Linear Algebra

1. Vector

   - Scalar Multiplication: Multiplying the vector by a real number
     If the vector x is represented by an arrow, then $\forall t \neq 0$, the vector tx is represented by an arrow in the same direction if $t > 0$, and in the opposite direction if $t < 0$.
     The length of the arrow tx is $|t|$ times the length of the arrow x
   - x and y are parallel if $y = tx$ for some $t \neq 0$

2. Fields

   (a) Definition of a field
   A field F is a set on which two operation addition and multiplication are defined so that $\forall x, y \in F$, $\exists$ unique elements $x + y \in F$ and $xy \in F$. The following conditions hold for all elements $a, b, c \in F$

   i. Commutativity of addition and multiplication

   ii. Associativity of addition and multiplication

   iii. Identity elements for addition and multiplication.
   $\exists$ distinct elements 0 and 1 in F such that

   $$0 + a = a$$

   $$1 \cdot a = a$$

   iv. Additive Inverse: $\forall a \in F$, $\exists c \in F$ such that

   $$a + c = 0$$

   Multiplicative Inverse: $\forall$ nonzero $b \in F$, $\exists d \in F$ such that

   $$b \cdot d = 1$$

   v. $a \cdot (b + c) = a \cdot b + a \cdot c$

   (b) Cancellation Law
   $\forall a, b, c \in$ field F, the following statements hold
   (a) If $a + b = c + b$, then $a = c$
   (b) If $a \cdot b = c \cdot d$, then $a = b$