STOR 614, Spring 2019
Homework Assignment No. 1

1. Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance from school j to neighborhood i is d_{ij}. Formulate an LP whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. You may ignore the fact that numbers of students must be integer.

2. A company produces and sells two different products. The demand for each product is unlimited, but the company is constrained by cash availability and machine capacity.

   Each unit of the first and second product requires 3 and 4 machine hours respectively. There are 20,000 machine hours available in the current production period. The production costs are \$3 and \$2 per unit of the first and second product, respectively. The selling prices of the first and second product are \$6 and \$5.40 per unit, respectively. The available cash is \$4000; furthermore, 45% of the sales revenues from the first product and 30% of the sales revenues from the second product will be made available to finance operations during the current period.

   (a) Formulate a linear programming problem that aims at maximizing net income subject to the cash availability and machine capacity limitations.

   (b) Solve the problem graphically to obtain an optimal solution.

   (c) Suppose that the company could increase its available machine hours by 2000, after spending \$400 for certain repairs. Should the investment be made?

3. Let $A \in \mathbb{R}^{m \times n}$, with $m < n$. Show that the system of linear equations $Ax = b$ either has no solution at all, or has infinite many solutions.

4. Suppose that A is an $m \times n$ matrix, whose rows are linearly independent. Prove that after a finite number of elementary row operations, the rows of the new matrix are still linearly independent.

5. Suppose that A is an $m \times n$ matrix, whose columns are linearly independent. Prove that after a finite number of elementary row operations, the columns of the new matrix are still linearly independent.

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6. By definition, a set $S \subset \mathbb{R}^n$ is an affine set if

$$x, y \in S, \alpha + \beta = 1 \quad \Rightarrow \quad \alpha x + \beta y \in S.$$

(a) Suppose that S is an affine set. Let $x_0 \in S$. Prove that $S_0 = S - x_0$ is a subspace.

(b) By definition, the dimension of the affine set S is the dimension of the subspace S_0. Find the dimension of the set

$$S = \{x \in \mathbb{R}^5 \mid x_1 - x_2 = 2, x_1 + x_2 - x_3 = 1\}.$$

Justify your answer.