# Non-Cartesian sampling

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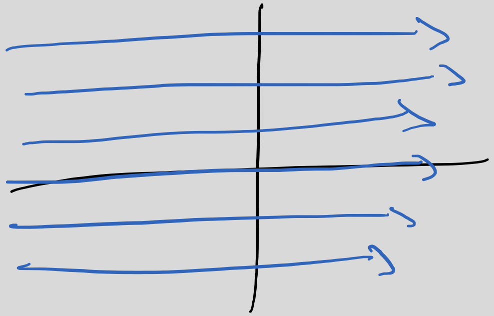
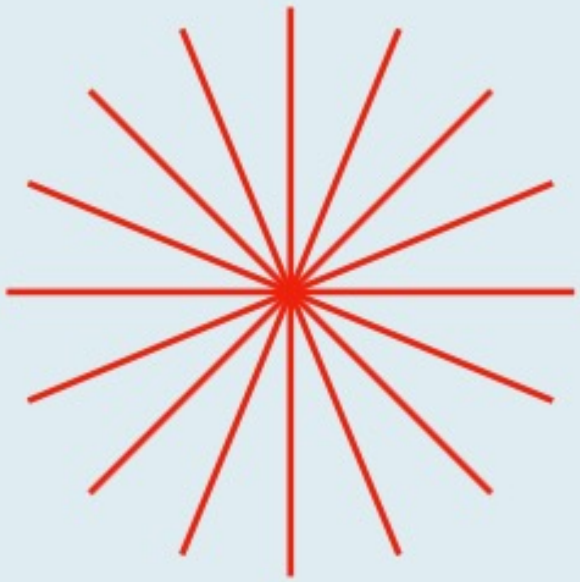
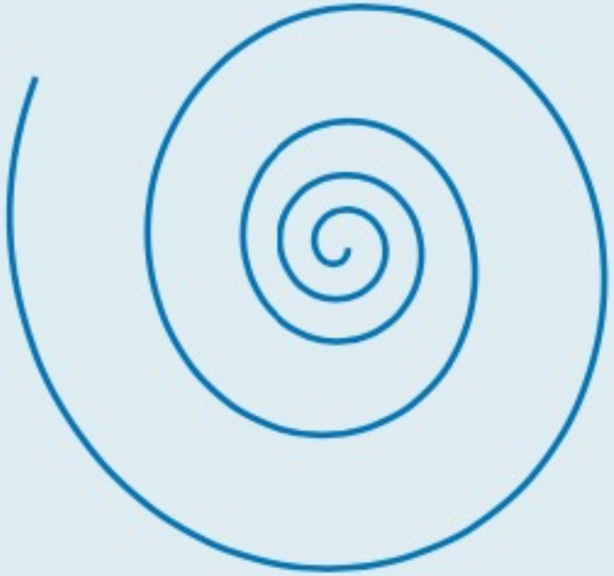


- Use gradients more efficiently to traverse k-space
- Acquire short-T2 (or T2*) components
- Can be designed to be silent
- Robust to motion by averaging in low frequency





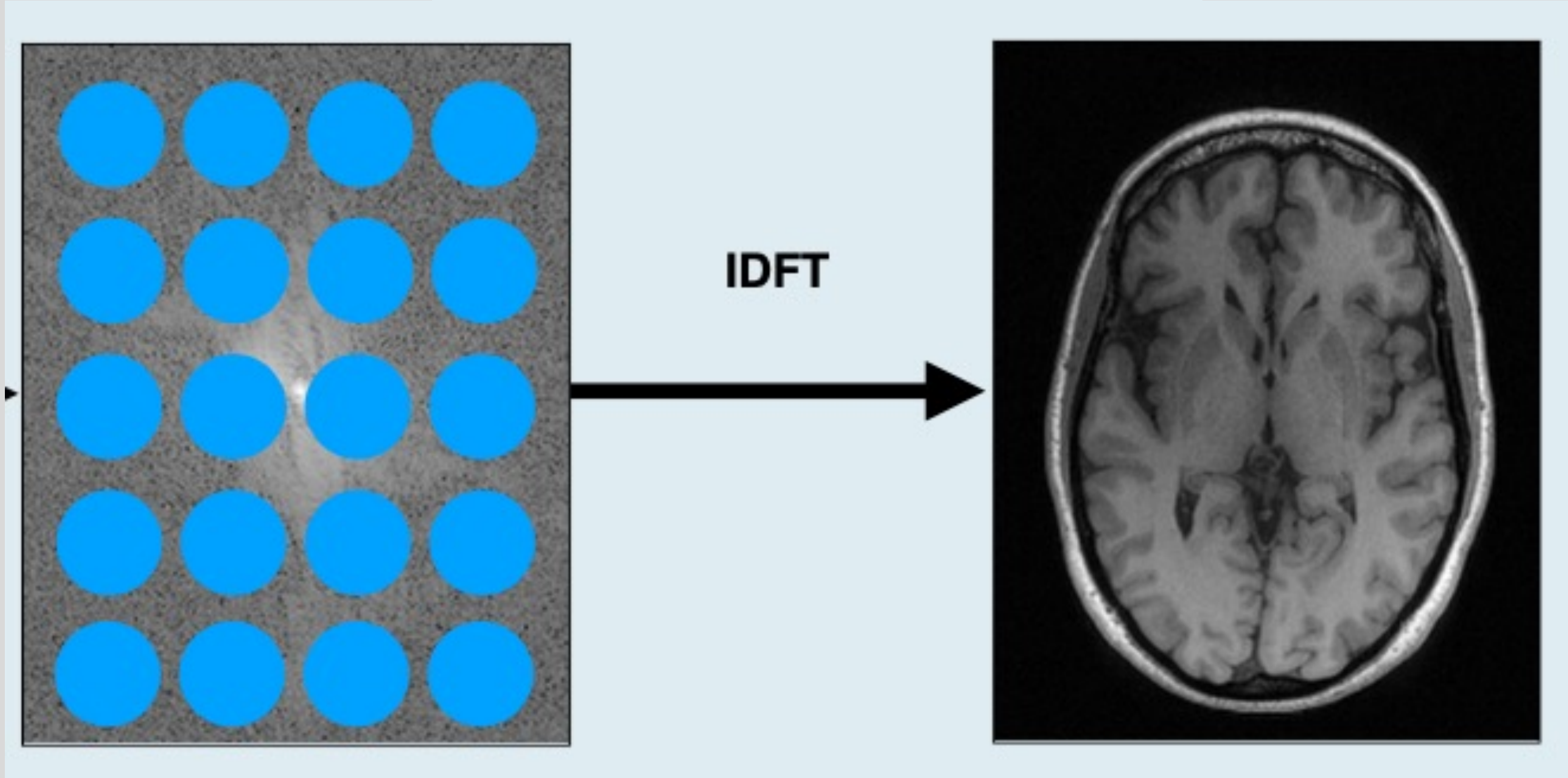
# Downsides of non-Cartesian sampling

- Reconstruction is more complicated
- Sensitive to gradient errors (delays, eddy currents)
- Sensitive to B0 errors

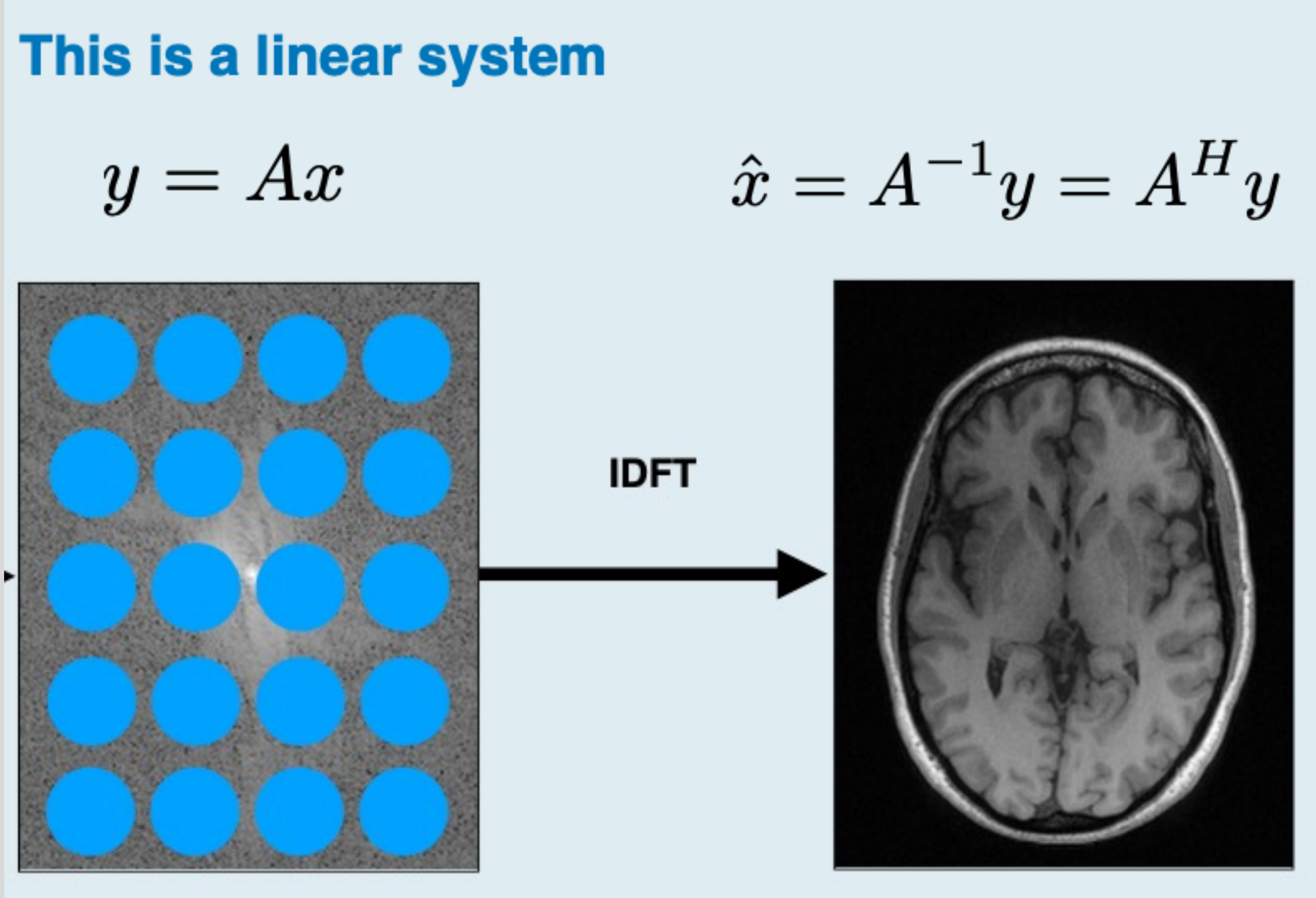
Good for PhD theses!

# Cartesian image reconstruction

$$\hat{x}[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y[k] e^{\imath 2\pi nk/N}$$



# Cartesian image reconstruction




FFT computes the inverse DFT in O(NlogN)instead of O($N^2$)

# Non-Cartesian image reconstruction

What is the same:

- Still a linear system, still interested in $A^{-1}$
- Exact solution given by inverse non-uniform DFT

What is different:

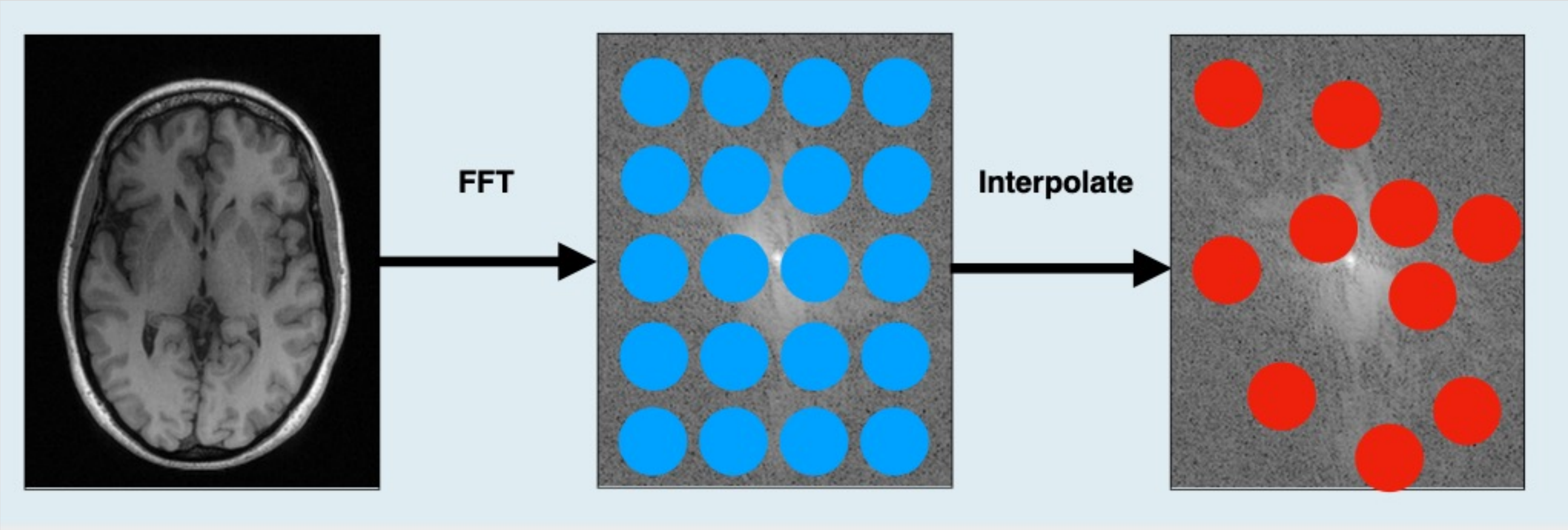
- No FFT for the inverse → $O(N^3)$ (direct inversion)
- No FFT for $Ax$ or $A^H y$ → $O(N^2)$ (iterative methods)

What can we do?

- Approximate it! → Non-uniform FFT (NUFFT)

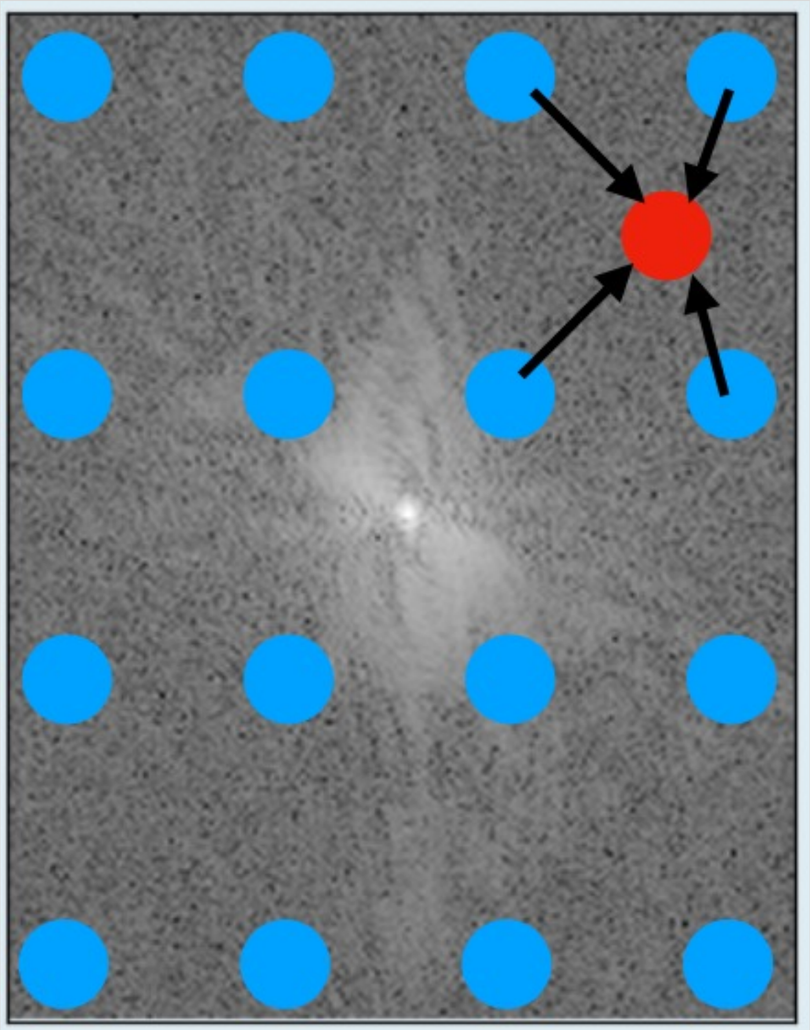
# Non-uniform Fast Fourier Transform (NUFFT)

- Idea: approximate the nuDFT with FFT followed by interpolation
- By Nyquist theorem, sinc interpolation is exact but takes $O(N^2)$ time
- **NUFFT:** Perform local interpolation with width << N (approximation)

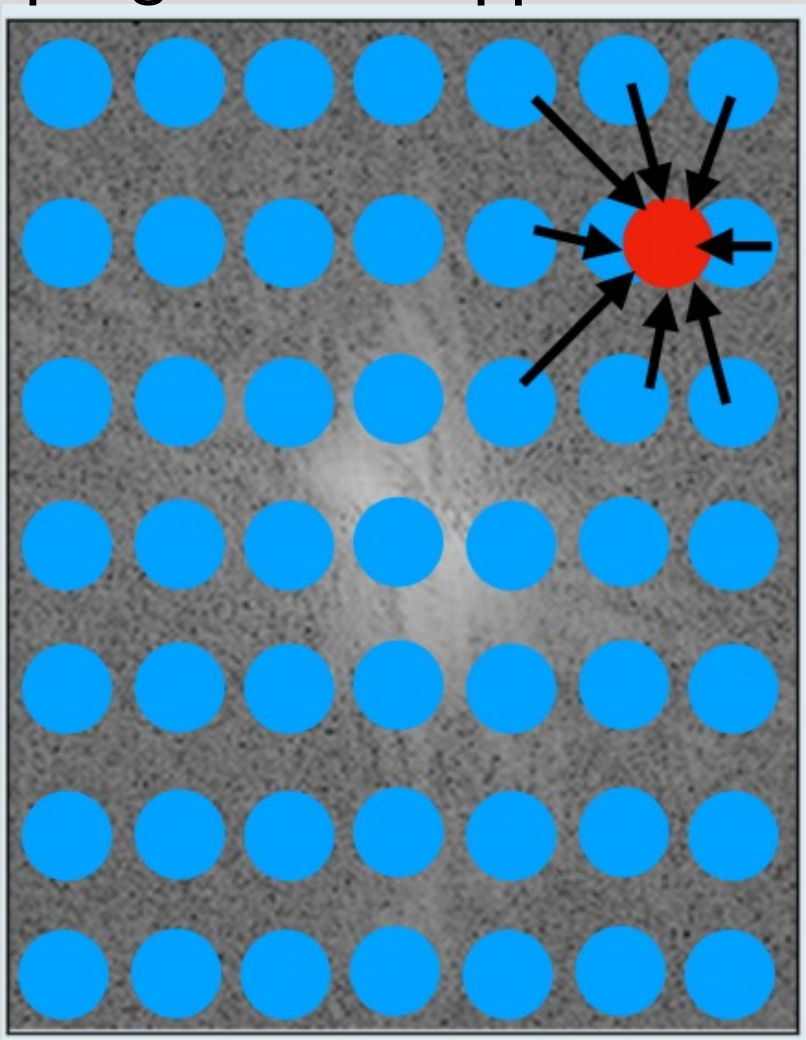


# Non-uniform Fast Fourier Transform (NUFFT)

Local interpolation

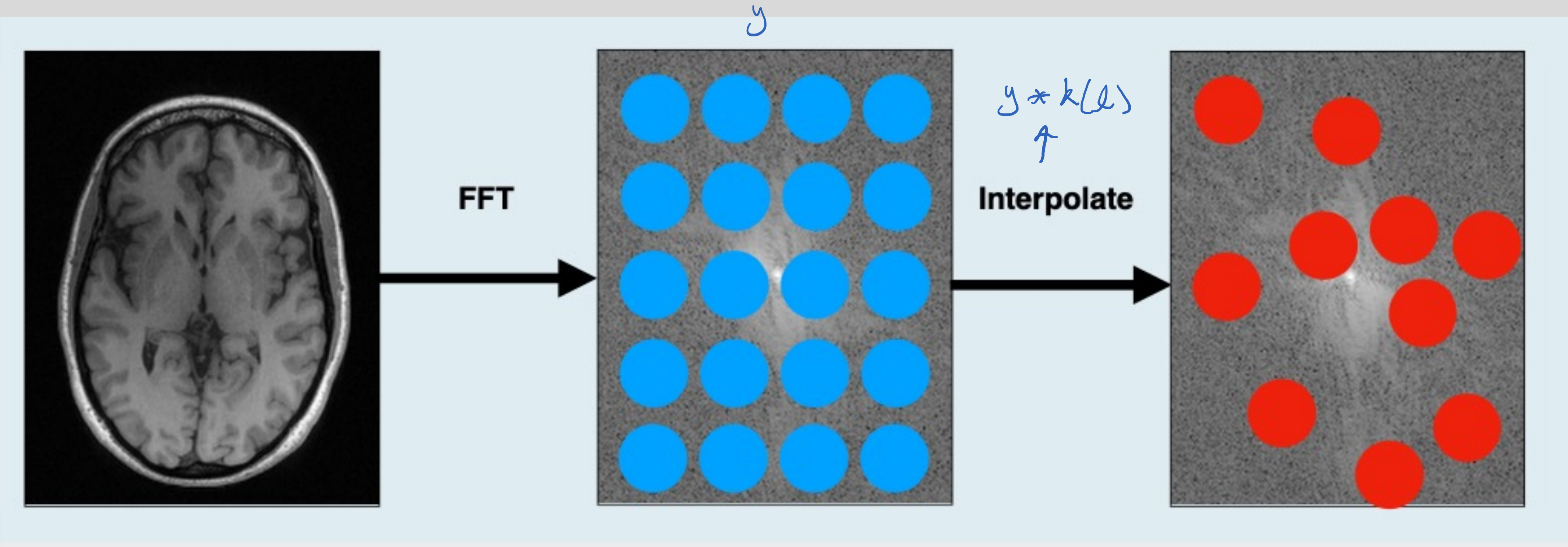


Over-sampling reduces approximation error



# Non-uniform Fast Fourier Transform (NUFFT)

- Interpolation in k-space creates image weighting (convolution property)

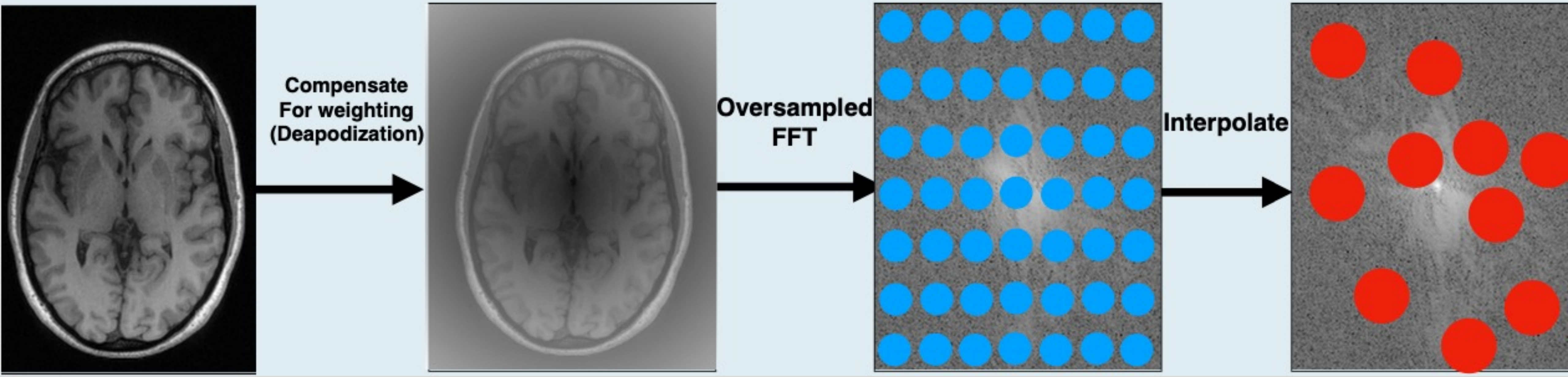


# Non-uniform Fast Fourier Transform (NUFFT)

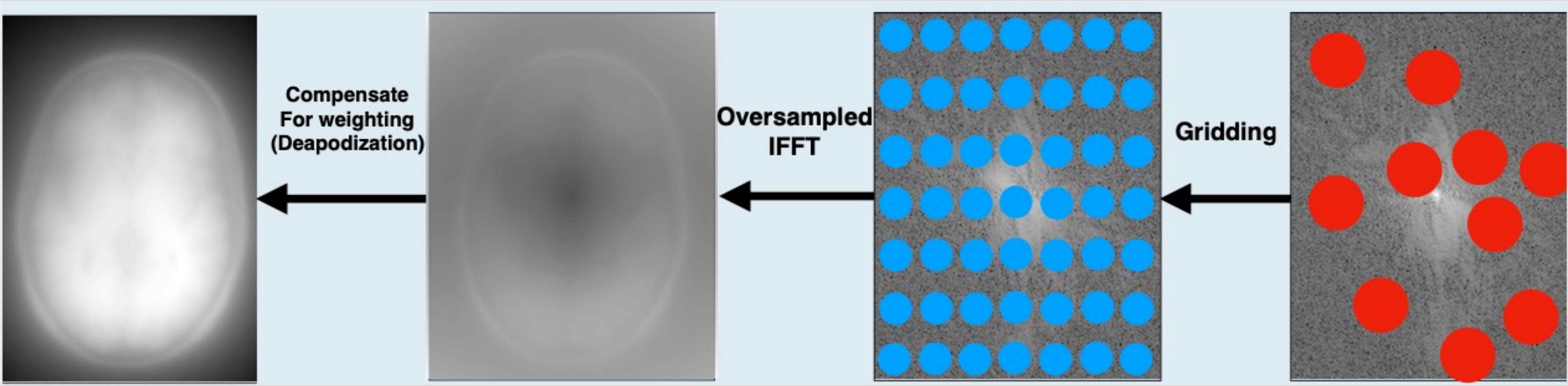
- Interpolation in k-space creates image weighting (convolution property)

$y = Ax$

$x$ $y$



# NUFFT Adjoint




$$A^{-1} \neq A^H$$

# Iterative reconstruction with NUFFT

- Need only A and $A^H$

$$\|Ax - y\|_2^2$$

$$x^{k+1} = x^k - \alpha A^H(Ax - y)$$

# Gridding and density compensation

- Called by the MRI community **Gridding Reconstruction**

$$A^{-1} \approx A^H D$$