# SE498 HW3 – Kalman filter (SP19)

( / 100 pts)

IMU (Inertial Measurement unit) is one of the most common approaches in the dead reckoning of mobile robots. We have discussed a lot about how a simple IMU-based localization system would fail over time. This homework is a close-up examination on why this is the case and how we can improve it.

1. Say, given an IMU sensor that only outputs the linear acceleration of a mobile robot, how can you obtain its displacement information p(t)? - (5 pts)

2. Treat your robot as a simple point-mass system with only one degree of freedom (only move forward or backward, you can control the actuators' acceleration), build a system model that acts according to the accelerometer reading (i.e. $u(t) = \mathrm{a\_measured(t)}$)
   a) Write down its state space representation (with the state being: $\mathrm{x = [p(t);\ v(t)]}$) and obtain the observability matrix of the system. - (15 pts)

$$\dot{x} = \mathrm{Ax + Bu :}$$
$$\mathrm{y\ = Cx :}$$

   b) Now, can you explain why having an IMU senor is not sufficient for localization? - (5 pts)

3. Now assume: $a\_measured(t) = \mathrm{a(t)\ +\ measurement\ noise(t)}$. Use the sample Matlab script and show that integral error (of robot position) diverges overtime under the influence of noise (integrate the time forward for 600 seconds). - (20 pts)
   System input: $\mathrm{a(t) = \sin(0.005 * t)}$
   Process noise variance: ${\sigma_\omega}^2 = \ 2.1 * 10^{-6}\ \mathrm{m^2}/_{s^4}$
   Measurement noise variance: $\sigma_z^2 = \ 3.4 * 10^{-3}\ \mathrm{m^2}/_{s^4}$
   ** Assuming there is no cross correlation between measurement and process noise.*
   ** All noises are assumed to be mean zero Gaussian noise.*
   ** You may use the discrete propagation in your implementation:*
   $\mathrm{x(t+1)} = expm(A * dt) * x(t) + B * u(t) * dt;$ or the c2d() function in Matlab

4. If, besides the IMU sensor (which updates at 100Hz), you are also given a GPS sensor that updates with a 1Hz frequency (with a measurement noise variance: $\sigma_g^2 = 5m^2$ ) can you fuse the two sensors' readings with a basic Kalman filter (Rewrite the state space representation with $\mathrm{x = [p(t);\ v(t); b(t)]}$ , and answer the question using the updated observability matrix) and why? - (10 pts)
   System input: $\mathrm{a(t) = 0}$
   ** b(t) is the bias, and you can use ε(t) as the "rate of bias change"*

5. Implement the Kalman filter algorithm.
   a) Plot the system’s position and velocity estimation against the ground truth, for 600 seconds. - (30 pts)

   b) Also plot the diagonal elements of the covariance matrix, for 600 seconds. (Use the same assumption from problem 3) - (10 pts)

6. What are the mathematical properties (characteristic) of the covariance matrix? Does your covariance matrix make sense? How does it behave? - (5 pts)

Reference reading: http://techteach.no/fag/seky3322/0708/kalmanfilter/kalmanfilter.pdf