# STOR 614
# Homework Assignment No. 3

1. Let $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$, where $A$ is an $m \times n$ matrix of full row rank. Suppose that $x^*$ and $y^*$ be two distinct basic solutions. Prove that $x^*$ and $y^*$ are adjacent if and only if there exist a basis for $x^*$ and a basis for $y^*$ such that the two bases share $m - 1$ elements. (Recall that two distinct basic solutions for a polyhedron are said to be adjacent, if they share $n - 1$ linearly independent active constraints.)

2. Consider the simplex tableau which shows a BFS $x^*$. Prove the following. Suppose this is a maximization problem.

   (a) If the reduced cost of every nonbasic variable is positive, then $x^*$ is the unique optimal solution.

   (b) If $x^*$ is the unique optimal solution and is nondegenerate, then the reduced cost of every nonbasic variable is positive.

3. Consider the problem

$$\begin{array}{llll} \max & z = & 2x_1 + x_2 & \\ \text{s.t} & & x_1 - x_2 & \leq 2, \\ & & x_1 + x_2 & \leq 6, \\ & & x_1, x_2 & \geq 0 \end{array}$$

   (a) Convert this LP into standard from and use the simplex method to solve it.

   (b) Draw the feasible set of this LP, and indicate the path taken by the simplex method.

4. Consider the simplex method applied to an LP in canonical form. Decide if each of the following statements is true or false. Justify your answer by a proof or a counterexample. (Note: we assume that the rules of the simplex method are strictly followed.)

   (a) An iteration of the simplex method may move the feasible solution by a positive distance while leaving the $z$-value unchanged.

   (b) A variable that has just left the basis cannot reenter in the very next iteration.

   (c) A variable that has just entered the basis cannot leave in the very next iteration.

   (d) If a simplex tableau shows a nondegenerate optimal solution, then this solution is the unique optimal solution.