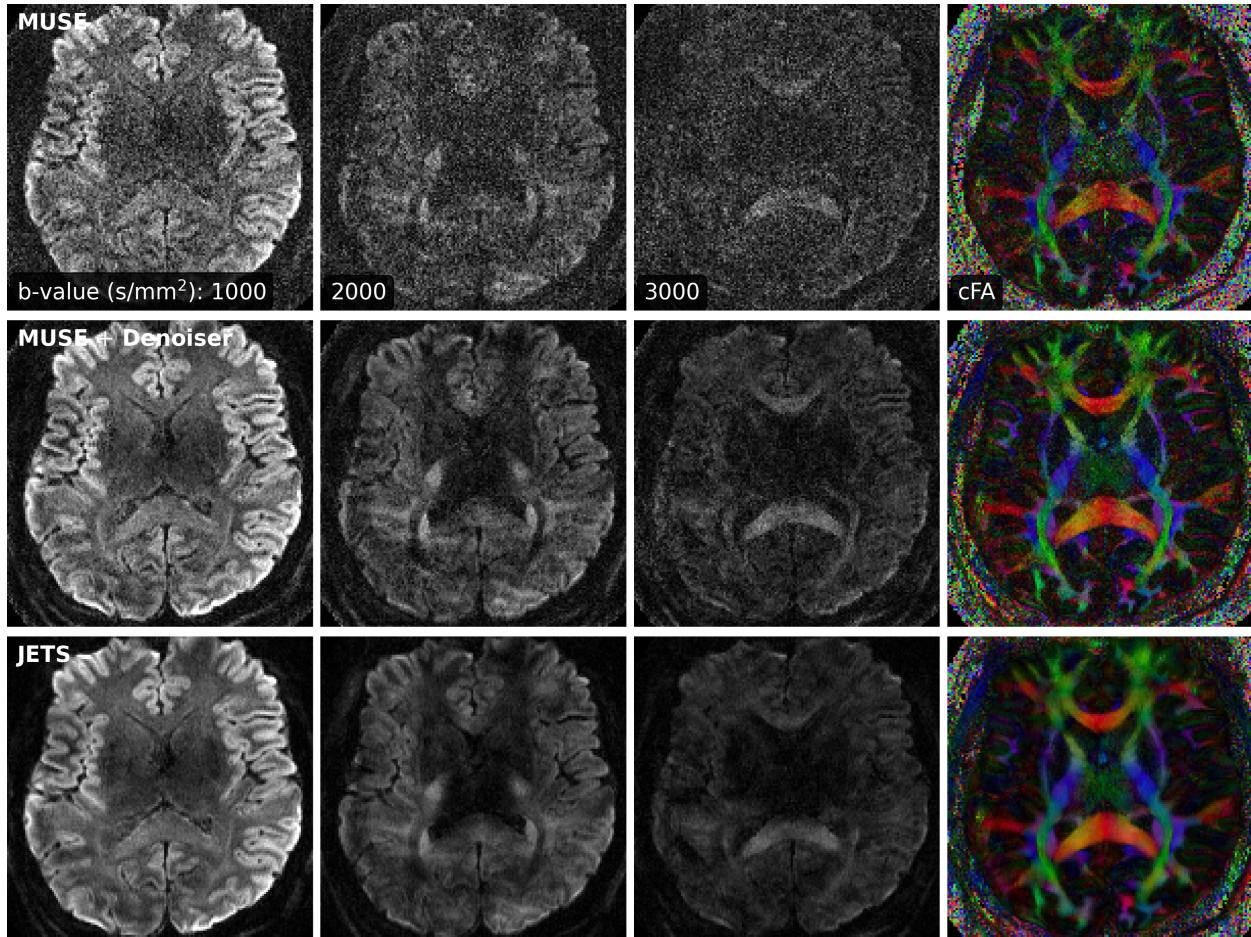
SI Figure S2: Reproducing Protocol #1. Quantitative validation of the proposed k_y -shift encoding sampling pattern based on 4-shot iEPI acquisition with 1 mm isotropic resolution. (Top) the 8th diffusion encoding and (bottom) mean DWI over 20 diffusion encodings. (1st column) JETS reconstruction of 4-shot iEPI acquisition is used as the ground truth. The 2nd and the 3rd column displays JETS reconstruction of retrospectively undersampled 1-shot acquisition without and with k_y shifting, respectively.



SI Figure S3: Reproducing Protocol #3. Reconstructed b_0 images from the 3-scan trace acquisition with the voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$.



SI Figure S4: Reproducing Protocol #3. Reconstructed TRACE images from the 3-scan trace acquisition with the voxel size $0.5 \times 0.5 \times 2.0 \text{ mm}^3$.



SI Figure S5: Reproducing Protocol #2. The FOV and bandwidth were adapted as 200 mm and 1086 Hz/pixel, respectively. Comparison of three-shell DWIs and cFA maps reconstructed by (top to bottom) MUSE, MUSE with the local-PCA denoiser, and the proposed JETS method, respectively.

Imaging Neuroscience #203: Responses to Editors and Reviewers

Reviewer #248

Authors have partially addressed my previous comments. A list of pending issues is listed below:

Major:

- 1) *The experiment is generally satisfactory, but its description is a bit confusing (especially point b) below:*

a) L125: "Protocol #1 with six-shot iEPI and without in-plane undersampling was implemented" then (L131) "We then retrospectively reduced the four-shot data to only one shot", guess there is a misprint in L125 which should be four-shot rather than six-shot?

b) L132: "with the proposed k_y shifting to simulate three-fold in-plane undersampling" → as there is no in-plane undersampling in Protocol #1 in the baseline data (according to Table 1) this sounds a bit confusing, if 1 out of 4 shots are used this would be four-fold in-plane undersampling, so unclear how you reach three-fold in-plane undersampling? Guess you mean something like "three-fold in-plane interleaving"? A few more details may help here.

c) L133: "JETS reconstruction was performed on all data" → "all data" is a bit vague, please reword.

a) Sorry about the misprint. We corrected it as four-shot.

b) Thank you for the question. You are right that "1 out of 4 shots are used", so it is four-fold in-plane undersampling. We corrected this in the manuscript.

c) Thank you. This sentence has been reworded.

- 2) *Fine, only a small detail, L61: "acquisition. SPA-LLR" → "acquisition and SPA-LLR"*

Done.

- 3) *There are some pending issues:*

a) *Authors could drop a line in text indicating why they use JULEPS rather than MUSSELS.*

b) *Results of MUSE + Denoiser look stronger than with previous denoiser. Indeed, now they are quite comparable to JETS (bit noisier but also bit sharper). A question is whether complex denoiser is applied before (replacing TV) or after shot-combination of MUSE? May make sense to try both approaches. Also, authors may want to draw a few lines about potential synergies between their LLR and the denoiser.*

c) *In previous version you also included comparisons for FA and different b-values but these seem missing here. I think that maintaining comparisons at different b-vals could be particularly useful.*

- a) Done.
- b) The complex denoiser is applied after the shot combination in MUSE. The local-PCA denoiser operates on spatial-diffusion matrices, whereas MUSE reconstructs one DWI at a time. Therefore, the denoiser is only applicable after the reconstruction of all DWIs, and is not possible before the completion of MUSE (e.g. before shot combination). We added "potential synergies between their LLR and the denoiser" into Discussion.
- c) We added back the three-shell data.

4) *Description of these experiments is misleading at times:*

- a) L300: "*According to (Cordero-Grande et al., 2019), it is suggested to keep the patch size roughly equal to the diffusion encoding length. In this 3-scan trace acquisition example, the diffusion encoding length is 4 (1 b0 plus 3 orthogonal diffusion directions). Among the tested block sizes, 3 is the closest to 4, and hence renders better denoising*" → *but in (Cordero-Grande et al., 2019), patch sizes are automatically estimated (have tried to find suggestion reported without success); whilst patch sizes inducing close to square matrices could be intuitively better for large matrices, that's not so clear for small matrices as here (4 columns); however, even in this case, patch sizes would be width*height, but here they are used interchangeably with widths. All of this needs detailed clarification or another thought.*
 - b) L311: "*which is prone to checkerboard artifacts even with the use of random shifting (Saucedo et al., 2017) in each ADMM iteration*" → *please review, as message of (Saucedo et al., 2017) seems different: "This result shows the robustness of the LLR-IRPA strategy, which maintains improved computational efficiency compared to CLEAR without introducing block artifacts", i.e., their method shows similar levels of artifacts than overlapped approaches (they even report a reduction in some experiments)* / P24(L308): "*the increment from one local patch to the next*" → *perhaps replace increment by "step" or "distance" for clarity?*
- a) Please refer to your minor comment 9) for responses.
 - b) Thank you for the suggestion. We agreed that LLR-IRPA improves the reconstruction for non-overlapping blocks. In our ablation experiments, the use of overlapped LLR better suppresses blocky artifacts. We rewrote this part.

6) *Abstract looks better now, but you may add a one-sentence summary of experimental results. Also, "developed the k_y " → "developed a k_y ".*

Thank you. We added the summary of experimental results in Abstract.

7) *Still unclear how do you select the slice / diffusion encoding of snapshots provided in Figs. 4-9. Also, caption of Fig. 5, "retrospectively" → "retrospectively"*

We tried to select representative slices. Figures 4 and 5 show a slice containing the globus pallidus with strong T_2 -weighted contrast. Figure 6 selects a slice with a suspicious lesion (the circular bright spot) within the left ventricle. Figure 8 displays three representative slices: (left) the lower brain region which identifies the B_1^+ field inhomogeneity, (middle) the middle brain slice which shows susceptibility artifacts in the frontal region, and (right) the upper brain slice which shows the ventricle.

- 8) *Understood regarding raw data. However, regarding source code, you say "we publish our source code (<https://github.com/ZhengguoTan/sigpy>)" but I see no reference to this work in that link. You may clarify by better documenting the repo / providing entry point of specific methods?*

Thank you for the comment. All our source codes have been pushed here: <https://github.com/ZhengguoTan/sigpy>. We will document this repo once the manuscript is accepted.

For an entry point of the proposed methods, please refer to the interactive demo at https://github.com/ZhengguoTan/demo_jets_diffusion_mri_7t. This earned 2nd Place in the MR-Pub Competition organized by the Reproducible Research Study Group in ISMRM 2023.

Minor:

- 6) *This comment is not satisfactorily addressed:*

- a) *Sentence "firstly slides [...] matrices" (L202) unclear, requested change ("you should reword and explicitly mention what dimensions are concatenated in rows and columns of matrices") not attended.*
 - b) *Reasons for not using 3D patches are not convincing. Authors are referring to SMS as reason behind, but this pertains to acquisition whilst patches are defined on image space, so usage of 3D (for instance to exploit redundancies between adjacent slices) could in principle be possible.*
- a) Thank you for the suggestion. We explicitly defined the spatial and diffusion dimension in the manuscript.
 - b) SMS acquires spatially separated slices, which means the slices in the joint reconstruction are not spatially continuous. Therefore, 2D patches are used for joint reconstruction. We removed the statement in the manuscript to avoid misunderstanding. But we agreed that after the reconstruction of all slices, 3D-patch-based denoising can be used as a postprocessing step. We put this into Discussion. On the other hand, please refer to the figure in https://github.com/ZhengguoTan/demo_jets_diffusion_mri_7t for the illustration of the construction of the forward model in our proposed reconstruction.

- 7) *Please, use a symbol rather than "input" (L215) for clarity of this expression.*

Thank you for the suggestion. We replaced "input".

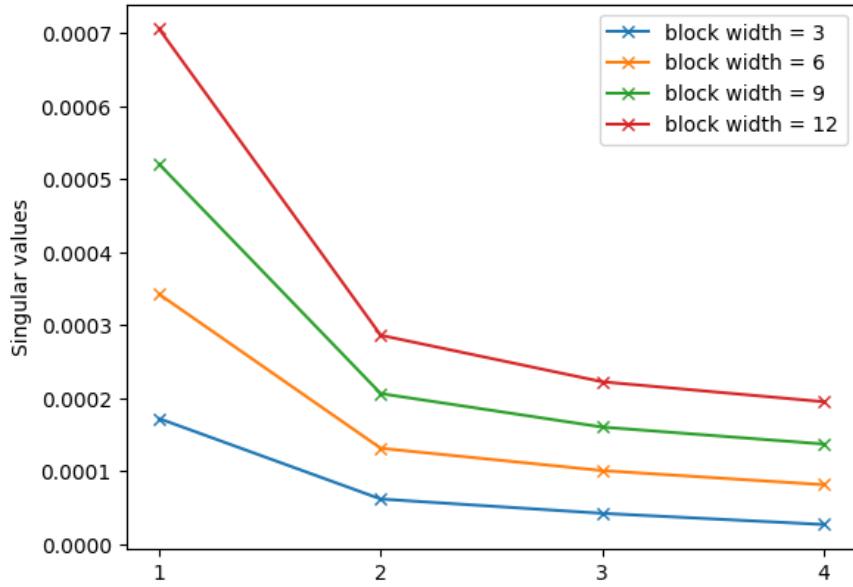


Figure 1: Mean singular values under different block widths. Since square blocks are used here, the block size is equal to the square of the width.

- 8) *This is only partly attended, when you say "efficient implementation" (L214, L217) → claim on efficiency does not seem supported from description... inverse density weighting is well-known for reconstructing original data levels back when slide-windowing, so this is just the standard method, not something particularly efficient.*

We removed the "efficiency" statement.

- 9) *This comment is not satisfactorily addressed. In new Fig. 6b we still see higher noise for larger block sizes, so my previous comment, "may appear counter-intuitive as small block sizes should aid with localization and therefore prevent blurring, at the price of less information for denoising? Can you clarify on reasons / potential hidden factors for this behaviour?" may still be relevant.*

Very good insight. This is connected to your major comment 4a) and we tried to attend them together.

Yes, you are right that patch sizes should be width × height. We corrected this in the manuscript.

The automatic estimation of the block size is described here (<https://mrtrix.readthedocs.io/en/dev/reference/commands/dwidenoise.html>), which suggests selecting "the smallest isotropic patch size that exceeds the number of DW images".

We provide another notebook for explanations (https://github.com/ZhengguoTan/demo_jets_diffusion_mri_7t/blob/main/demo_llr.ipynb). As shown in Figure 1, larger block sizes lead to larger singular values, and thus also result in higher noise when the singular value thresholding value is kept consistent for different block sizes. Therefore, different block sizes require different singular value thresholds.

11) Sorry, I don't see that this has been attended.

We added the row "slice thickness" in Table 1.

12) (New content) Reg. Eq. 6, L. "where the index k denotes the iteration" → this iteration was not in previous manuscript, unclear what it refers to.

It refers to the iteration step of the iterative phase smoothing procedure.

Reviewer #249

I was pleased to see that the Authors have put significant effort into this revision. However some major concerns I had regarding the description of the method and its potential reproducibility still remain.

Major

Number of subjects

I am happy that the Authors now state that they measured three subjects in the Methods section, but the presented data still only seem to show a single subject (e.g. in Fig. 9). Perhaps the Authors used a different subject for each separate experiment? Nevertheless, I would really like to see data from more than one subject for each experiment, even if this is put into an Appendix or Supplementary Information. Reproducing results in more than one subject is important to evaluate whether a method gives good results outside of a single (often "best") subject. If the main results do really each rely only on a single subject, I would strongly argue that each result should be reproduced in another subject before publication.

Thank you for your comment. Because of the measurement time constraint, we used a different subject for each separate experiment.

We recruited another subject to reproduce the results. Please refer to Supplementary Information for the results.

Checkerboard artefact suppression (pg. 15)

(1) *I thank the Authors for providing the code that let me understand what the T and T^H operators are doing. However I now think that expressing the operation of T and T^H as matrix multiplication is very misleading. I would urge the Authors to rethink how they express this part. Probably it is e.g. better to write $T(x)$ instead of Tx , unless the authors explicitly define what the multiplication means in this non-standard operation.*

Thank you for looking into the code. We changed to $T(x)$ instead.

(2) *Further, I think T^H needs to be better explained and justified. Where is it used? Are all the blocks reconstructed separately and then recombined for computational efficiency? Or is it just used when calculating the nuclear norm? It seems that T^H is notation from the python package used by the Authors, but I think it is not very useful notation for those unfamiliar with that package. If I followed everything correctly, then what is really needed is an inverse of T (i.e. T^{-1}), but this is non-unique for overlapping blocks (one could have any combination of weights to recombine the overlapping elements as long as they sum to one). The Authors decided to take equal weights for each repeated elements, i.e. to take the mean, and I agree that this is probably the best choice to avoid checkerboard artefacts. However it would be much clearer (at least to me) if the authors expressed this in words, making clear that they take the mean of the overlapping elements, rather than in terms of an operator " T^H " and "multiplication" operations which are not defined by the Authors. The Authors could then*

point explicitly to the scripts they have written for details.

First, the singular value thresholding (nuclear norm regularization) is calculated from the blocks, i.e., $T(x)$. Second, after the thresholding, T^H is used to recombine the blocks to obtain the updated DW images.

- (3) *If the operations really do need to be expressed in such detail, then it would perhaps be good to be explicit that the operation to get the "divisor" just gives the element-wise denominator for the mean.*

Thank you for the suggestion. We rewrote the description.

- (4) *The claim that this way of recombining blocks is efficient is not proven. I would suggest removing this claim.*

OK. We removed this statement.

Minor

My minor comments are mainly regarding the wording of some of the new text, which doesn't read as natural English to me. There are also some points which I didn't quite understand which the Authors could consider elaborating on further.

Sincerely thank you for your correction on the language. We have corrected all of them and highlighted the correction in the manuscript. Therefore, no response was provided in the following comments concerning language issues.

- (5) pg. 3: *Techniques on the correction of... ← Techniques for the correction of...*

- (6) pg. 11: *Different reconstruction methods, i.e., JETS, MUSE, and JULEP were compared. ← Different reconstruction methods, specifically JETS, MUSE, and JULEP, were compared.*

- (7) pg. 12: *A possibility is to perform magnitude average... ← One possibility is to perform magnitude averaging...*

- (8) pg. 13: *This can be incorporated to our formulation... ← This can be incorporated into our formulation...*

- (9) pg. 14: *Please define the nuclear norm in Eq. (7). This would probably also help alleviate some of the problems with the description of the checkerboard artefact suppression mentioned above.*

Thank you. We defined the nuclear norm.

- (10) pg. 15: *"This work employed blipped-CAIPI SMS (Setsompop et al., 2012), in which spatially separated slices are simultaneously excited and acquired. Therefore, 2D instead of 3D patches were used to construct the spatial-diffusion matrices." I don't understand how it follows from using multiband that 2D patches are used. Wouldn't this apply to all 2D sequences?*

Thank you for pointing this out. Here we reconstructed multiband slices jointly, but the multiband slices are spatially separated. This prohibits the use of 3D patches for the LLR regularization. We removed this part to avoid this confusion.

- (11) pg. 18: As show in Fig. 3... \leftarrow As shown in Fig. 3...
- (12) pg. 18: ...the ripple-like phase artifact disapper after five iterations. \leftarrow ...the ripple-like phase artifact disappears after five iterations.
- (13) Fig. 4: Please define what the arrows represent in the image caption.

Done.

- (14) pg. 21: retrospecitvely \leftarrow retrospectively
- (15) pg. 21: structural \leftarrow Structural
- (16) Fig. 5: Please define what the arrows represent in the image caption.

Done.

- (17) pg. 22: as pointed \leftarrow as indicated
- (18) pg. 22: On the contrary \leftarrow In contrast
- (19) pg. 22: First, we varied the regularization strength λ from 0, to 0.08, and to 0.16. \leftarrow First, we varied the regularization strength λ . We tested values of 0, 0.08, and 0.16.
- (20) pg. 22: "i.e., reduced noise" What does this mean? Perhaps you need to be more explicit?

Done.

- (21) pg. 24: ...with the use of 5-shot acquisition. \leftarrow ...with the use of a 5-shot acquisition.
- (22) pg. 24: When compared to the single-shot acquisition... \leftarrow When compared to a single-shot acquisition
- (23) Fig. 8: The TRACE image at the bottom right seems to show aliasing near the bottom. Do you have an explanation for this?

This may be caused by B_1 field inhomogeneity from the lower brain region.

- (24) pg. 29: ...MRI protocols in 7 T, e.g., 3-scan trace acquisition... \leftarrow ...MRI protocols at 7 T, e.g., a 3-scan trace acquisition...
- (25) pg. 30: "The standard distortion correction method is known as TOPUP (Andersson et al., 2003), which acquires two scans with opposing phase-encoding directions to obtain the field inhomogeneity map and then performs conjugate phase reconstruction to correct for distortion." The B_0 map does not need to come from TOPUP. Why not just collect a B_0 map from multiecho data and incorporate it into the reconstruction? Often the multiecho data used to create SENSE maps for coil combination is used for this purpose.

Thank you for the suggestion. We added this into Discussion.

- (26) pg. 30: "Beyond this SVT-based machine-learning reconstruction, one may seek to learn a q -space prior as the regularizer..." I don't understand what this sentence is trying to say. Is it a suggestion that machine learning could be used to learn a q -space prior?

Yes, it is a suggestion that machine learning could be used to learn a q -space prior.

Reviewer #250

All points of my critique have been addressed adequately.

Thank you for your review.