Kinematic Equations
Differential Driven Robot

$$v = \frac{v_R + v_L}{2}$$
$$\omega = \frac{v_R - v_L}{B}$$
$$R = B \cdot \frac{(v_R + v_L)}{(v_R - v_L)}$$
$$v = \omega \cdot R$$

Kinematic equations for a differential driven robot

Use B = 0.5

- Baseline between wheels, B
- Velocity left wheel, v_L
- Velocity right wheel, v_R
- Vehicle angluar velocity, ω
- Vehicle forward velocity, v
- Vehicle turn radius, R (can be negative)

Time Continuous Kinematic Model
Differential Driven Robot

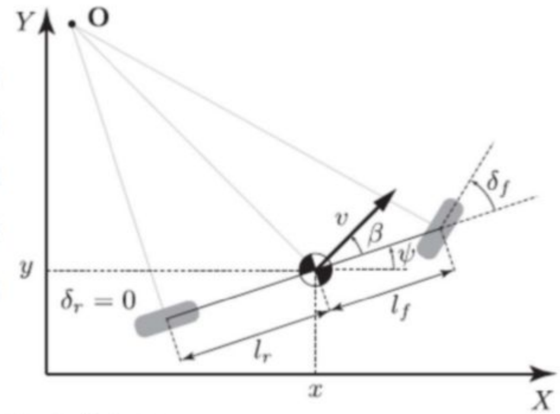
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_G = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Time continous kinematic model

- Differential driven robot (wheelchair)
- Vehicle orientation, θ
- Vehicle position, (x, y)
- Vehicle change in position, $(\dot{x}, \dot{y})$
- Forward velocity, v
- Turning rate, ω

Kinematic Bicycle Model

$$\dot{x} = v\cos(\psi + \beta) \quad (1a)$$
$$\dot{y} = v\sin(\psi + \beta) \quad (1b)$$
$$\dot{\psi} = \frac{v}{l_r}\sin(\beta) \quad (1c)$$
$$\dot{v} = a \quad (1d)$$
$$\beta = \tan^{-1}\left(\frac{l_r}{l_f + l_r}\tan(\delta_f)\right) \quad (1e)$$



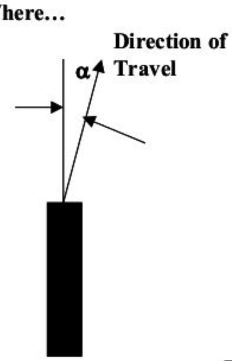
where x and y are the coordinates of the center of mass in an inertial frame (X, Y). ψ is the inertial heading and v is the speed of the vehicle. l_f and l_r represent the distance from the center of the mass of the vehicle to the front and rear axles, respectively. β is the angle of the current velocity of the center of mass with respect to the longitudinal axis of the car. a is the acceleration of the center of mass in the same direction as the velocity. The control inputs are the front and rear steering angles δ_f, and a. Since in most vehicles the rear wheels cannot be steered, we assume $\delta_r = 0$.

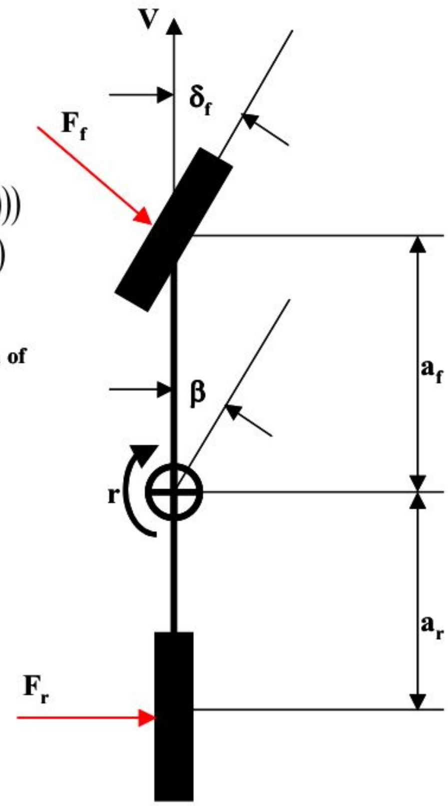
$l_f = 1.07$
$l_r = 0.936$

2D Dynamic Bicycle Model

$$\alpha_f = (\beta - \delta) + \operatorname{atan}\left(\frac{a_f r\cos\beta}{V}\right)$$
$$\alpha_r = \beta - \operatorname{atan}\left(\frac{a_r r\cos\beta}{V}\right)$$
$$F_f = D_f\left(\sin\left(C_f \operatorname{atan}\left(E_f \operatorname{atan}(\alpha_f B_f) + (\alpha_f B_f)(1 - E_f)\right)\right)\right)$$
$$F_r = D_r\left(\sin\left(C_r \operatorname{atan}\left(E_r \operatorname{atan}(\alpha_r B_r) + (\alpha_r B_r)(1 - E_r)\right)\right)\right)$$
$$\dot{\beta} = \frac{F_f + F_r}{mV} - r$$
$$\dot{r} = \frac{(a_f F_f - a_r F_r)\cos(\beta)}{I_z}$$
$$\dot{\theta} = r$$
$$x = \int V\cos(\beta + \theta)dt$$
$$y = \int V\sin(\beta + \theta)dt$$

Where…





Variables

α_f = slip angles of front wheels

α_r = slip angles of rear wheels

a_f = distance from center of gravity to front axle

a_r = distance from center of gravity to rear axle

β = slip angle of the center of gravity

F_f = cornering force on the front wheels

F_r = cornering force on the rear wheels

D, C, E, B = Magic tire formula variables

δ = steering angle

V = vehicle speed

I_z = mass moment of inertia about the center of gravity z axis

r = angular velocity about the CG

θ = angle of orientation

m = mass of the vehicle

x = vehicle CG's global x axis position

y = vehicle CG's global y axis position

** CG is Center of Gravity

Variables

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Bf = 0.242;
Cf = 1.352;
Df = 2751.69;
Ef = -0.392;
Br = 0.24;
Cr = 1.29
Dr = 3113.08;
Er =  0.507;
af = 1.07;              %CG to front axle m
ar = 0.936;             %CG to rear axle m

Ixx = 552.718;          %Moment of Inertia kg*m^2

Mass = 645;             %Mass of platform kg
fzf = 2951.8;           %Force from weight front N
fzr = 3373.49;          %Force from weight rear N
length = 2.81;          %Length of vehicle m
g = 9.807;              %Acceleration due to gravity m/s^2
b = 1.107;              %Height m
a = 1.545;              %Width m
```