SE 498

Assignment 3

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1. Only linear acceleration -> displacement p(t):

- In theory, double integration of the linear acceleration will provide you p(t).

$$\mathbf{v}_I = \int \mathbf{a}_I,$$
$$\mathbf{r}_I = \iint \mathbf{a}_I$$

- In practice, there are two more points to mention.

First, intuitively, we need to use the discrete format for integration.

$$\mathbf{v}_I[k+1] = \mathbf{v}_I[k] + T\mathbf{a}_I[k]$$
$$\mathbf{r}_I[k+1] = \mathbf{r}_I[k] + T\mathbf{v}_I[k].$$

Second, however, usually we cannot obtain an accurate displacement result by simply using the method above. The main reason is that the double integration result has a high sensibility to noise and the drift of raw linear acceleration data, which means a very little measurement error in linear acceleration will accumulate to be a large error in displacement as time goes by. Without nicer rate gyros or an external reference like GPS, accurate dead-reckoning is usually not possible.

Thus, a better way to deal with only raw linear acceleration data is to use IIR filters designed using polynomial regression with an exponentially fading memory. That would give a smoothed acceleration data. Finally, with the refined data, we can do double integration to get the displacement.

Nevertheless, the accuracy of this kind of displacement will still drop dramatically as time goes by. Thus, a really good solution is to combine linear acceleration with other sensors and use data fusion.

2.

a) State space equation:

x_dot = [0 1; 0 0] * x + [0; 1] * u

y = [c1 c2] * x   (set C = [c1 c2], the specific coefficients depend on which value you are observing.)

Observability matrix of the system: O = [C; CA] = [c1 c2; 0 c1]

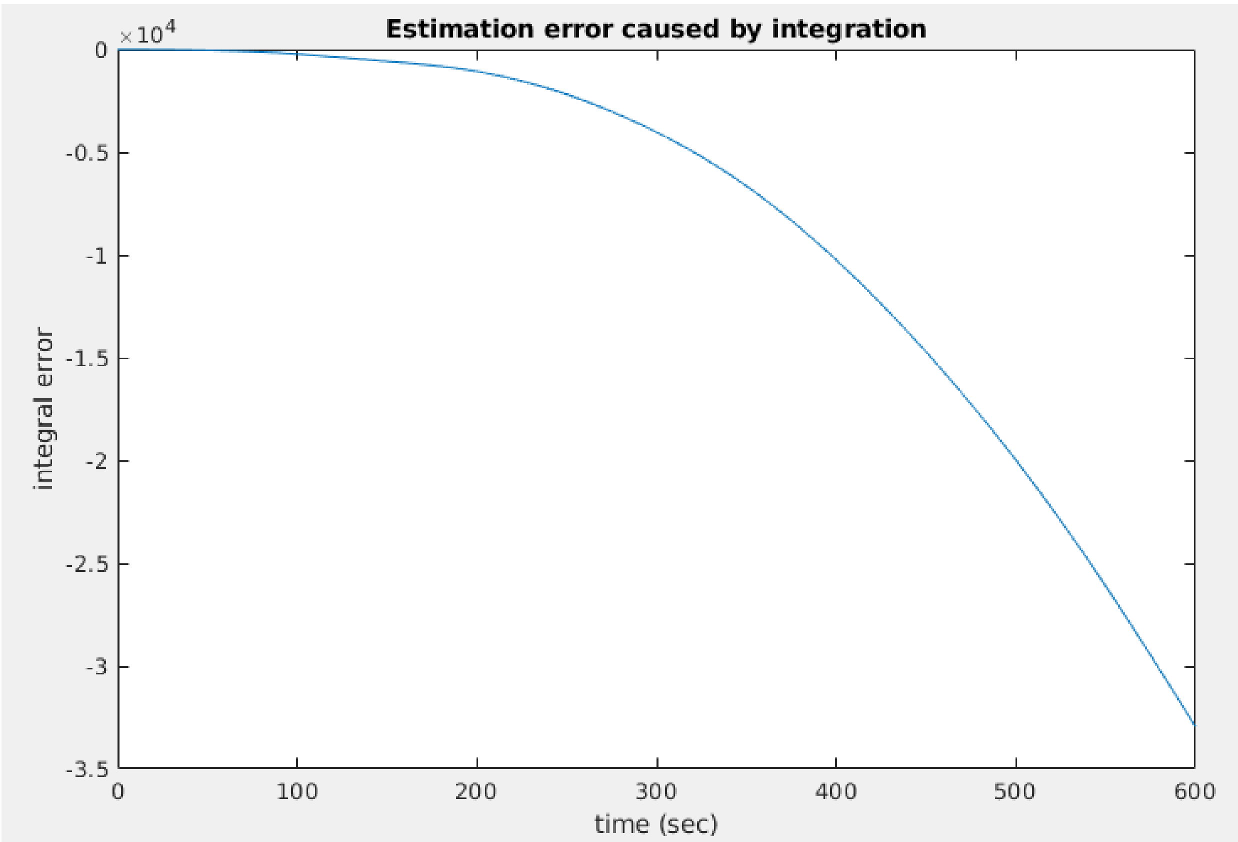
When c1 != 0, rank(O) = 2 thus it is observable; When c1 = 0, rank(O) < 2, thus the system is not observable no matter what c2 is.

b) According to a), we need to measure p(t) to make the system observable for observing all the system states for localization. However, as talked in question 1, using the raw linear acceleration data of IMU to give the double integration for the displacement measurement results is not accurate, because of the high sensitivity to the noise and drifts, thus a single IMU is not sufficient for localization.

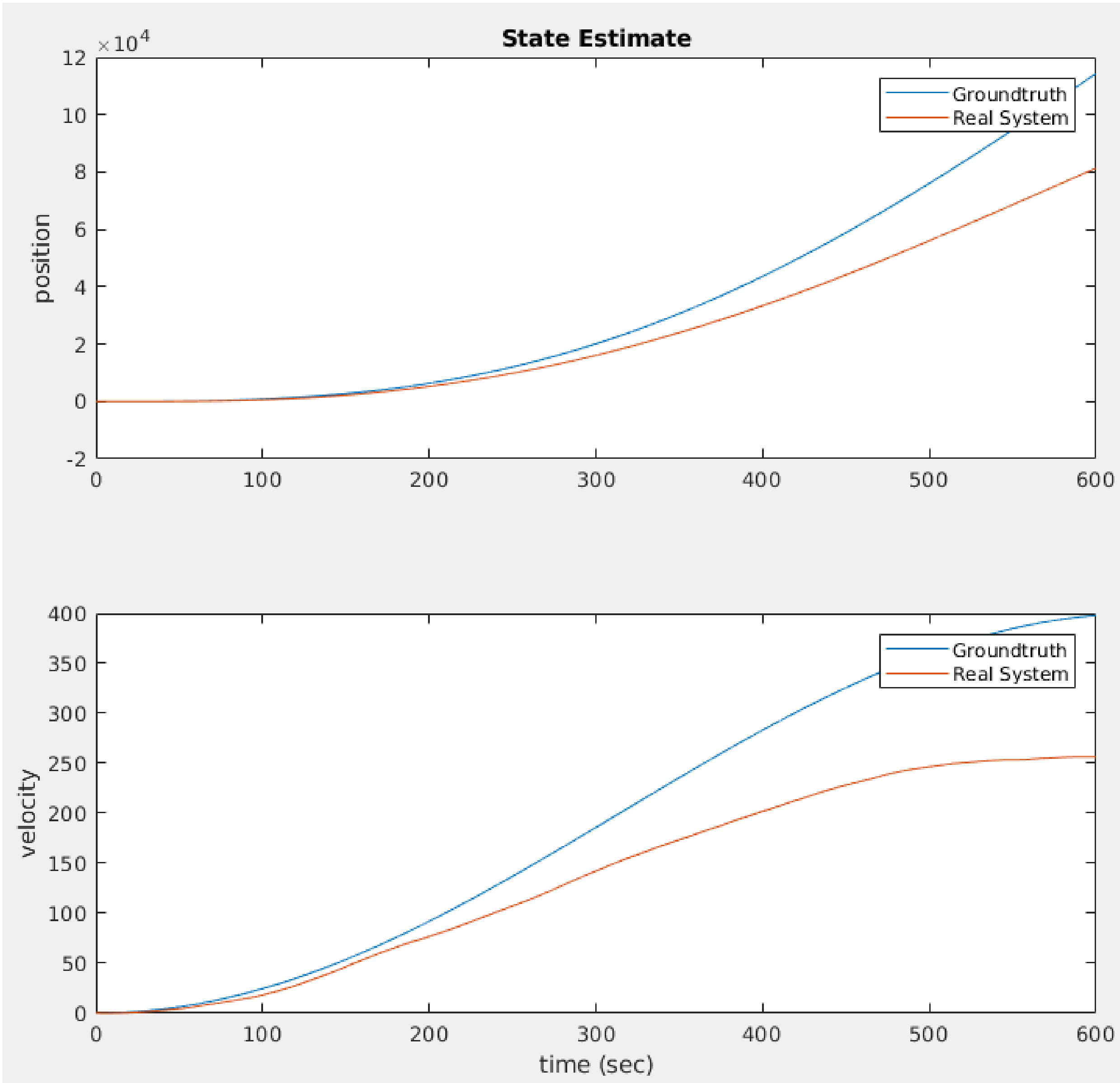
3.

Every Simulation can be different because of the Random Variable of Gaussian noise, however, I picked up a typical one to show the divergence of error and state estimates.

Estimation Error diverges as time goes by:



State Estimates:



4.

State vector: $x(t) = \begin{bmatrix} p(t) \\ v(t) \\ b(t) \end{bmatrix}$, Input Vector: $u(t) = 0$

Process Noise: $w \sim N(0, \sigma_w^2 = 2.1 * 10^{-6})$;

Measurement Noise for IMU: $z \sim N(0, \sigma_z^2 = 0.0034)$;

Measurement Noise for GPS: $g \sim N(0,5)$;

Time interval for IMU update: $dt = 0.01s$

Time interval for GPS update: $T = 1s$

Position update: p(t) = p(t-1) + v(t-1) * T+ (u(t)-b(t-1)) * T^2/2

Velocity update: v(t) = v(t-1) + (u(t)-b(t-1)) * T

Acceleration Bias: b(t) = b(t-1) + z

State Matrix: $A = \begin{bmatrix} 1 & dt & -\frac{T^2}{2} \\ 0 & 1 & -T \\ 0 & 0 & 1 \end{bmatrix}$

Input Matrix: $B = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 0 \end{bmatrix}$

Measurement Matrix: C = $[1 \quad 0 \quad 0]$

System Covariance Matrix: Q = $\begin{bmatrix} \frac{T^4}{4}\sigma_g^2 & \frac{T^3}{2}\sigma_g^2 & 0 \\ \frac{T^3}{2}\sigma_g^2 & T^2\sigma_g^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Measurement Covariance Matrix: R=$[\sigma_z^2]$

Kalman Filter Equations:

Prediction (100Hz): $x_k' = Ax_{k-1} + Bu_{k-1}, P_k' = AP_{k-1}A^T + Q$

Update (1Hz):$K_k = P_k C^T (CP_k' C^T + R)^{-1}, x_k = x_k' + K_k(y_k - Cx_k'), P_k = (I - K_k C)P_k'$

Rewrite to State Space representation: $\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} w + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z\,, y = [1 \quad 0 \quad 0]x$

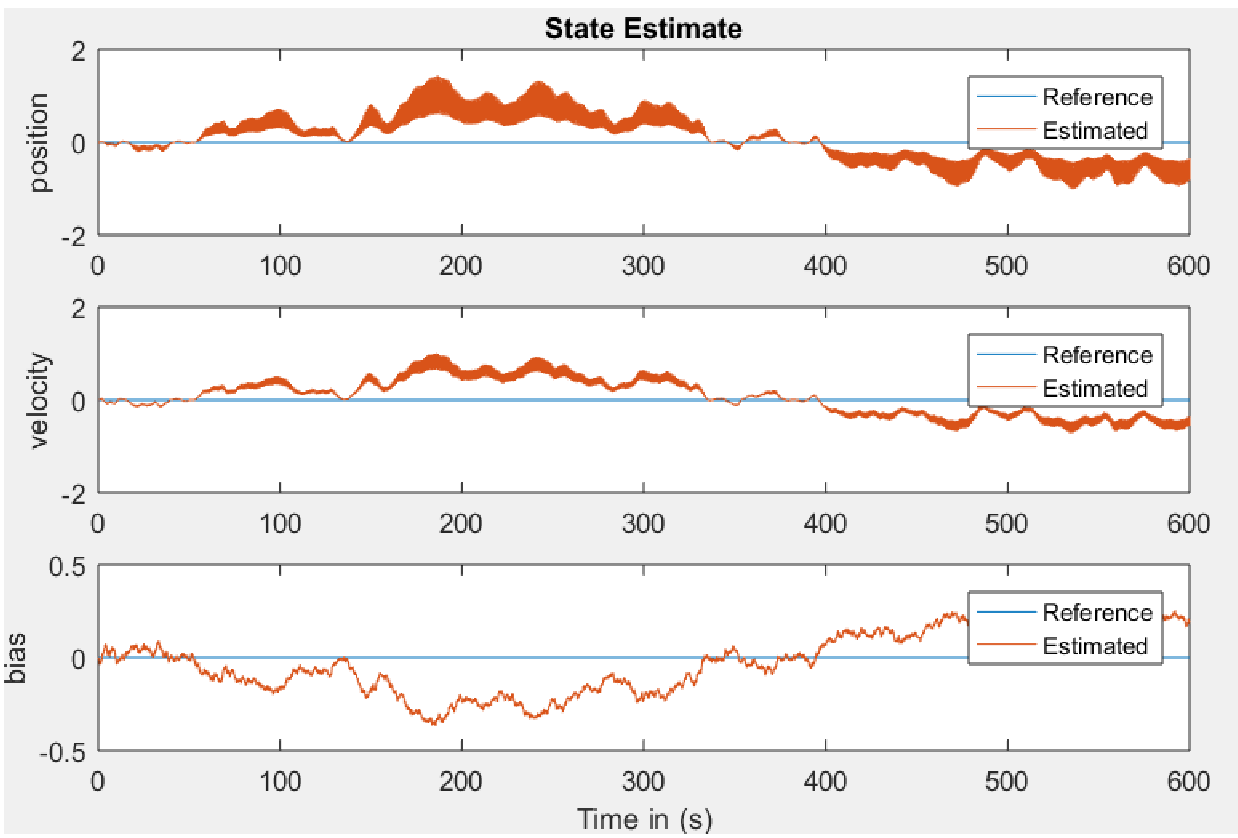
Observability Matrix: $M_{obs} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & dt & -\frac{dt^2}{2} \\ 1 + dt - \frac{dt^2}{2} & 2dt & -2dt^2 \end{bmatrix}$

Yes, the system is completely observable since the observability matrix is of full rank, thus IMU + GPS can be sufficient for localization.

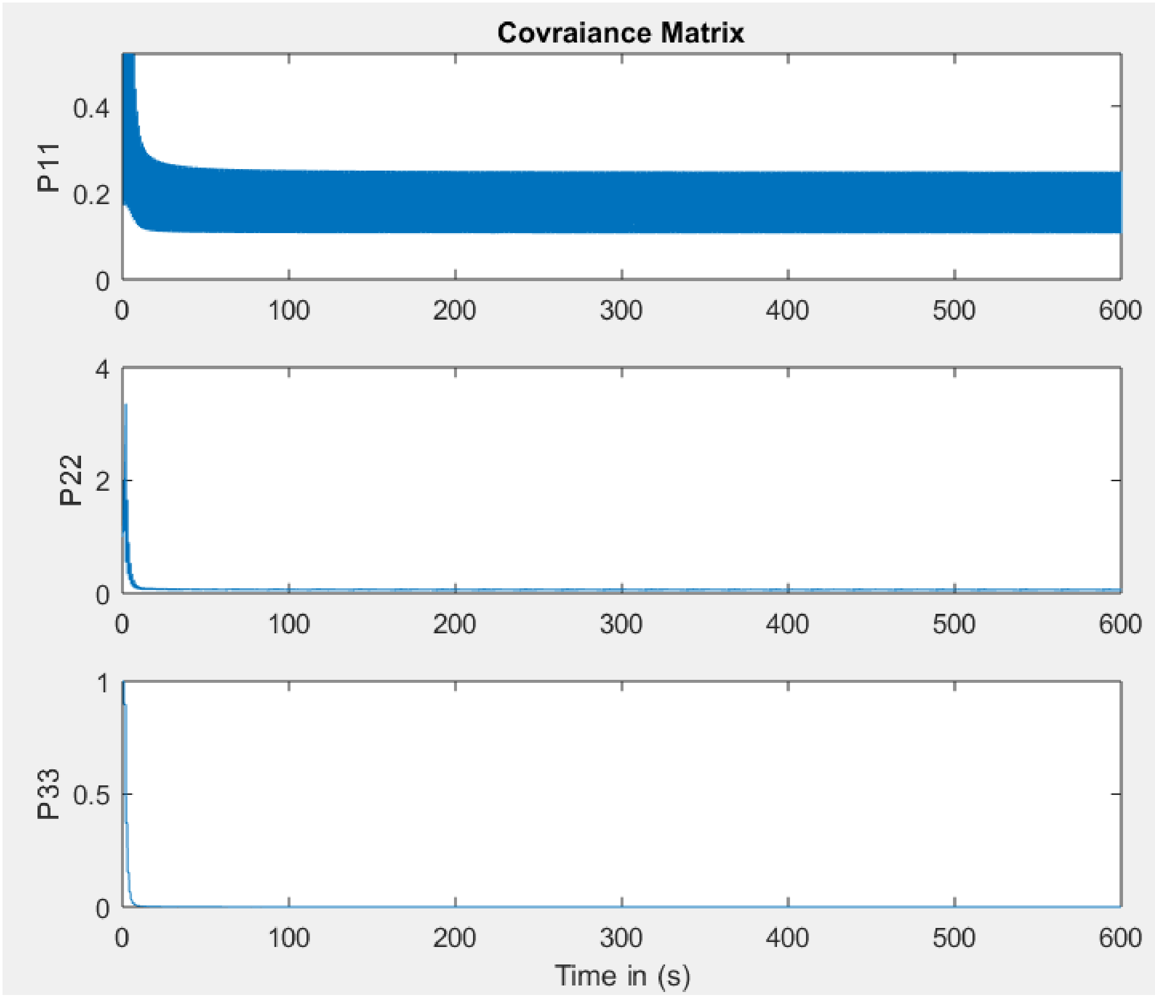
5. Sdf

- When input is 0 with process noise and measurement noise in IMU and GPS:

a)

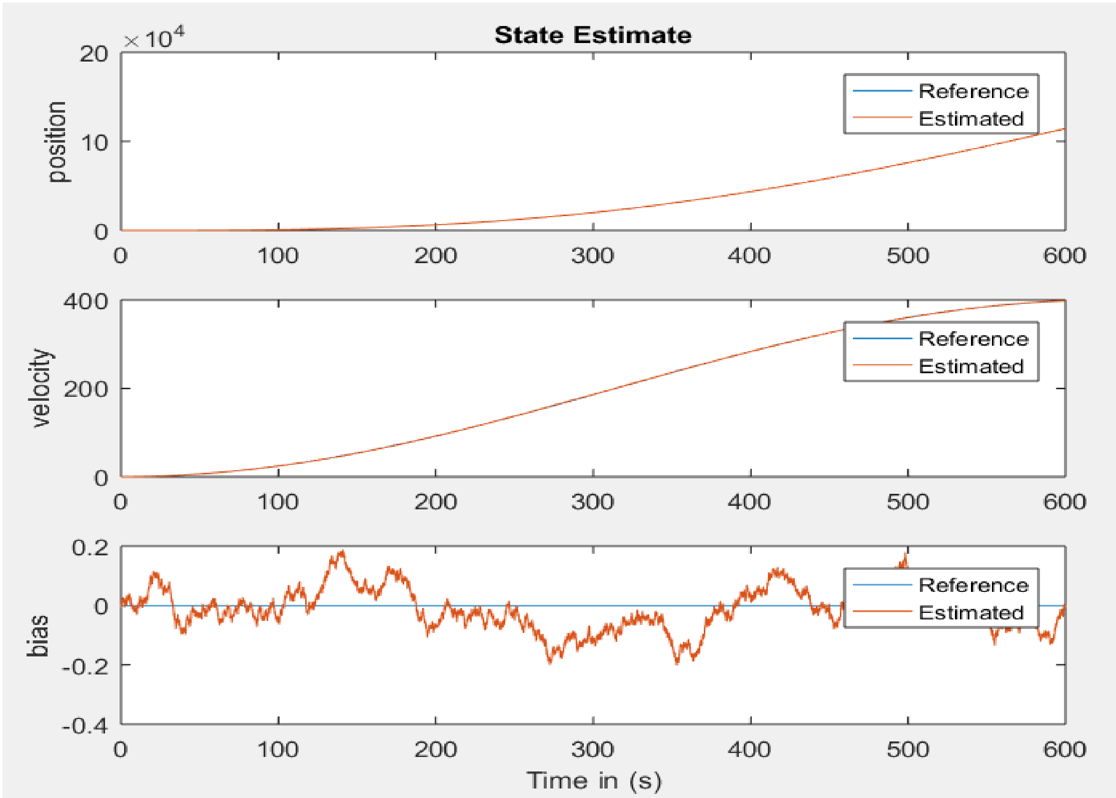


b)

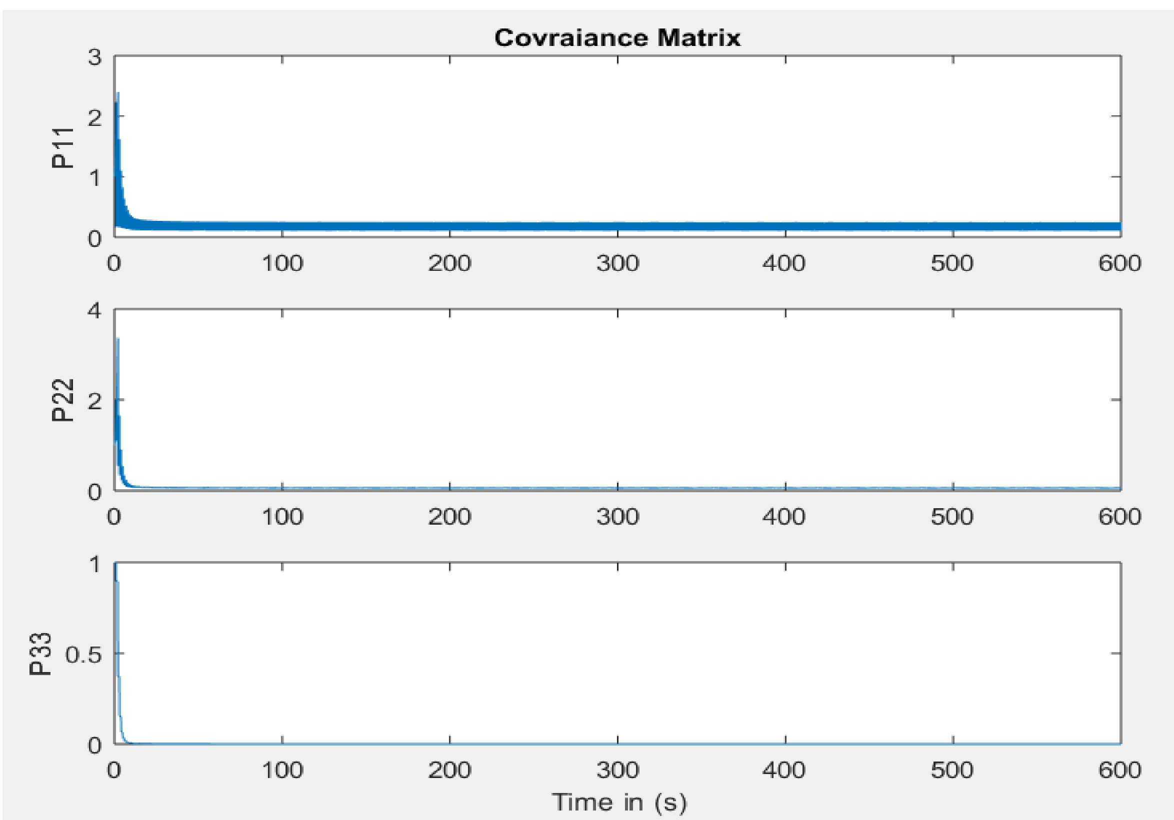


- When input is a(t) = sin(0.005*t) with process noise and measurement noise in IMU and GPS:

a)


State Estimate
position
$\times 10^4$
velocity
bias
Reference
Estimated
Time in (s)

b)


Covraiance Matrix
P11
P22
P33
Time in (s)

6.

Math properties of covariance matrix:

1. the diagonal elements should be the variance for corresponding item thus should be all positive;

2. Cov^T = Cov, it should be a symmetrical matrix.

My covariance matrix makes sense for these properties. And the diagonal elements, i.e. variance of noise, converge to their positive constant (which should be just the variance we set for the Gaussian noise) quickly and stay there.

END