# STOR 614 - Linear Programming, Spring 2019

# Homework No. 7

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May 22, 2019

**Problem 1.**

By Theorem 4 in Lecture Notes 14, for any $b \in B$, the LP has an optimal solution if and only if there does not exist an extreme ray $d$ of the feasible set such that $c^T d < 0$. Since changing $b$ does not change the extreme rays of the feasible set of LP, the LP has an optimal solution for each $b \in B$.

**Problem 2.**

**(a)**

By definition,

$$f(b) \geqslant (y^*)^T b,$$

$$f(b^*) = (y^*)^T b^*.$$

Thus,

$$f(b) - f(b^*) \geqslant (y^*)^T (b - b^*).$$

**(b)**

For any $x \geqslant 0$, let $b = Ax$. Then,

$$c^T x \geqslant f(b) \geqslant f(b^*) + (y^*)^T (b - b^*) = f(b^*) + (y^*)^T Ax - (y^*)^T b^* \tag{1}$$

Since the dual feasible set is nonempty, $f(b^*) > -\infty$.

From Equ. (1),

$$
\begin{aligned}
& c^T x - (y^*)^T A x \geqslant f(b^*) - (y^*)^T b^* > -\infty, \text{ for all } x \geqslant 0 \\
\Longrightarrow\ & c^T - (y^*)^T A \geqslant 0
\end{aligned}
$$

i.e., $y^*$ is a dual feasible solution. Thus, $f(b^*) \geqslant (y^*)^T b^*$.

Take $x = 0$ in Equ. (1), we get $f(b^*) \leqslant (y^*)^T b^*$.

Thus, $f(b^*) = (y^*)^T b^*$, i.e., $y^*$ is a dual optimal solution at $b = b^*$.

**Problem 3.**

**(a)**

Extreme points: $(0, 4)$.

Extreme rays: $(1, -2)$, $(0, 1)$.

**(b)**

Extreme points: none.

Extreme rays: $(1, -2)$, $(-1, 2)$.