Step-by-Step Reconstruction Using Learned Dictionaries

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Declaration of Financial Interests or Relationships

Speaker Name: Jonathan I Tamir

I have no financial interests or relationships to disclose with regard to the subject matter of this presentation.

Links

- Compressed sensing MRI overview:

  https://www.ismrm.org/19/program_files/WE22.htm

CS-MRI Talk

- Hands-on examples:

  https://github.com/utcsilab/dictionary_learning_ismrm_2020

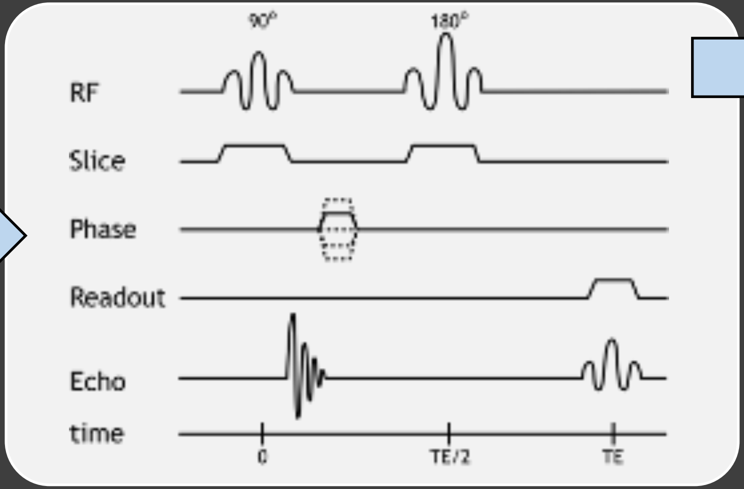
Hands-on code

F Ong et al., SigPy: https://sigpy.readthedocs.io/en/latest/

MRI Background

Patient in MRI scanner

Pulse sequence controls MRI signal



Measurements are collected

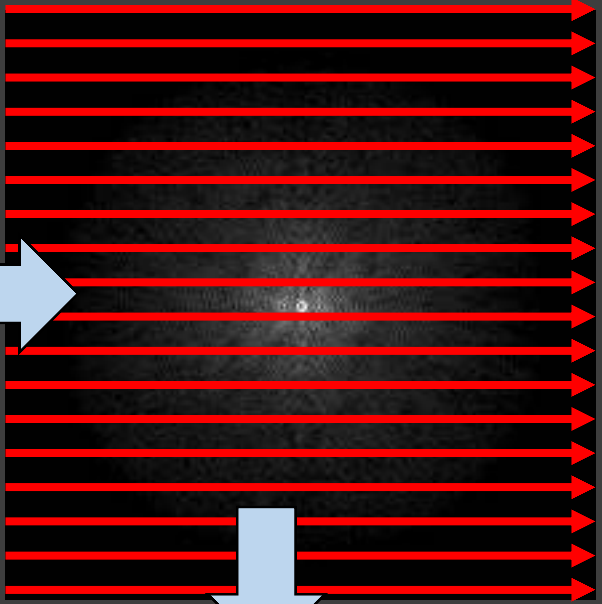
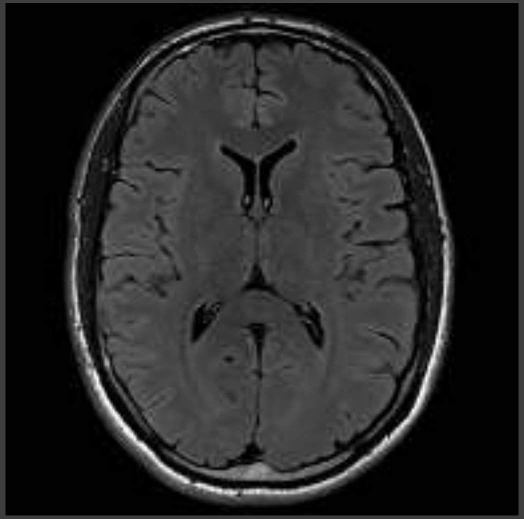


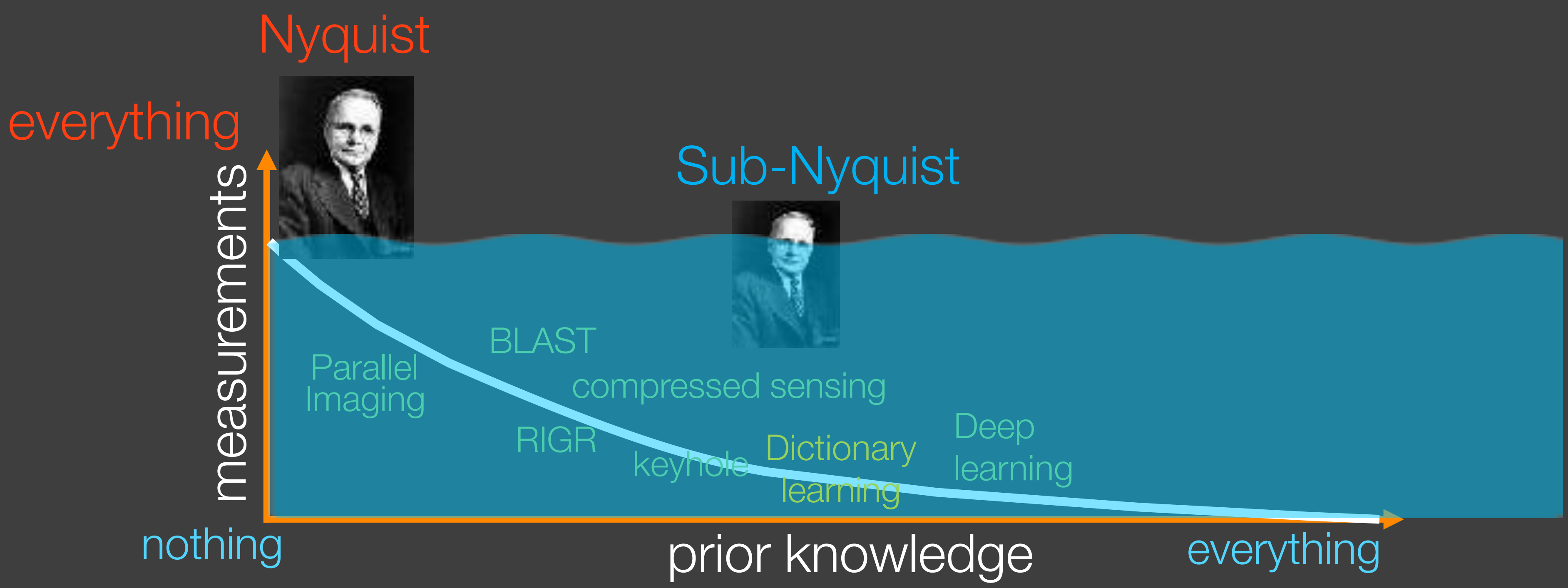
Image is reconstructed



https://www.mountnittany.org/assets/images/krames/Image_201608102119_2786.jpg
http://xrayphysics.com/se_diag_wgrad.png

Data redundancy

Redundancy reduces sampling requirements
(The more you know, the less you need)

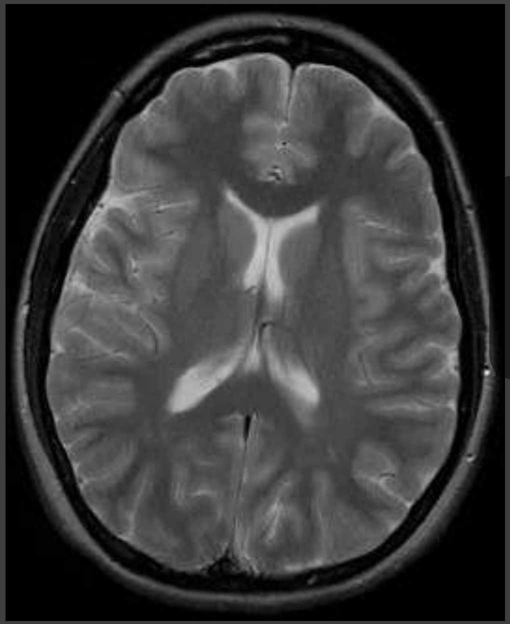


From M. Lustig

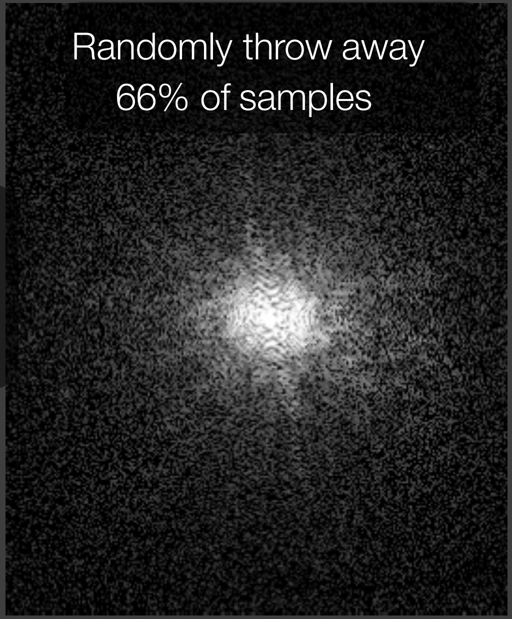

Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging

Michael Lustig, David Donoho, and John M. Pauly

Compressed sensing MRI

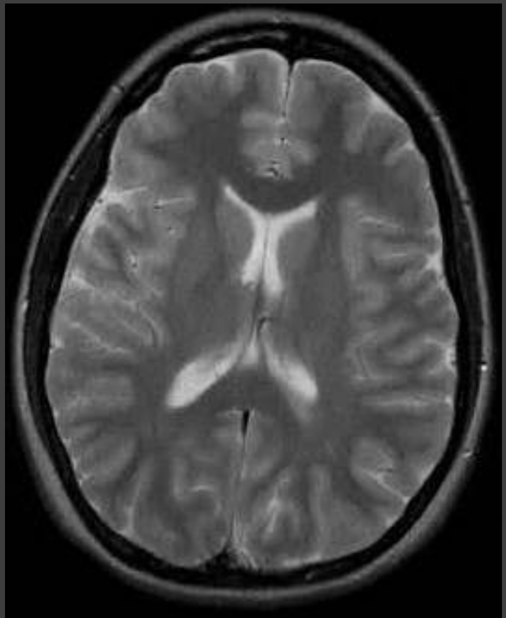


Fourier transform



standard recon

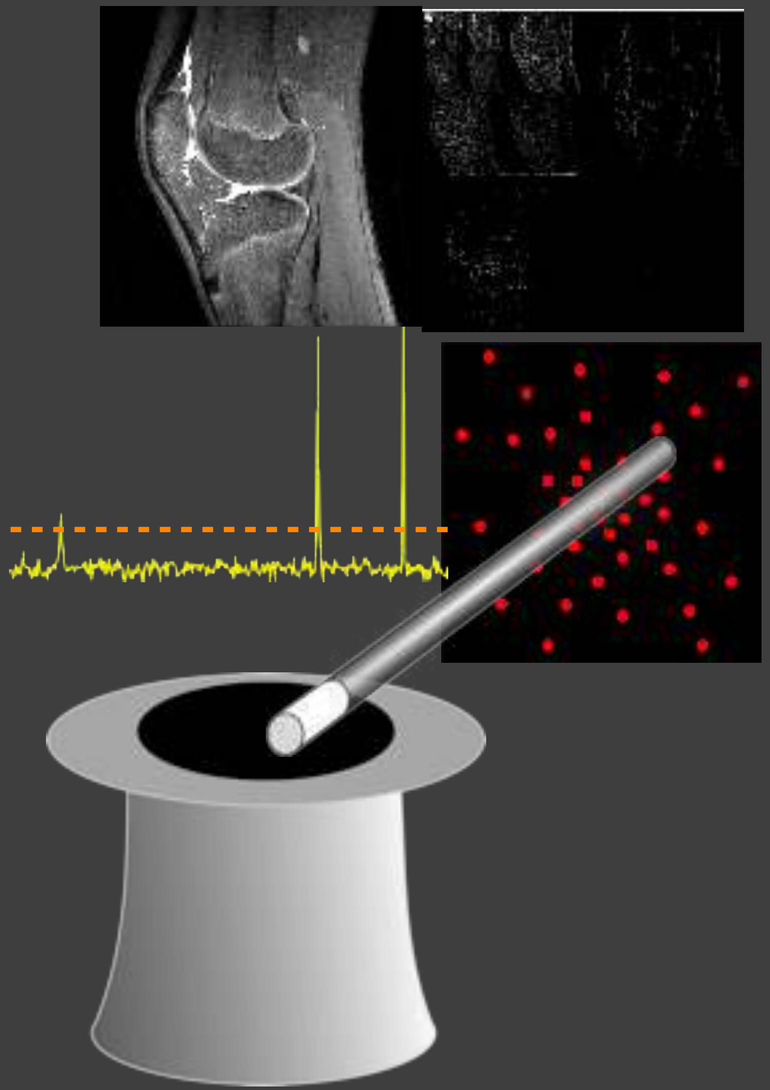
compressed sensing



From Miki Lustig

Compressed sensing recipe

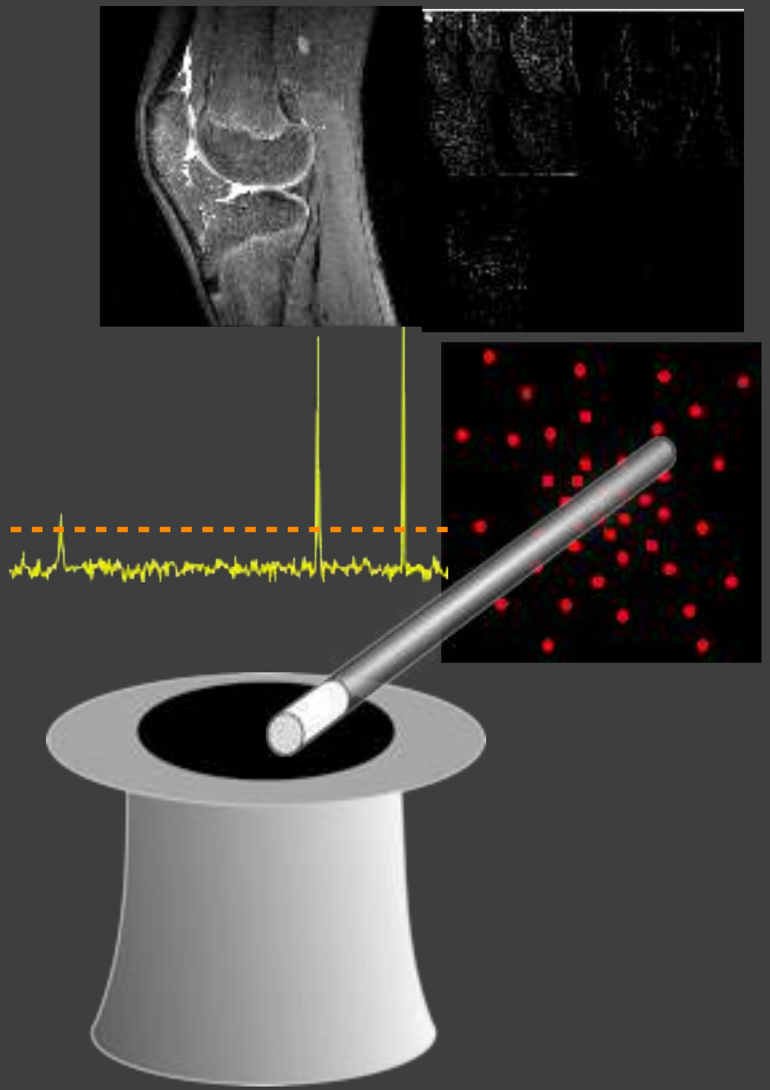
1. Sparse signal model
2. Incoherent sensing operator
3. Non-linear reconstruction algorithm



EJ Candes et al., IEEE SPM, 2008
M Lustig et al., IEEE SPM, 2008

Compressed sensing recipe

1. Sparse signal model
2. Incoherent sensing operator
3. Non-linear reconstruction algorithm



EJ Candes et al., IEEE SPM, 2008
M Lustig et al., IEEE SPM, 2008

Sparse signal modeling

- Assumption: x is a K-sparse signal (K << N)
  - Make M (K < M < N) incoherent linear measurements

$$\mathbf{E} \in \mathbb{C}^{M \times N}$$

$$\mathbf{y} \in \mathbb{C}^{M} \quad = \quad \boxed{M \times N} \quad \mathbf{x} \in \mathbb{C}^{N}$$

K non-zeros

Measurements
(e.g. k-space)

Incoherent encoding operator
(e.g. under-sampled Fourier Transform)

Input signal
(e.g. image)

S Chen, D Donoho. “Basis Pursuit,” Technical Report, Department of Statistics, Stanford University.
Tibshirani, R. (1996). J. Royal. Statist. Soc B., Vol. 58, No. 1, pages 267-288).
Cai and Wang (2011). Orthogonal Matching Pursuit for Sparse Signal Recovery With Noise. IEEE TMI.

Sparse signal modeling

- Assumption: x is a K-sparse signal (K << N)
  - Make M (K < M < N) incoherent linear measurements
  - Enforce sparsity during reconstruction

Greedy methods

$$\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{E}\mathbf{x}||_2^2 \text{ subject to } ||\mathbf{x}||_0 \leq K$$

Relaxation methods

$$\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{E}\mathbf{x}||_2 + \lambda||\mathbf{x}||_1$$

S Chen, D Donoho. "Basis Pursuit," Technical Report, Department of Statistics, Stanford University.
Tibshirani, R. (1996). J. Royal. Statist. Soc B., Vol. 58, No. 1, pages 267-288).
Cai and Wang (2011). Orthogonal Matching Pursuit for Sparse Signal Recovery With Noise. IEEE TMI.

Sparse signal modeling

- What if the signal (image) is not sparse (in the pixel domain)?

not sparse

not sparse

not sparse

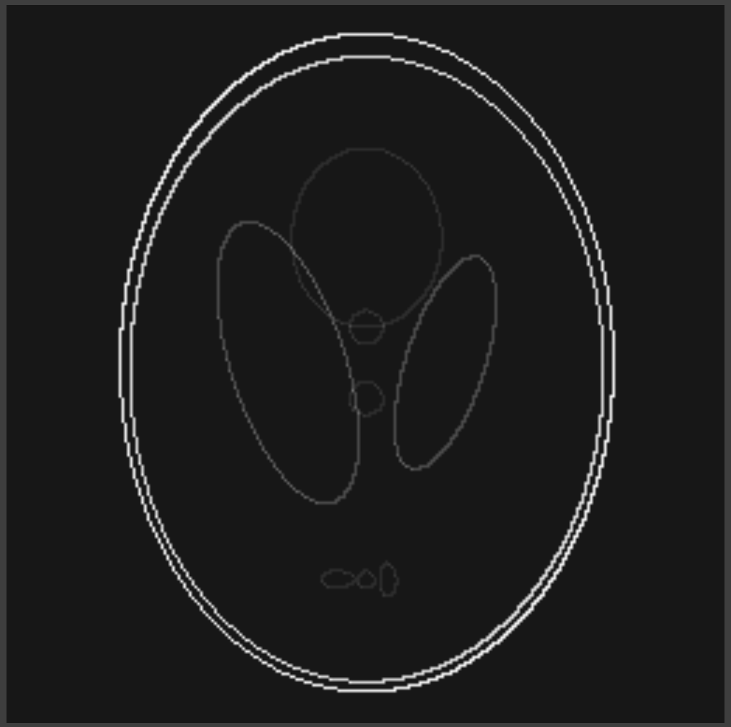
not sparse

Transform sparsity

- Most medical images are sparse in an alternative representation

not sparse in pixel domain

sparse edges

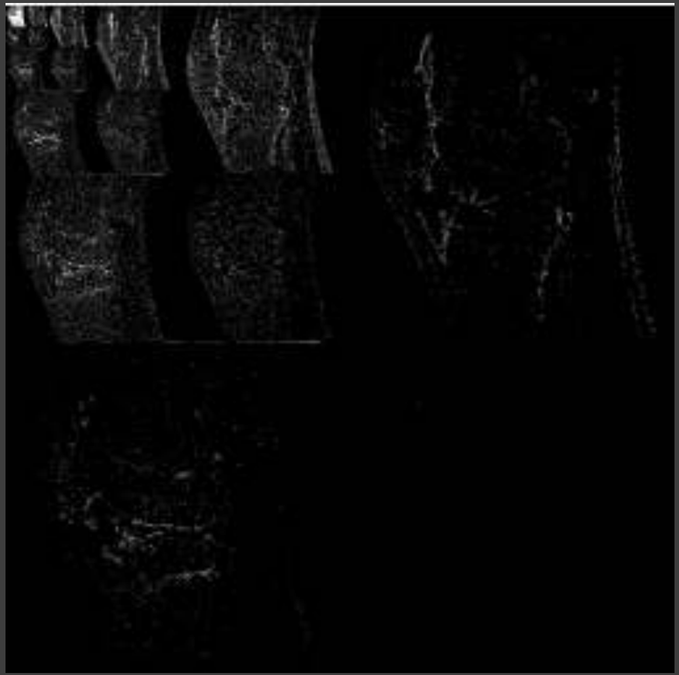


Transform sparsity

- Most medical images are sparse in an alternative representation

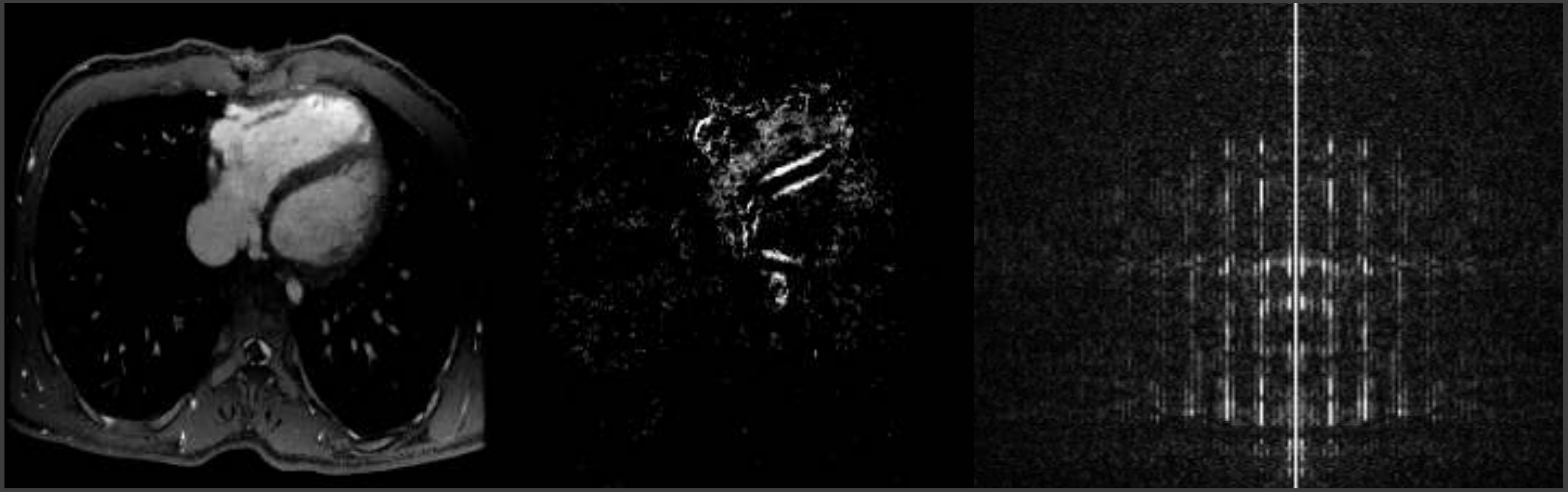
not sparse in pixel domain

sparse in wavelet domain



Transform sparsity

- Most medical images are sparse in an alternative representation



From Joseph Cheng

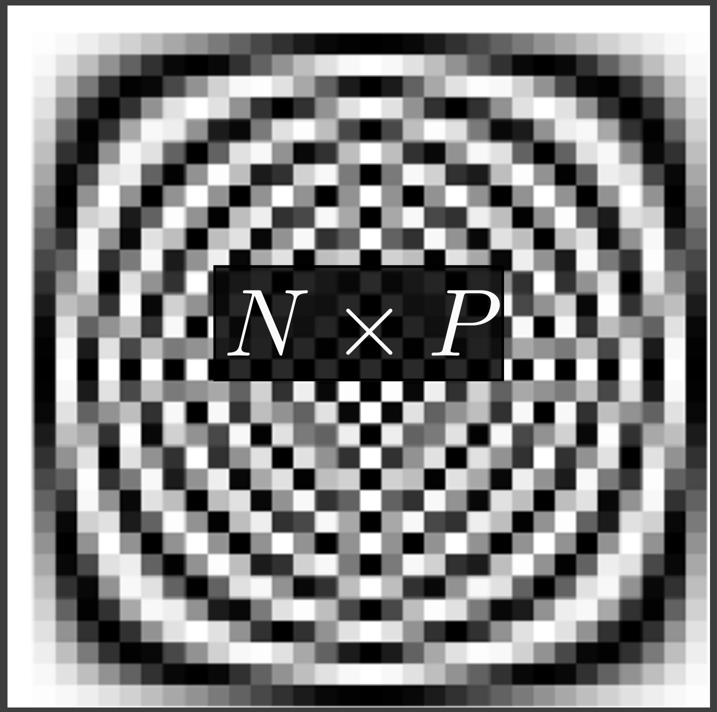
Transform sparsity

- Represent the original signal as sparse in a transform domain

$$\mathbf{D}$$

$\mathbf{x} \in \mathbb{C}^N$

$=$



$\alpha \in \mathbb{C}^P$

Sparse coefficients

Dictionary: prototype atoms make a signal

Transform sparsity

- Represent the original signal as sparse in a transform domain
- Enforce sparsity on transformed coefficients

Greedy methods

$$\min_{\alpha} \frac{1}{2}||\mathbf{y} - \mathbf{ED}\alpha||_2^2 + \lambda||\alpha||_1$$

Relaxation methods

$$\min_{\alpha} \frac{1}{2}||\mathbf{y} - \mathbf{ED}\alpha||_2^2 \quad \text{subject to } ||\alpha||_0 \leq K$$

R Rubinstein, AM Bruckstein, M Elad, Dictionaries for Sparse Representation Modeling,
Proc IEEE, vol 98, no 6, p 1045-1057, 2010.

Transform sparsity

- Problem:
  - Sparsity is only as good as our transform
  - Need to choose the right transform for each signal

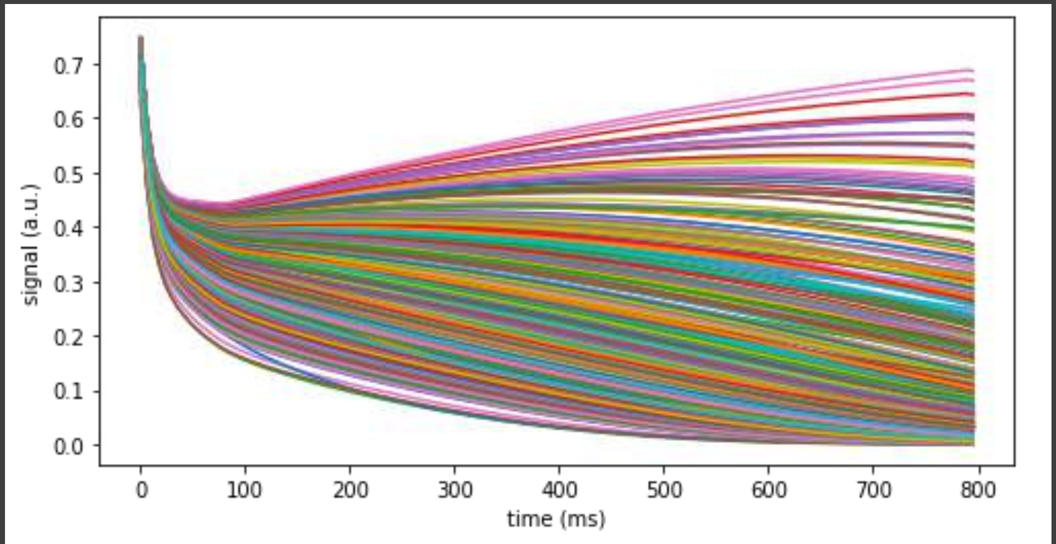
- Solution?
  - Learn the transform! → Dictionary Learning

R Rubinstein, AM Bruckstein, M Elad, Dictionaries for Sparse Representation Modeling,
Proc IEEE, vol 98, no 6, p 1045-1057, 2010.

Dictionary learning

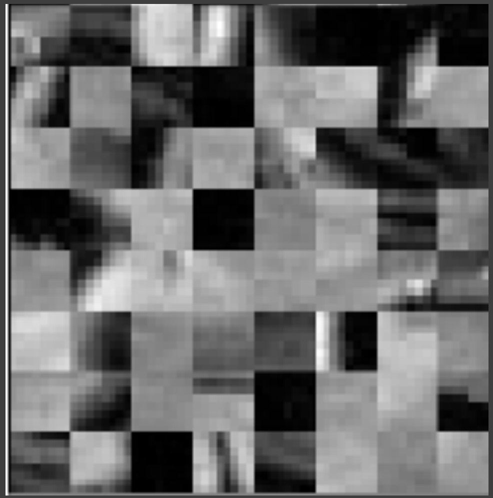
Given training data (1D, 2D, N-D,…):

Temporal relaxation curves



Images

Patches



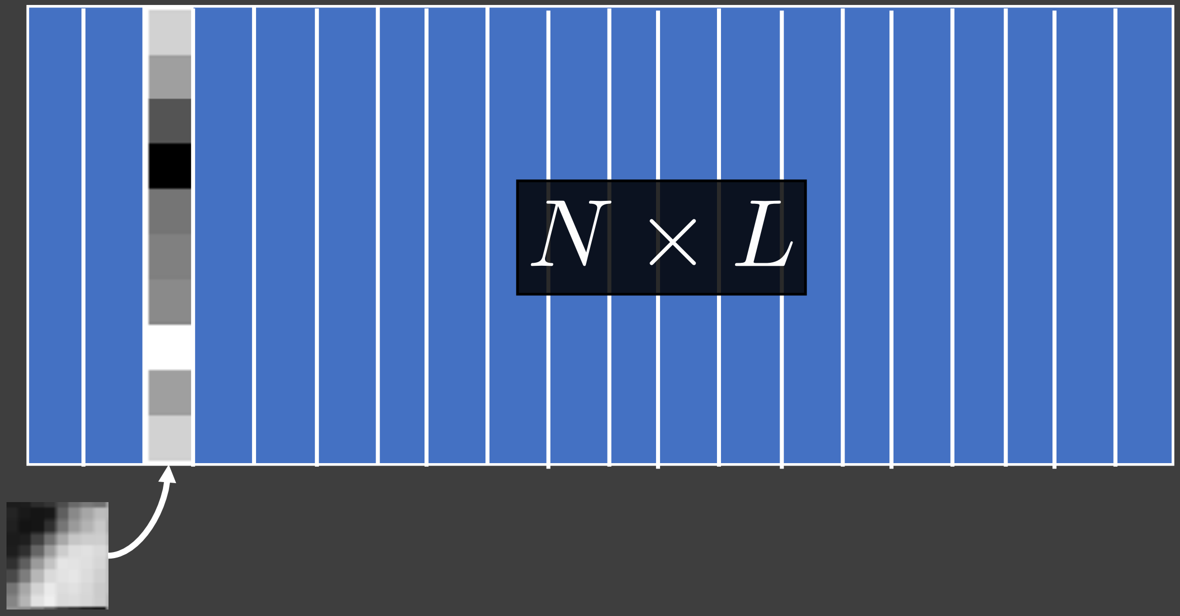
R Rubinstein, AM Bruckstein, M Elad, Dictionaries for Sparse Representation Modeling, Proc IEEE, vol 98, no 6, p 1045-1057, 2010.

Dictionary learning

Given training data: form a data matrix

Data matrix

$\mathbf{X}$



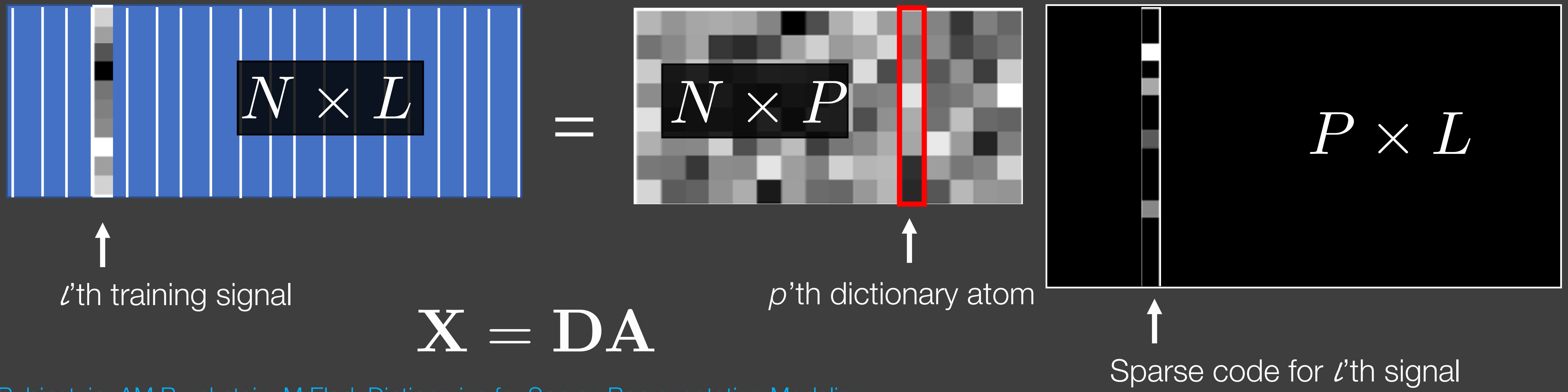
L training examples
Each length-N

ℓ'th training signal, vectorized into a column

Dictionary learning

Given training data: form a data matrix

- Jointly learn the dictionary and the sparse representation



$$\mathbf{X} = \mathbf{DA}$$

R Rubinstein, AM Bruckstein, M Elad, Dictionaries for Sparse Representation Modeling, Proc IEEE, vol 98, no 6, p 1045-1057, 2010.

Dictionary learning

Given training data: form a data matrix

- Jointly learn the dictionary and the sparse representation

$$\min_{\mathbf{D},\mathbf{A}} \frac{1}{2}||\mathbf{X} - \mathbf{DA}||_2^2 \quad \text{subject to } ||\alpha_l||_0 \leq K \quad l = 1, ..., L$$

$$||\mathbf{d}_p||_2 \leq 1 \quad p = 1, ..., P$$

L: number of training examples
P: number of dictionary elements (atoms)

R Rubinstein, AM Bruckstein, M Elad, Dictionaries for Sparse Representation Modeling, Proc IEEE, vol 98, no 6, p 1045-1057, 2010.

Dictionary learning

Given training data: form a data matrix

- Jointly learn the dictionary and the sparse representation
- Approach: alternating minimization

Step 1: Update coefficients (sparse coding)

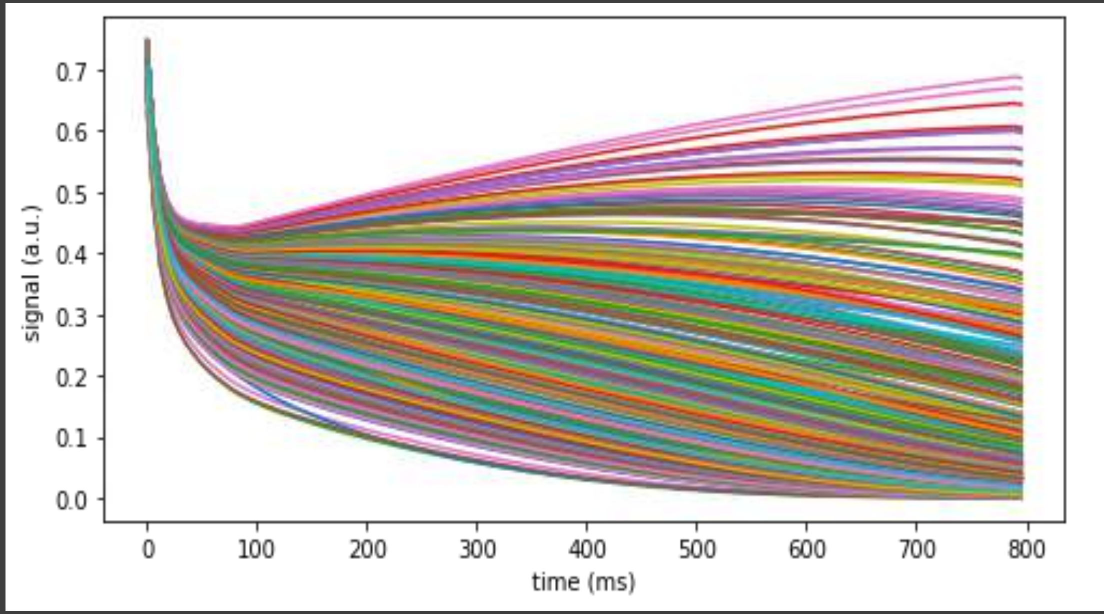
$$\min_{\mathbf{A}} \frac{1}{2}||\mathbf{X} - \mathbf{D}\mathbf{A}||_2^2 \quad \text{subject to } ||\alpha_l||_0 \leq K$$
$$l = 1, ..., L$$

Step 2: Update dictionary

$$\min_{\mathbf{D}} \frac{1}{2}||\mathbf{X} - \mathbf{D}\mathbf{A}||_2^2 \quad \text{subject to } ||\mathbf{d}_p||_2 \leq 1$$
$$p = 1, ..., P$$

Dictionary learning: example

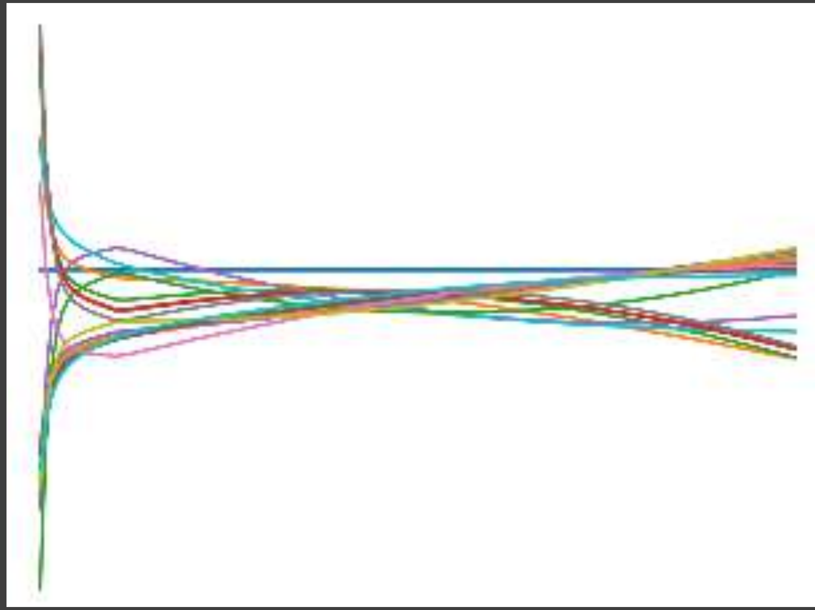
Temporal relaxation curves



N = 130

$\mathbf{X}$

=



P=20

$\mathbf{D}$

K=2

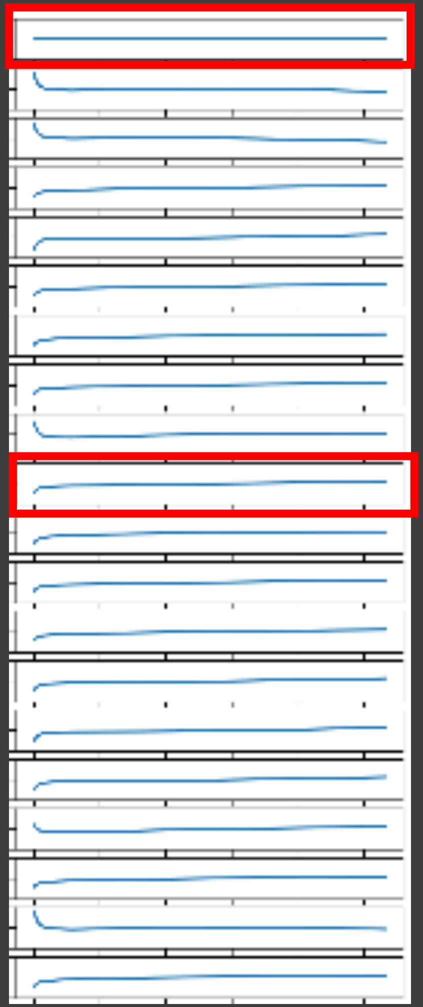
α

M. Doneva et al., MRM 2010. doi: 10.1002/mrm.22483
J.I. Tamir et al., MRM 2016. doi: 10.1002/mrm.26102

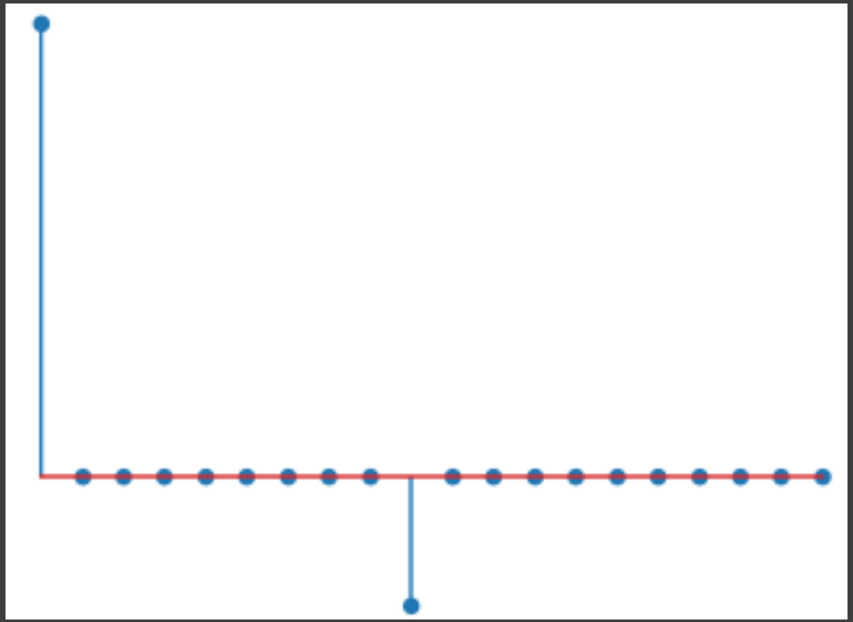
Dictionary learning: example

1

10

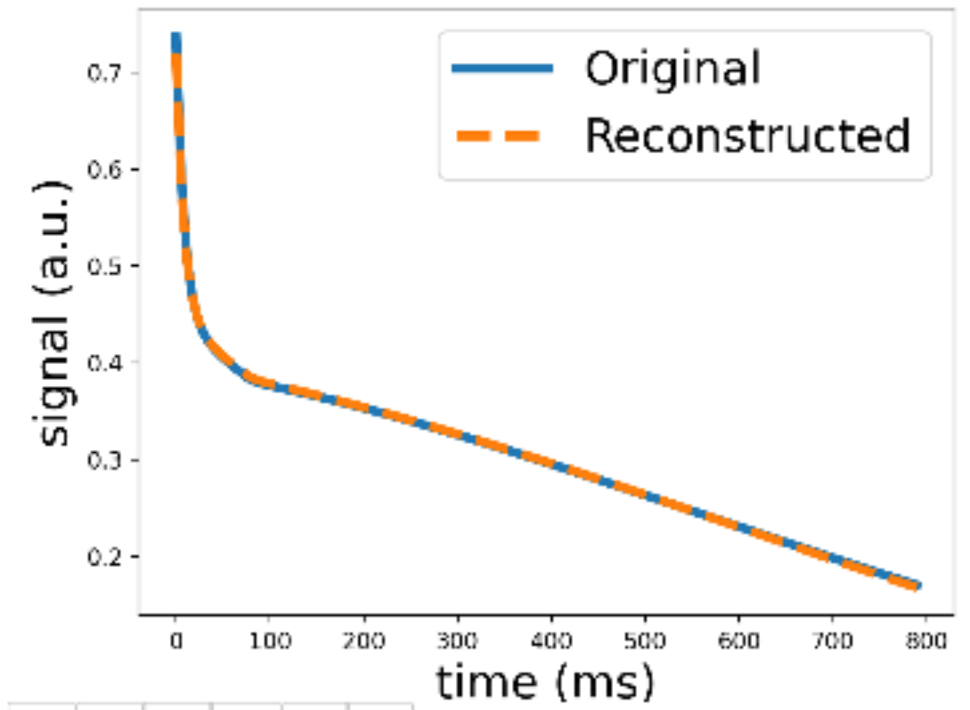


$\mathbf{D}$



α

$=$



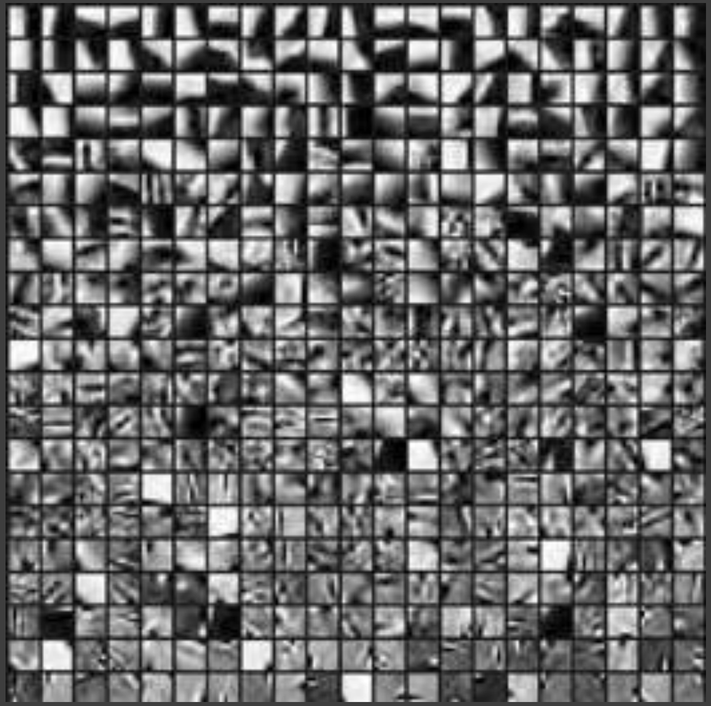
$$\mathbf{d}_1\alpha_1 + \mathbf{d}_{10}\alpha_{10} = \hat{\mathbf{x}}$$

Dictionary learning: example

Image patches

$\mathbf{X}$ N = 8 x 8

=



$\mathbf{D}\alpha$ P=441

K=7

Aharon et al., IEEE TSP, 2006, doi: 10.1109/TSP.2006.881199

From image space to k-space

- We can learn the dictionary directly from training examples…
  - At “inference time”, solve the sparse coding problem

- But we can **also learn** the dictionary directly from the MRI data!

M. Doneva et al., MRM 2010. doi: 10.1002/mrm.22483
Ravishankar et al., IEEE TMI, 2011. doi: 10.1109/TMI.2010.2090538

Dictionary learning MRI

$$\min_{\mathbf{x},\mathbf{D},\mathbf{A}} \frac{1}{2}||\mathbf{y}-\mathbf{E}\mathbf{x}||_2^2 + \frac{\lambda}{2}||\mathbf{x}-\mathbf{R}\left(\mathbf{D}\mathbf{A}\right)||_2^2 \quad \text{subject to}$$

$$||\alpha_l||_0 \leq K$$

$$||\mathbf{d}_p||_2 \leq 1$$

$$l = 1, ..., L$$
$$p = 1, ..., P$$

Data consistency

Dictionary fit

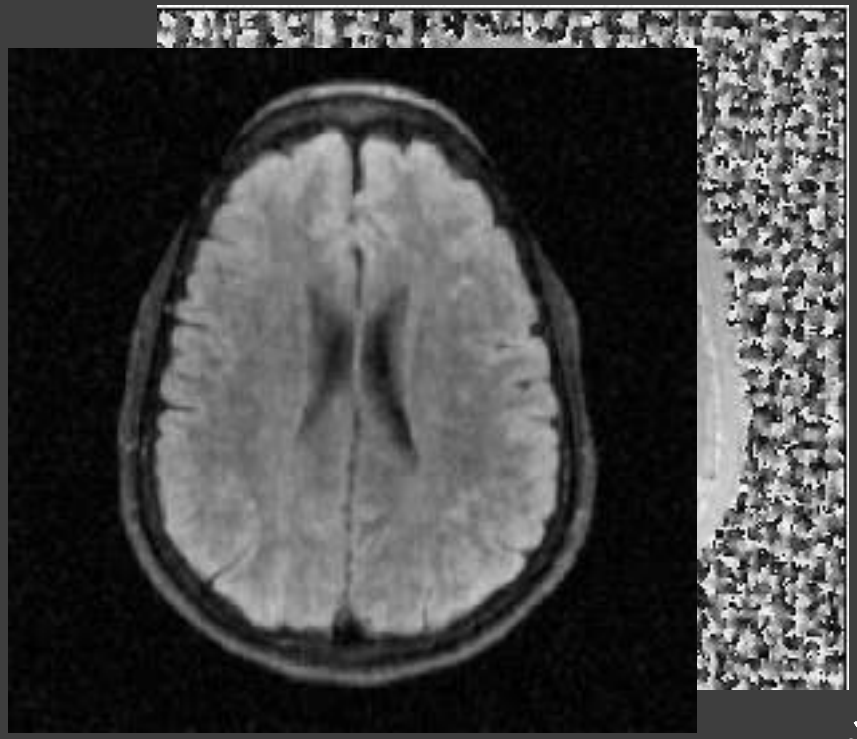
Converts from data matrix (patches) to image

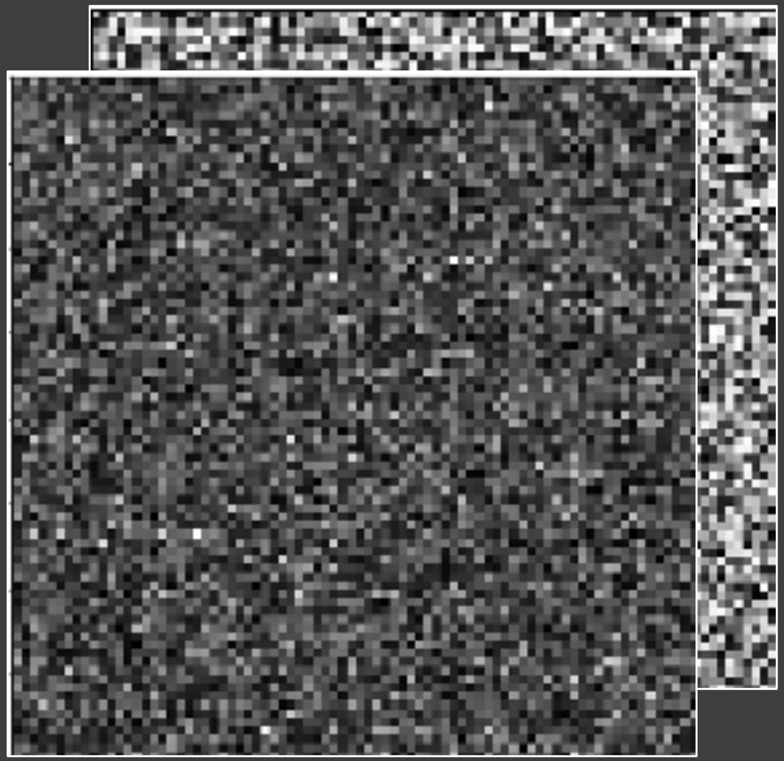
Solution: → Alternating minimization (again!)

M. Doneva et al., MRM 2010. doi: 10.1002/mrm.22483
Ravishankar et al., IEEE TMI, 2011. doi: 10.1109/TMI.2010.2090538

Dictionary learning MRI

Step 0. Initialize image and dictionary

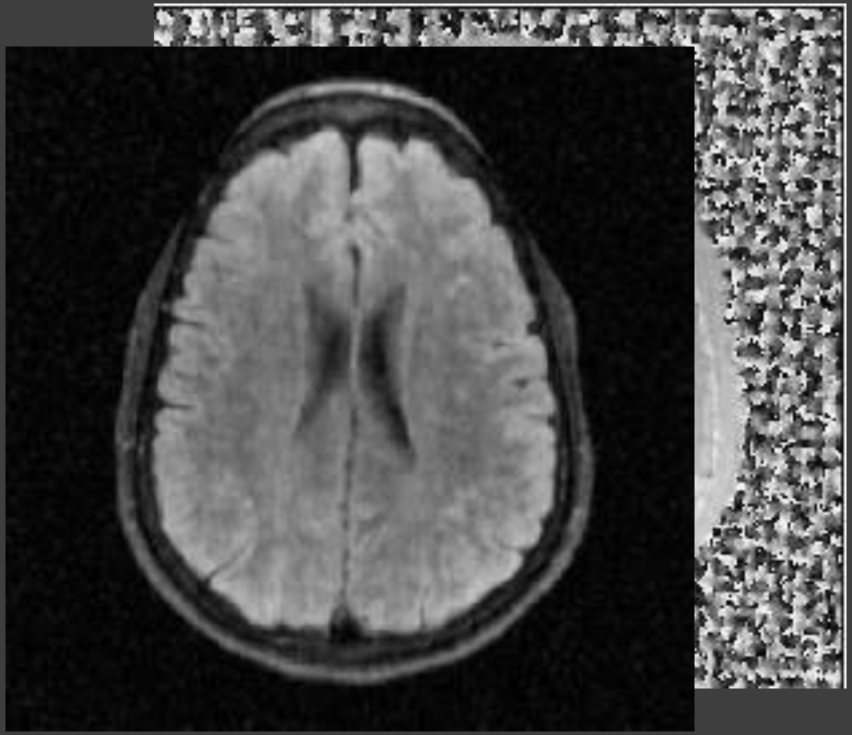




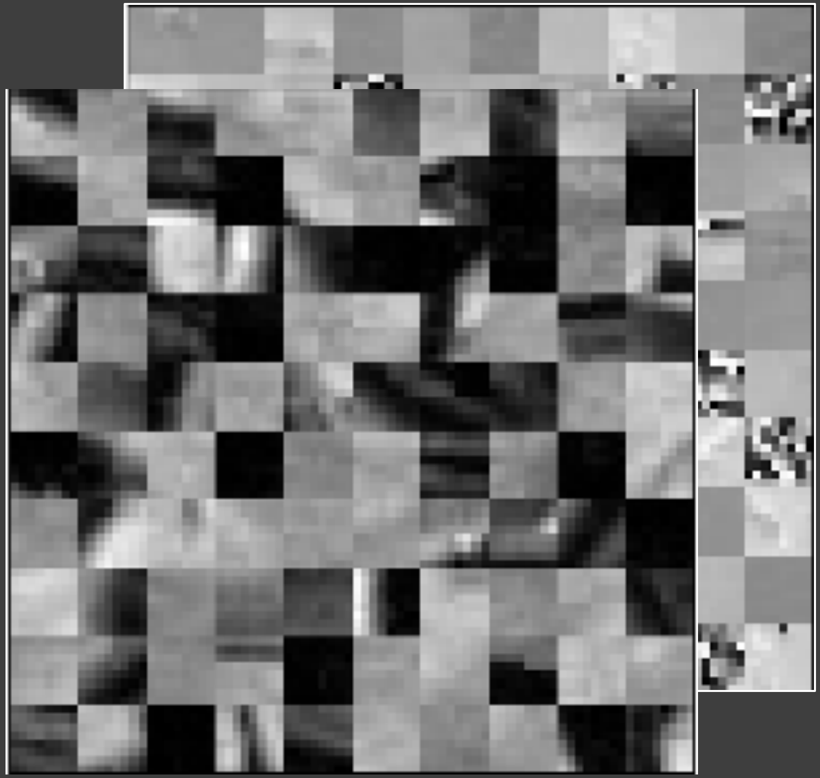
Dictionary learning MRI

For iterations 1:T

Step 1. Extract patches from image



$\mathbf{x}$



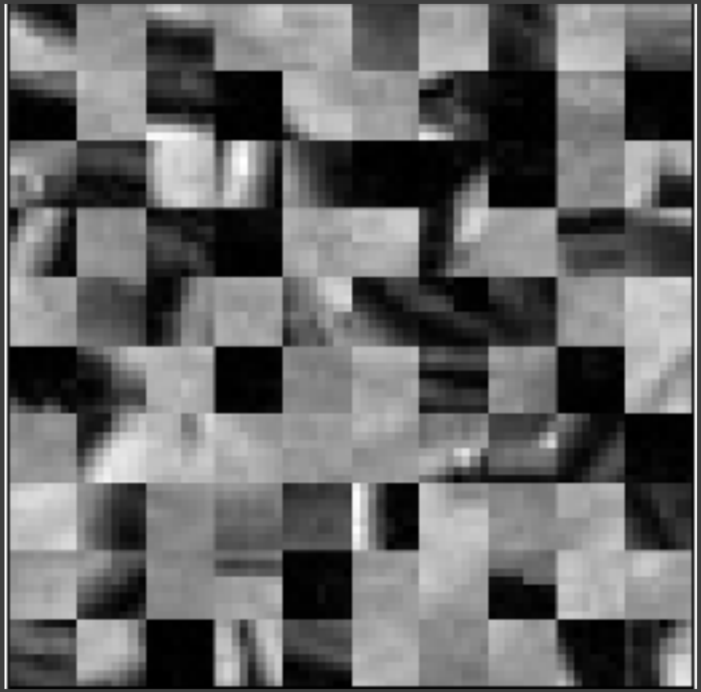
$\mathbf{X} = \mathbf{R}^T \mathbf{x}$

(mag + phase)

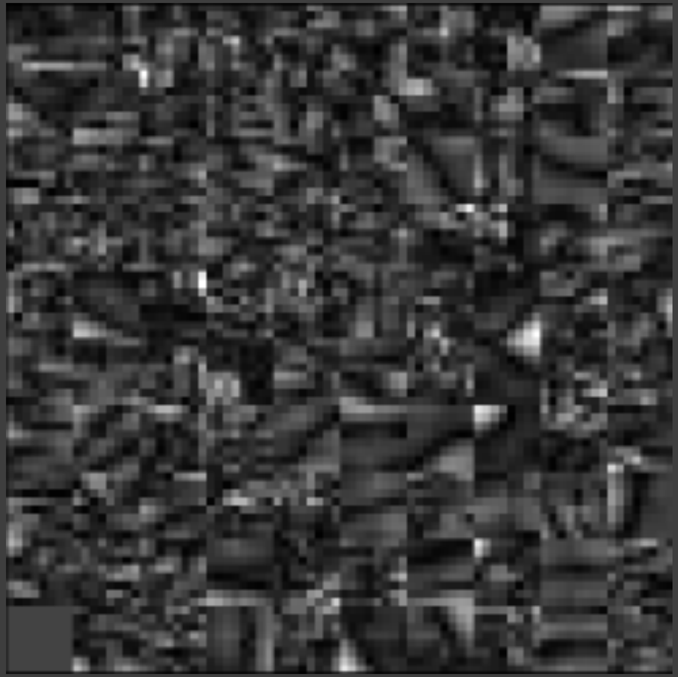
Dictionary learning MRI

For iterations 1:T

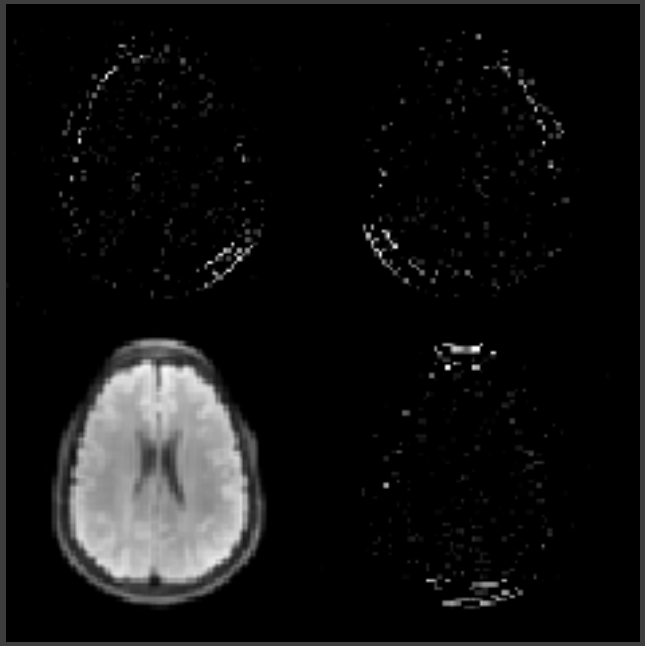
Step 2. Fit dictionary and sparse codes to patches



$\mathbf{X}$



$\mathbf{D}$

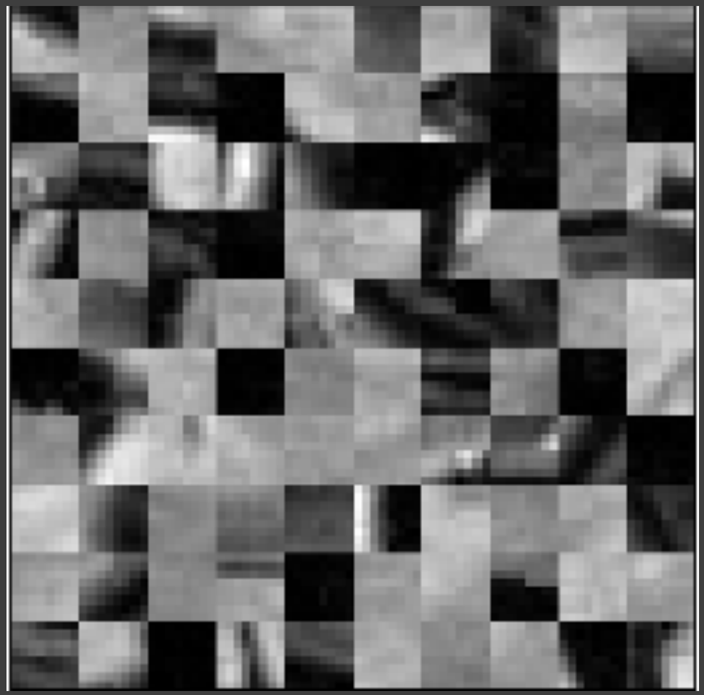


$\mathbf{A}$

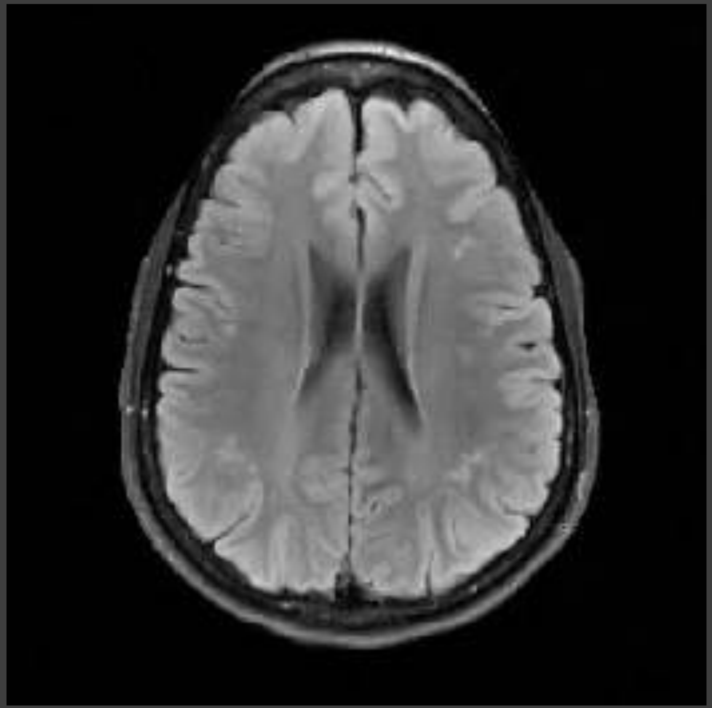
Dictionary learning MRI

For iterations 1:T

Step 3. Reshape and average patches into image



$$\mathbf{DA}$$

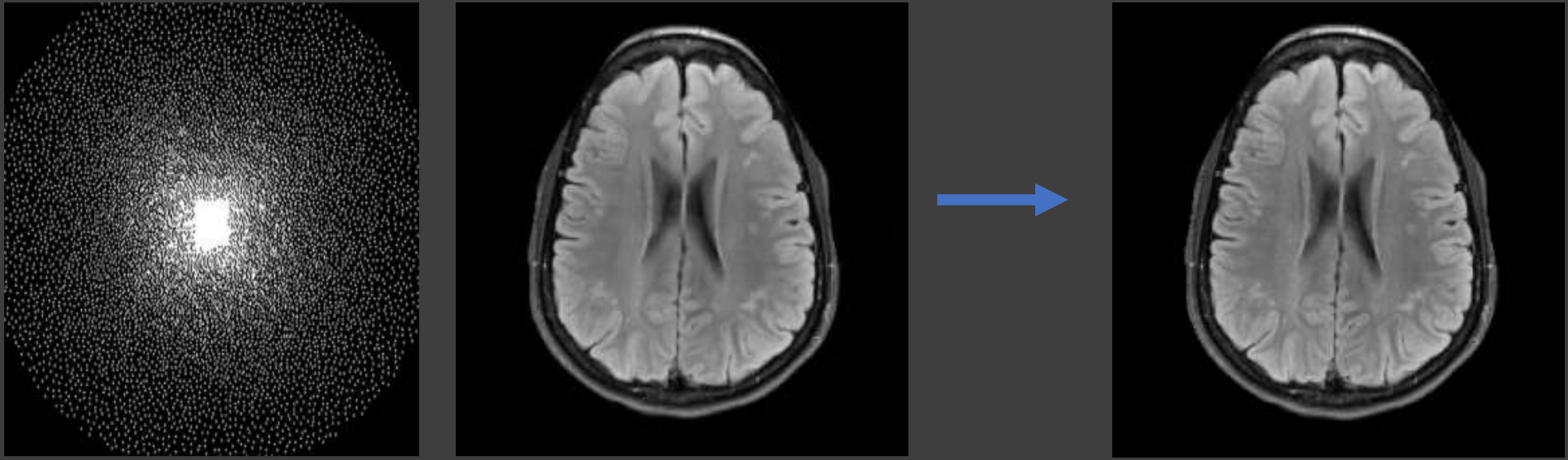


$$\mathbf{x}_D = \mathbf{RDA}$$

Dictionary learning MRI

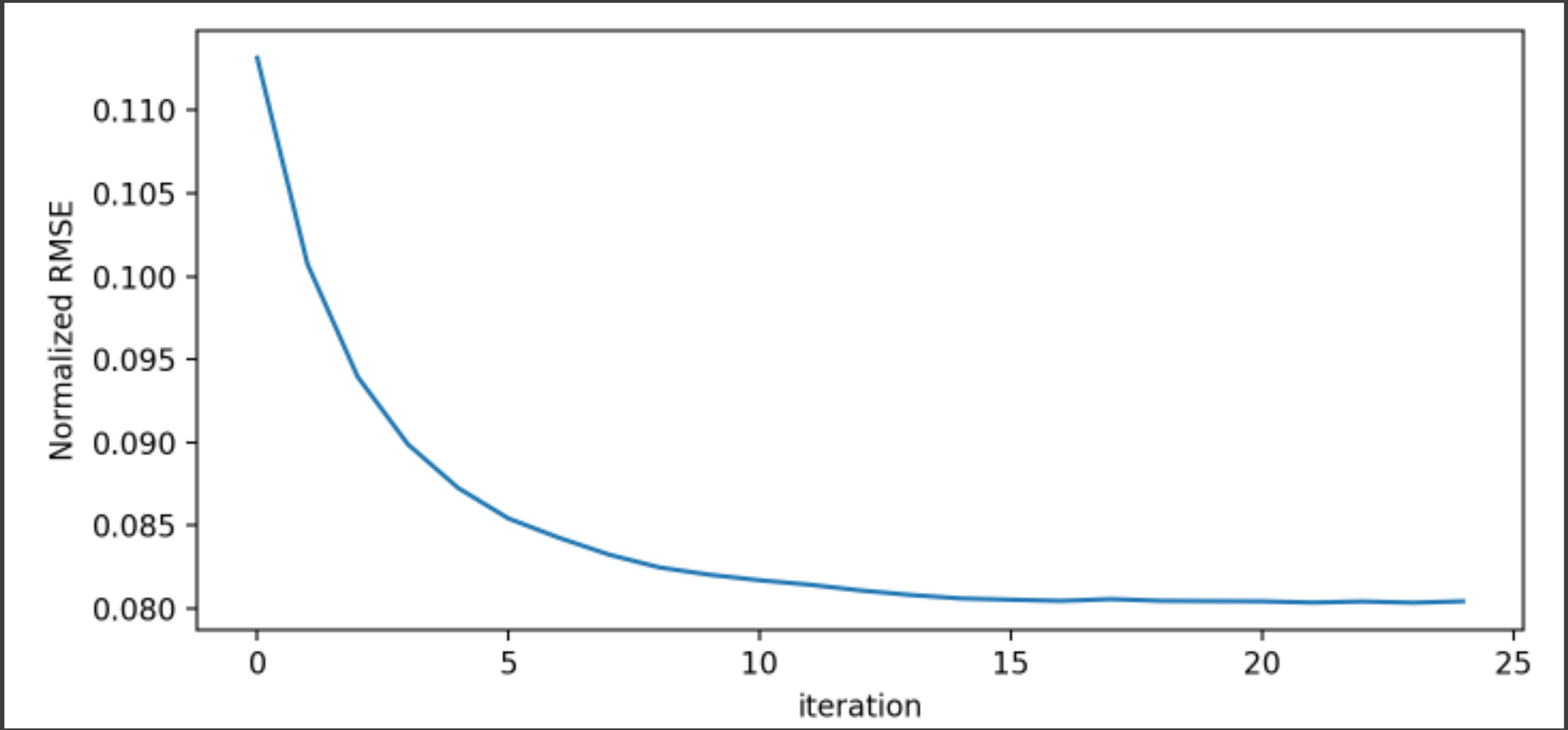
For iterations 1:T

Step 4. Enforce data consistency with dictionary fit



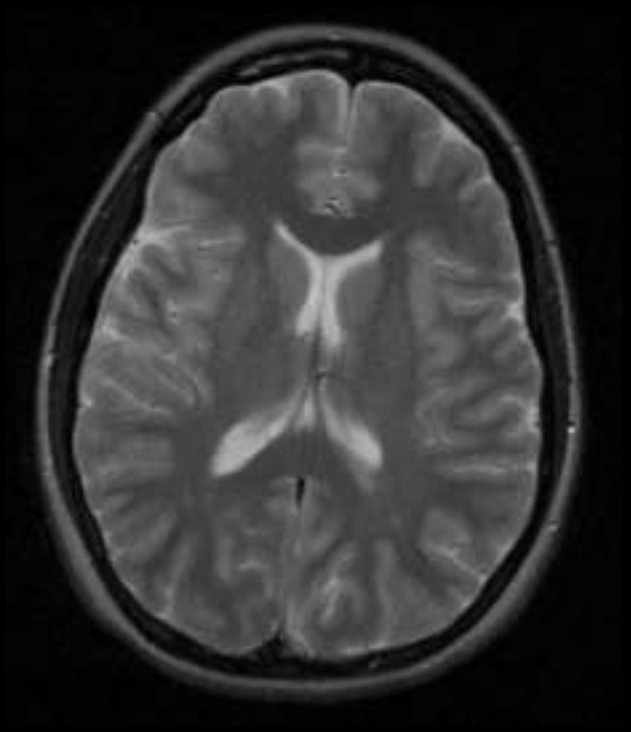
$\mathbf{y}$ $\quad$ $\mathbf{x}_D$ $\quad$ $\mathbf{x}$

Dictionary learning MRI

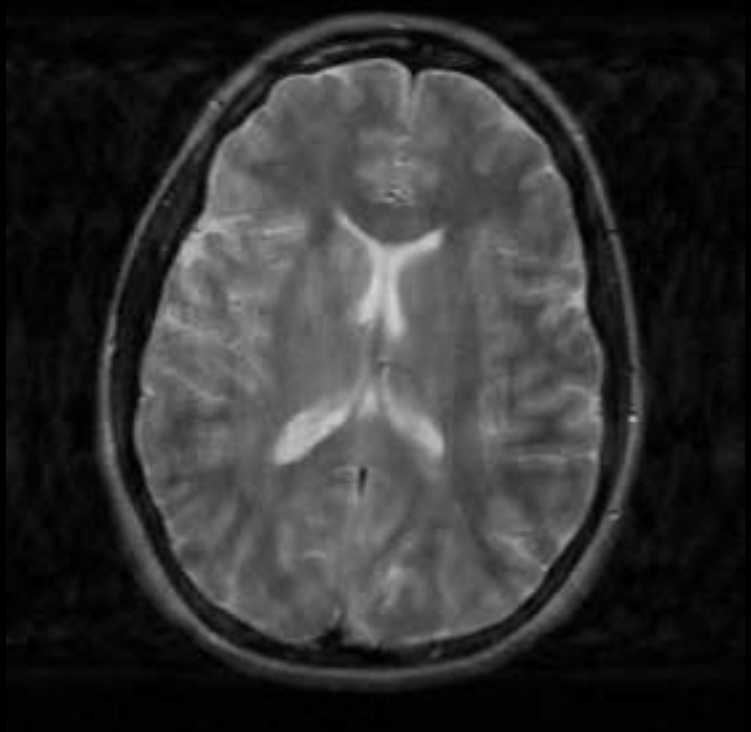


Error decreases with number of iterations

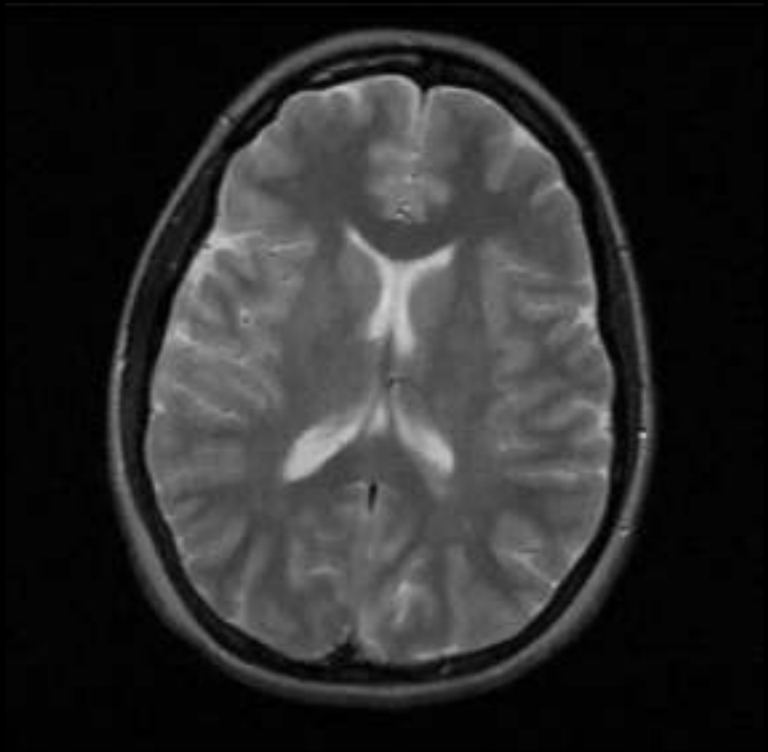
Dictionary learning MRI



Ground-truth



CS-MRI



Dictionary Learning

Ravishankar et al., IEEE TMI, 2011. doi: 10.1109/TMI.2010.2090538

5x under-sampling, single-coil

The Devil's in the details…

? How should I remove "bad" atoms?

? How big should the dictionary be?

? How sparse should the signal be?

? Should I update the dictionary all at once?

? How should I initialize the dictionary?

? How should I enforce sparsity?

Hands-on dictionary learning

- Tutorial code for dictionary learning:

  https://github.com/utcsilab/dictionary_learning_ismrm_2020

Tutorial Code

- Based on SigPy, a Python toolbox for iterative signal processing

  https://sigpy.readthedocs.io/en/latest/

SigPy

From local to global modeling

- Dictionary learning: local representation
  - The dictionary is built to represent local patches
  - Each patch is a sparse combination of the dictionary
  - The patches are reshaped and averaged to form the image

- Can we create a sparse global model?

From Frank Ong

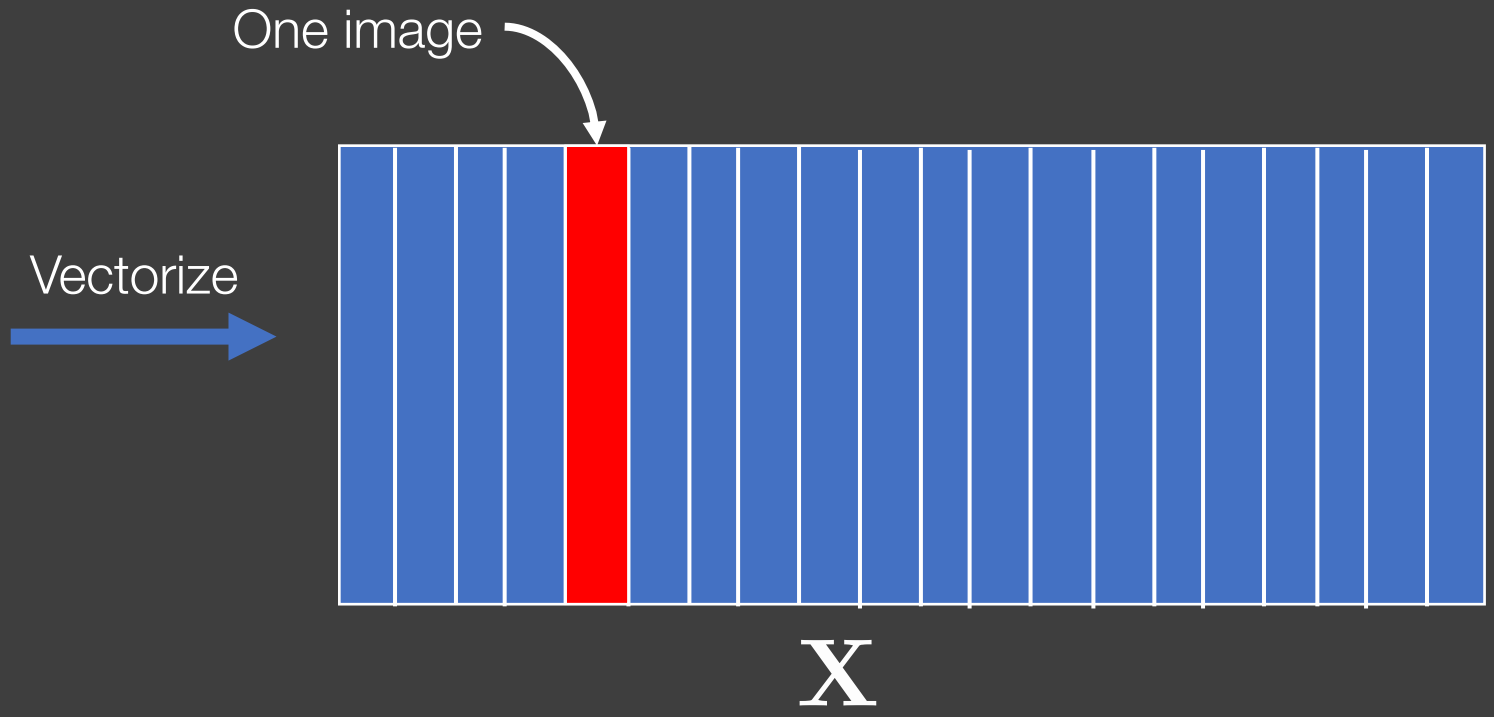
Ref: [1] Zeiler et al. CVPR 2010, [2] Quan et al, ISBI 2016, [3] Elad et al., TMI 2006

From local to global modeling

- Can we create a sparse global model?
  - Could we represent the image as a sparse combination of *images?*
  - Our dictionary is N x P, where N is the number of pixels!!

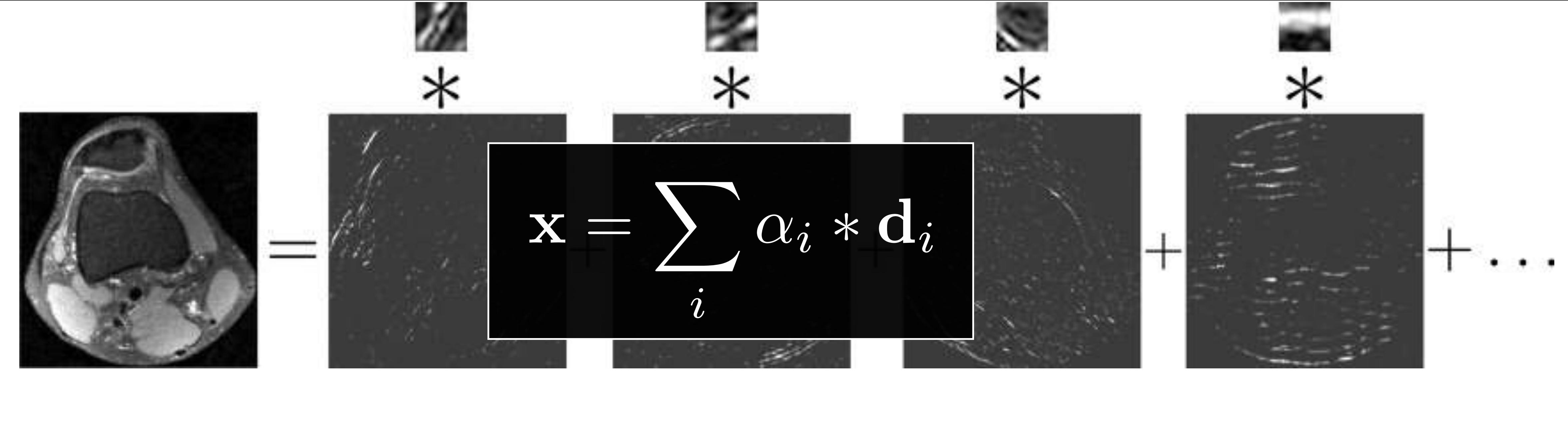
    → Not feasible!

Images



Alternative: convolutional sparse coding

- Represent the full image as a sum of dictionary filters convolved with sparse coefficients



$$\mathbf{x} = \sum_i \alpha_i * \mathbf{d}_i$$

From Frank Ong

Ref: [1] Zeiler et al. CVPR 2010, [2] Quan et al, ISBI 2016, [3] Elad et al., TMI 2006, [4] F Ong et al., ISMRM 18

Convolutional sparse coding (CSC)

- In k-space, over multiple data sets:

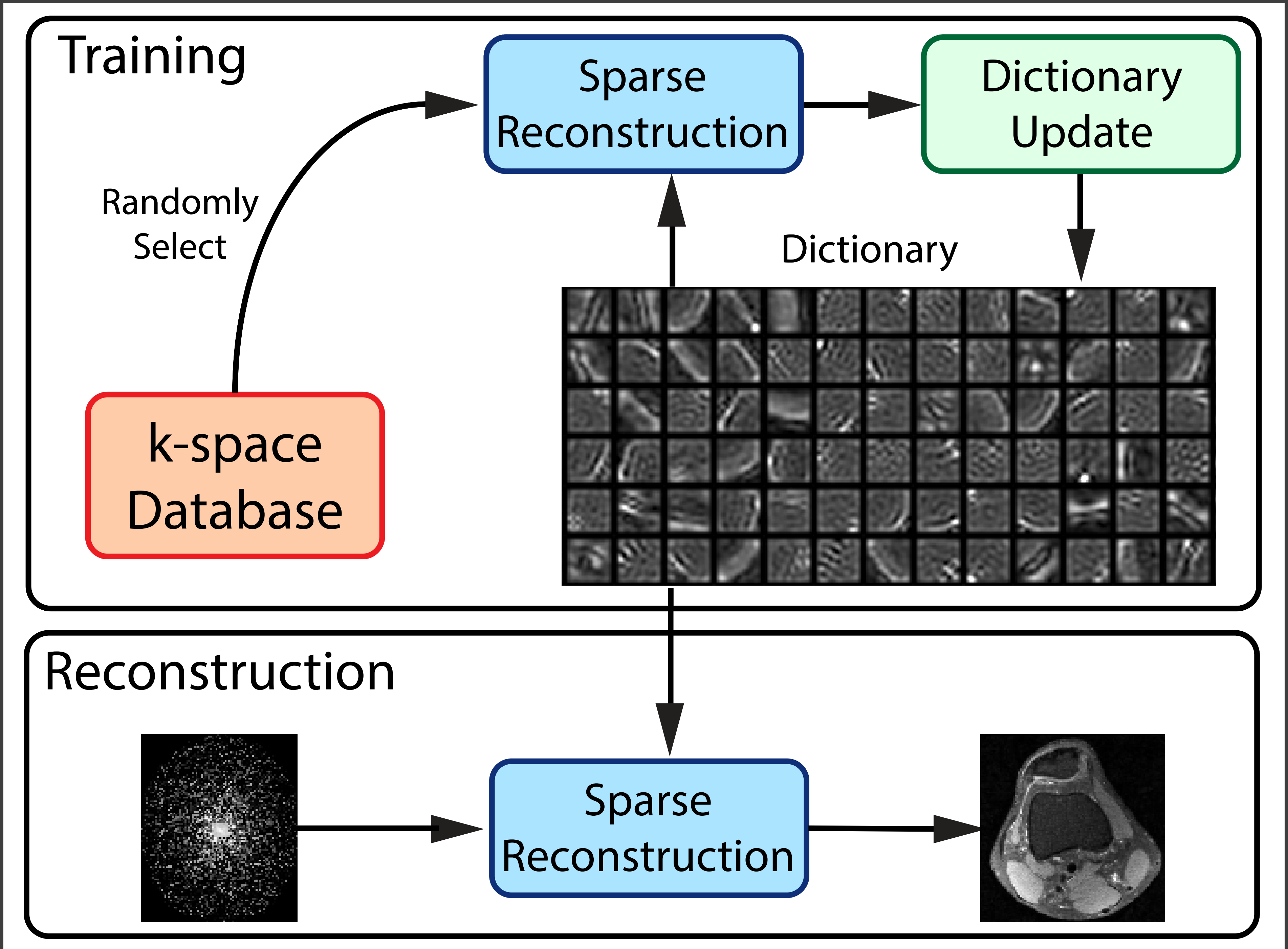
$$\min_{\alpha_j, \mathbf{D}} \sum_j \frac{1}{2} ||\mathbf{y_j} - \mathbf{E}_j \sum_i \alpha_{ij} * \mathbf{d}_i||_2^2 + \lambda ||\alpha_{ij}||_1$$

Same filters for all datasets

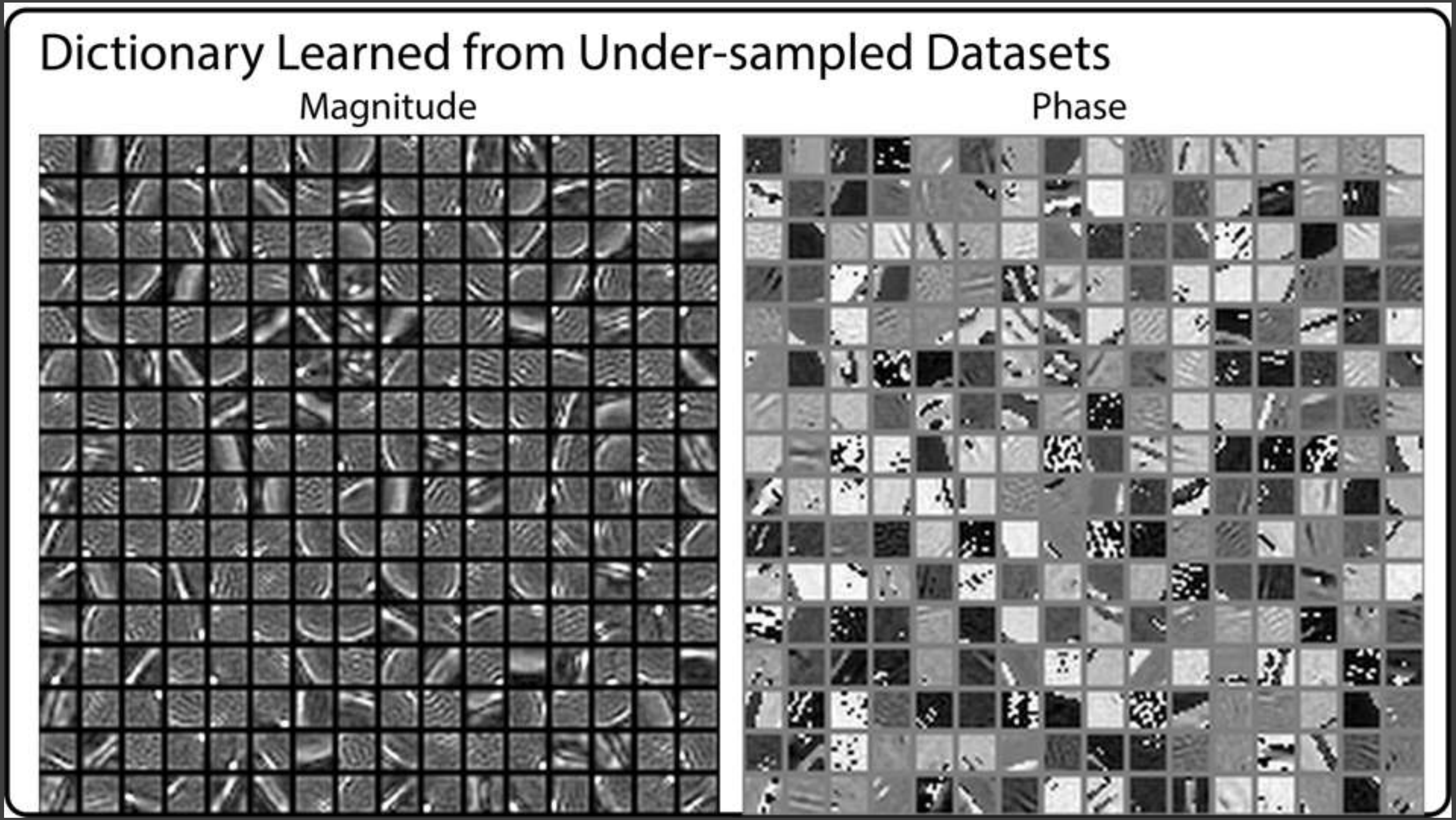
j'th k-space dataset

i'th sparse channel for j'th dataset

Ref: [1] Zeiler et al. CVPR 2010, [2] Quan et al, ISBI 2016, [3] Elad et al., TMI 2006, [4] F Ong et al., ISMRM 18

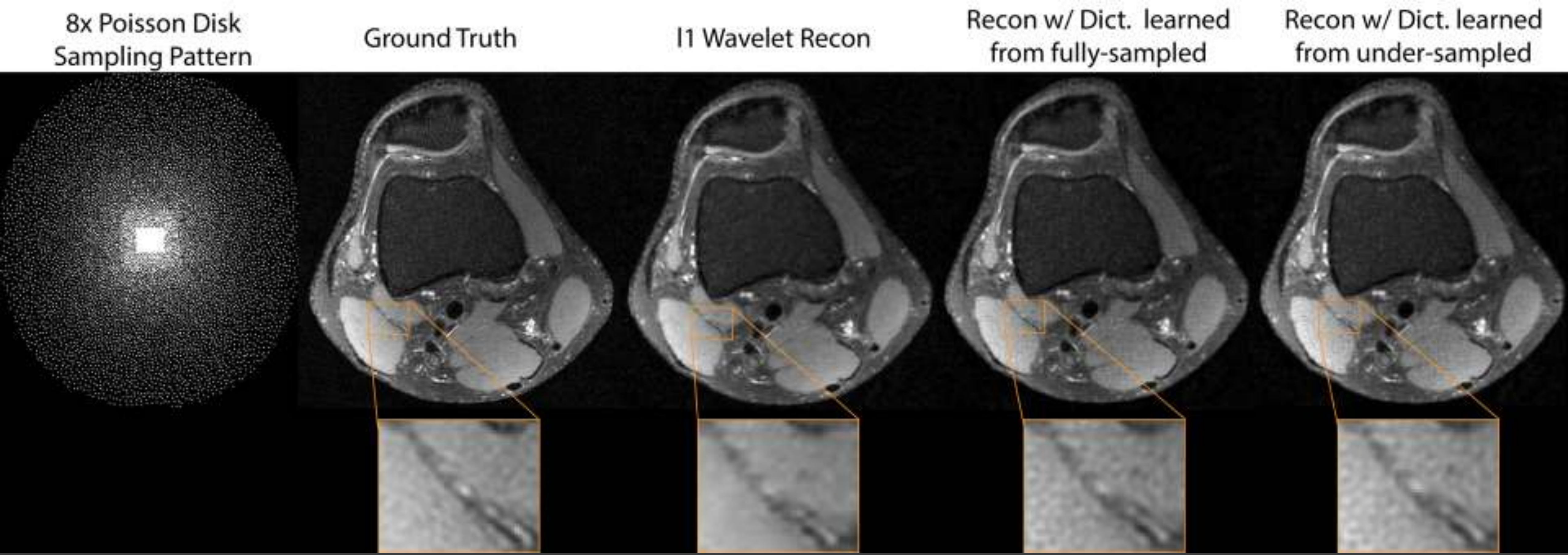


From Frank Ong
F Ong et al., ISMRM 18



From Frank Ong
F Ong et al., ISMRM 18

CSC MRI



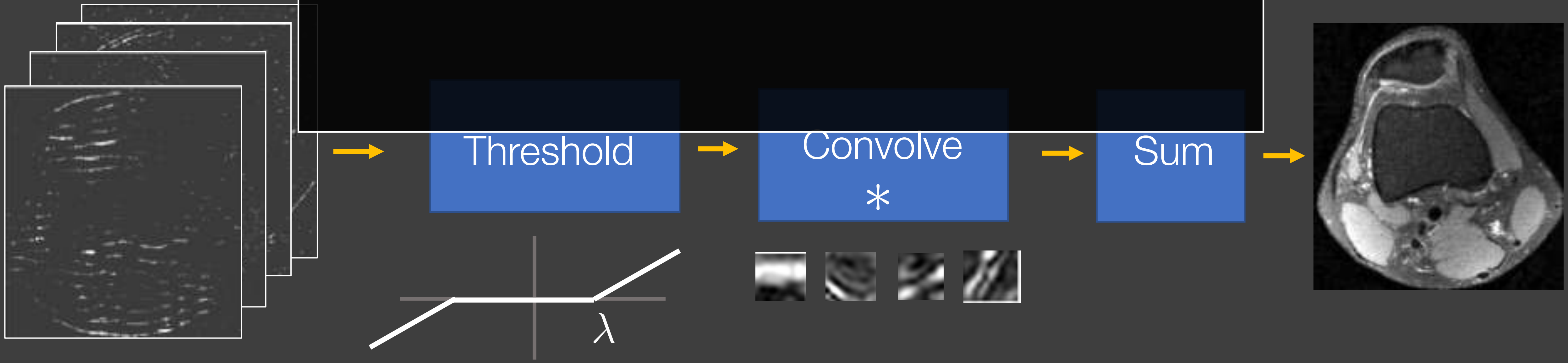
From Frank Ong
F Ong et al., ISMRM 18

From CSC to deep learning

CSC recovery algorithm:

- Threshold coefficients to enforce sparsity
- Convolve multi-channel coefficients with multi-channel filters
- Sum across channels to form the image
- Repeat for T iterations

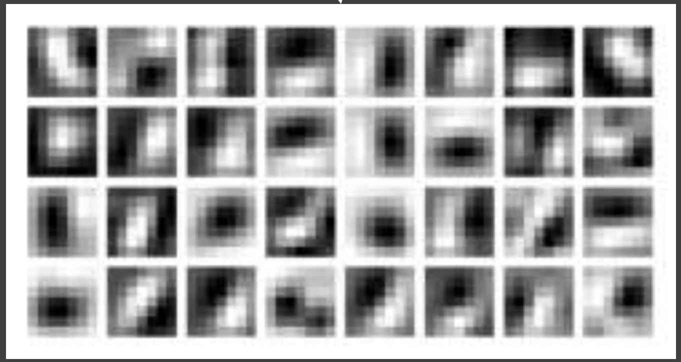
One layer of a convolutional neural network?



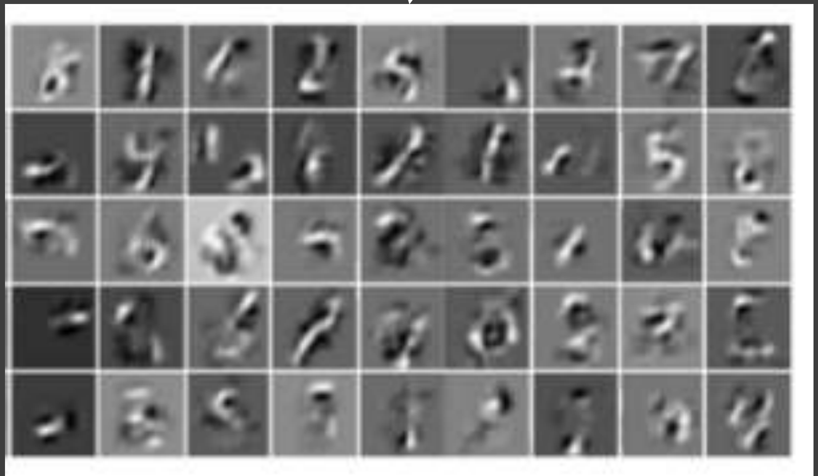
Multi-layer convolutional sparse coding

- Apply CSC multiple times, learn filters at each stage
- Like deep learning, but with performance guarantees!!

CSC Block → CSC Block → … → CSC Block →



dictionary filters



Sulam et al., IEEE TSP, 2018. DOI: 10.1109/TSP.2018.2846226

Summary

- Dictionary learning extends compressed sensing to learned sparsifying transforms

- Application to MRI is straightforward

- Choice of hyper-parameters greatly impacts performance

- Connections between Dictionary Learning (DL) and Deep Learning (DL) in more than just initials www.jtsense.com

Slides/images
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