STOR 614, Spring 2019
Homework Assignment No. 4

Note: *You can use Matlab or similar software for computation. For example, you can compute the simplex tableaus using formulas such as* $A_B^{-1}A$ *instead of manually conducting the elementary row operations. You still need to write down the full simplex tableaus if the question asks for an application of the standard (two-phase) simplex algorithm.*

If A *is an* $m \times n$ *matrix in Matlab, and* B *is a vector with* $B_i \in \{1, \cdots, n\}$*, then* $A(:, B)$ *in Matlab gives the submatrix of* A *containing its columns with indices in* B*. See the following Matlab commands for an example.*

```
>> A=[1 2 3 4 5; 6 7 8 9 10; 11 12 13 14 15];
>> B=[1 2 3];
>> A(:, B)
ans =
    1  2  3
    6  7  8
   11 12 13
```

1. Use the two-phase simplex algorithm to solve the following problems. It suffices to find an optimal solution, if one exists.

   (a)
$$\begin{array}{llrcl} \min & z = & x_1 + x_2 & & \\ \text{s.t} & & 2x_1 + x_2 + x_3 & = & 4, \\ & & x_1 + x_2 + 2x_3 & = & 2, \\ & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

   (b)
$$\begin{array}{llrcr} \min & z = & 2x_1 + 3x_2 & & \\ \text{s.t} & & \frac{1}{2}x_1 + \frac{1}{4}x_2 & \leq & 4, \\ & & x_1 + 3x_2 & \geq & 20, \\ & & x_1 + x_2 & = & 10, \\ & & x_1, x_2 & \geq & 0 \end{array}$$

2. Consider the following LP with a single equality constraint:

$$\begin{aligned} \min \ & \sum_{i=1}^{n} c_i x_i \\ \text{s.t.} \ & \sum_{i=1}^{n} a_i x_i = b \\ & x_i \geq 0, \ i = 1, \cdots, n \end{aligned}$$

   (a) Derive a simple test for checking the feasibility of this problem.

   (b) Assuming that the optimal value is finite, develop a simple method for obtaining an optimal solution directly.

3. Consider the standard form polyhedron $P = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\}$. Suppose that the matrix A, of dimensions $m \times n$, has linearly independent rows. Let x, y be two different basic feasible solutions. If we are allowed to move from any basic feasible solution to an adjacent one in a single step, show that we can go from x to y in a finite number of steps.

4. Show that in using the two phase simplex method, if an artificial variable becomes nonbasic, it need never again become basic. Thus, when an artificial variable becomes nonbasic, its column can be eliminated from the tableau.

5. Use the revised simplex method to solve the following LPs.

   (a) Use (x_4, x_6, x_1) as the initial basis.

$$\begin{array}{llrcl}
\min & z = & 2x_1 + 3x_2 - 4x_3 + 3x_4 + x_5 - 4x_6 + 6x_7 & & \\
\text{s.t} & & x_2 + 2x_3 + x_4 + x_5 \qquad - 5x_7 & = & 2, \\
 & & 2x_2 + x_3 \qquad - 2x_5 + x_6 \qquad & = & 1, \\
 & & x_1 + x_2 - 2x_3 \qquad + x_5 + \qquad 3x_7 & = & 3, \\
 & & x & \geq & 0
\end{array}$$

   (b) Use (x_1, x_2, x_6) as the initial basis.

$$\begin{array}{llrcl}
\min & z = & 14x_1 - 19x_2 \quad + 21x_4 + 52x_5 & & \\
\text{s.t} & & x_1 \qquad - x_4 + x_5 + x_6 & = & 3, \\
 & & x_1 + x_2 \qquad - x_4 + 3x_5 \qquad & = & 4, \\
 & & x_1 + x_2 + x_3 - 3x_4 \qquad + x_6 & = & 6, \\
 & & x & \geq & 0
\end{array}$$