Team Control Number

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T1 ________________ F1 ________________

T2 ________________ F2 ________________

T3 ________________ F3 ________________

T4 ________________ F4 ________________

Problem Chosen

B

2012 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Summary

With the increasing popular of river rafting, more times of trips is required, and that means it will increase the possibility of meeting between different travelers in the river. Obviously this is not conducive to the the travelers to enjoy the wild experience. In fact, there is a contradiction between the increasing number of trips and the need of field experience. Classifying the traveling by different duration and making a reasonable arrangement, a balance of these two contradictory can be achieved.

In order to get the best time arrangements for boat launching, we mainly use the simulation by computer program, and establish a simulation model. Firstly, we make a great simplification of the actual issue, classifying the duration into 3 kinds (which are 7 nights, 8 nights and 15 nights), and all the kinds of traveling duration have the same launch rate, simulating by the C program. But it doesn't rule out the possibility of boat passing (that is the contact between groups). Then get a preliminary simulation result: the total groups it can launch every year for the six months is 188, and the daily maximum carrying capacity of the river about 13. Secondly, we expand this simulation model, letting the number of the input values, duration of different ways of travel, can be arbitrary, making the possibility that launch rate of different traveling ways can be arbitrarily assigned and different (not the same).And consider the possibility of boat passing, that is the possibility is zero ,only according to the average speed of different boats. Let different ways of travel launch by it's own launch rate, ruling out the boats that can make boat passing. We write this program with Mathematica. According to the actual situation, the input value of the program can have more than 8000 kinds. Through the evaluation of program results we have found the best solution. And the results is: 200~300 boats each year and 35 boats of the daily maximum carrying capacity of the river. At last, we have transformed the result into a monthly class launching date schedule.

A Simulation of the Boat Launching

Introduction

Our models are based on the solution of the Grand Canyon. Big long river is a river which is about $225 miles$, and if visitors want to enjoy scenic views and exciting white water rapids, they have to travel down the river.[1] In order to let visitors have a good time, we build a model to find a best way to travel. We collect some data of the Grand Canyon trip to build a model.

In our model we divide boats into three parts: long time trip, not so long time trip and short time trip. In long time trip, visitors will travel for 13 to 17 nights; in not so long time trip, visitors will travel for 8 to 10 nights; and in short time trip, visitors will stay for 6 to 8 nights. Then we take 15 as long time trip time, and take 9 as not so long time trip time and 7 as short time trip time. We arrange the order of three trip on this river to make the model optimization.

Assumptions

- Every group must camp every night, and the camper site must different.
- Every group can pass camper site.
- Every group must stay in their boat and can't change their boat, every boat keep a speed without change.
- People in a group must stay together and use one traffic tool.
- The number of people in every season can't change.
- Each group may meet others on the way, but it is not likely.

The setting of variables

L	the length of the river
y	the number of the camper sites
T_C	the period of one travel
t_{LT}, l_L	the nights and daily trips of long time trip
t_{NT}, l_N	the nights and daily trips of not so long time trip
t_{ST}, l_S	the time and daily trip of shot time trip
k	the long time trip will begin at k day

Models

1.The analysis of the problem

The key problem of this question is how to optimize the number of boat on the river. And find the best way to arrange the boat on the river to make the visitor enjoy the view and attract more tourist.

In fact, we must consider many things as the prerequisite. There are many things will influence the results. Firstly the trip only can come true in a six month period because the rest year is too cold for river trips. Secondly, visiters can't beyond the maximum carrying capacity of this river during these six months. Every group must camp on the river, but no two sets of campers can occupy the same place at the same time. Each group contact with other groups on the river with minimal even they don't contact with each other. And lastly, we will use camper sites fully.

And here, we divide visitors into three parts. The speed of oar-powered rubber rafts is $4mph$, and the speed of motorized boats is $8mph$. According to our calculation, we use oar-powered rubber rafts to have along time trip, and use motorized boats to have a short time trip, during the not so long time trip, take oar-powered rubber rafts or motorized boats is reasonable.

There are y camp sites on the Big Long River, and distributed fairly uniformly throughout the river corridor. In order to let every group arrive at camp site before night come, and that no two set of camper can occupy the same site at the same time. So the number of the group will not more than y, that will ensure every group has a rest place. The earliest boat will exit the river at Final Exit at the day time. So the maximum carrying capacity of the river will be $y+1$. [2]The longest trip on the river is $18night$, so the maximum camp site on the river is 18, and the maximum carrying capacity of the river is 19. The camp site distributed uniformly throughout the river corridor, and the length of the river is $L = 225miles$, so the distance between any two site will not more than $\frac{L}{y} = 12.5miles$. The oar-powered rubber rafts travel between two sites will stay $\frac{12.5miles}{4mph} \approx 3h$, and the motorized boats will stay for $\frac{12.5miles}{8mph} \approx 1.56h$. We can arrange the order of the three kinds of boats to achieve the majorization.

We can let three boats go in a same day, and don't let these three boats stay at one site. If we increase the number to 4 or more, then the leave time between two boats alongside at least will be $2\times1.56h+3h=6.12h$, so it's not better for the arrangement.

In order to find a better way to arrange the time of the boats leave, we analyze the problem in a period. We arrange the short time trip primarily, so it will have few boats on the river. From the first day, let a boat go per 15 day. So the saturation of the river is 1. Then arrange the not so long time trip. From the first day, let a boat go per 9 days. From the conclusion all above, we can know that this kind of boat will never meet the long time trip boat, and they will never stay in a same camp site at the same time. So now the saturation of the river is 2. Lastly, we will arrange the short time trip boat. And let it set up per week, and the same situation, it will never contact with other group on the river and the camp site. And now the saturation of the river is 3.

So in a large period, any group will never contact with each other on the river or at the camp site.

If several boats will be set up in a same day, we will let motorized boat go first is that the order of the boats leave is the short time trip boats go first, then is the not so long time trip boats, and the long time trip boats in the final time. In this way, the possibility to meet on the river will be minimum. If the last boat can't arrive at Final Exit in the last day, this boat will not allow be set up. So after a period, there will be $N=\frac{T}{t_{ST}}+\frac{T}{t_{NT}}+\frac{T}{t_{LT}}=57$ groups attend the river trip. After the first period, let we arrange the second period. We can control boats to arrange the camp site, that will meet camp of two groups' needs. According to this way, the long time trip boats will arrival at the number k day, and it will meets a motorized boat everyday. According to the following equations we get that k can minimum take 8.

$$\begin{cases} l_i=\frac{L}{t_i}\left(i=LT,NT,ST\right) \\ tl_{ST}>\left(t+k\right)l_{LT} \\ tl_{ST}<L \\ \left(t+k\right)l_{LT}<L \end{cases}$$

So at the beginning of the 8^{th} day, the first boat will meet the other boats. In order to have the most groups, we take the 5 as the period. So during 15days, it will have 3 big periods. And during 180 days, it will has 171 groups visit the Big Long River. Base on the basic assumption, we only consider the conflict of the single period and double period. We analyze the three period conflict by the following schedule.[3]

2.The preliminary arrangement

In order to get a reasonable arrangement about the boats, we analyze the problem by using C program. From the result of the model, we find that the maximum carrying capacity of the river is 9. But it's far away from the maximum carrying capacity in assumption. So we revise our premise, and assume there will have 5 periods in $15\ nights$.

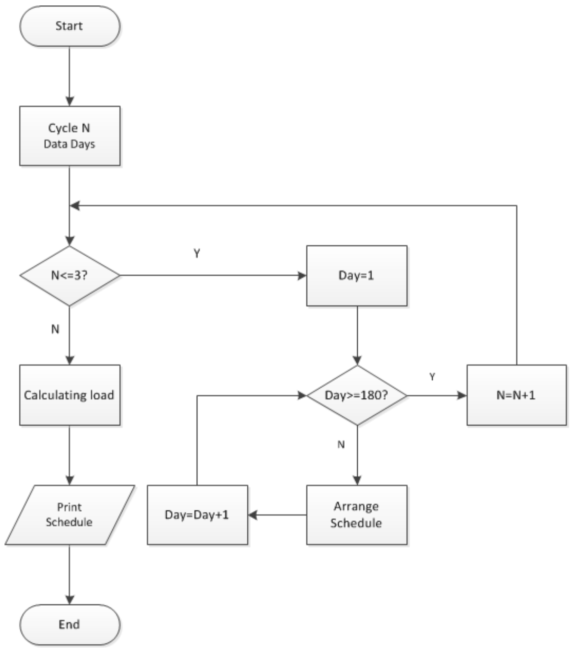

Fig.1 Program flow chart

After revise the premise, we get the maximum carrying capacity will be 13, now it's closer to the maximum carrying capacity. And in the maximum carrying capacity, during$18.180 nights$ there will be 236 groups on this river.

3.Schedule optimization analysis

According to the schedule, we find that set up time of the long time trip and not so long time trip boat are same. The long time trip time is 15 days and the not so long time trip's is 9 days, so 3 is just their common divisor. During 15 days, there are five times set up of not so long time trip boat. And $t_{LT} - t_{NT} = 6 nights$, so the long time boat will not be catch up by not long time boat in the $7^{th}\ nights$. But two of the not long time boats set up during the first six days will catch the long time boats up. In the same day not long time boats set up first, so there is only one not long time boat will catch the long time boats which set up on the third day.

In the same way. The period of long time boats and short time boats are 9 days and 7 days. If the departure time of the not long time boats is 2 to 3 days earlier than short time boats, and the short time boats will never catch up. If the time is less than 2 days, the not long time boats will be catched up. Under the statistics, there are 77 short time boats catch the not long time boats in total, the number of the not long time boats is 57, so there are $\frac{77}{57} \approx 1.35$ short time boats will catch one not long time boat, so we can decrease the number of the short time boats.

Because $t_{LT} - t_{ST} = 8 nights$, so in the early days there are at least 3 short time boats catch the long time boats up. The frequency of the short time boats' set up is higher, so in the later period, there will be a mount of boats catch the long time boats up. So we should reduce the frequencies of the short time boats.

In conclusion, we can reduce the frequencies of the boats, so any two boats will contact with each other with minimum . Under certify the largest number every year, reduce the number of short time boats as many as possible, and any two short time boats will not set up continuously. After optimizing, there are 188 boats set up in total, and the maximum carrying capacity of the river is 13.

4.Model expand

We simplify the problem prodigious in front of the process, and divide the trip time into three parts. Then use three determined time to stand for these three kinds of trip. Assuming the frequency of the boats set up is unchanged, we simulate this problem by using C program, and get a preliminary result. The carrying capacity of the river and the probability of the boats contact with each other on the river are closely related with the time and the frequency of the boats set up. To contain more visitors on this river is our goal, but we also should consider visitors with minimum contact with other groups on the river. We only make a simple plan for the boats set up time of different kinds of boats, but it doesn't contain the probability of visitors meeting on the river. In order to get a more reasonable project about the time, increasing the carrying capacity and make full use of camp sites as can as possible, and reduce the probability of visitors meeting on the river, we make a simulation program by Mathematica ,to make the simulation process more reasonable.

We don't consider the traffic tools, the only difference in travel is the travel time or the average speed.

It will never happen on the river that one boat catch up the others, if we calculate the average only, the probability of boats contact with other groups with minimum.

The frequency of boats set up can different between different kinds of boats, the night of the visitors on the river also can be different.

The best way to arrange the groups on the river and don't need to consider the number of groups is stay only one group on the river, that will make the visitors enjoy the scenic view and exciting white rapids fully. But we have to take the maximum carrying capacity of the river into consideration, so the idea above is stupid. In order to meet more visitors' needs, we will use the model that different kinds of groups set up at the same time. Because different kinds of trip have different average speed. If they start at the same time, on the way to the Final Exit they will never contact with each other. In order to make the probability of the boats meeting each other minimum, the phenomenon of one catch up another will never happen. After consider all aspects factors, we simulate the process of boats set up on the Big Long River by Mathematica program, modify the different input values and analyze for the simulation result to find the best scheduling way.

4.1 The Simulation Program

Input value of the program

1. the total number of the schedule day;
2. the travel time and frequency of the boats set up;
3. the schedule in the first day of different trips (1 sand for set up, and 0 stand for don't set up);

Output value of the program

1. The total number of the boats during the whole time;
2. The schedule of the set up of the different trips boats everyday, and the total number of the boats on the river everyday;

```
process[180, input, initial]

374
```

<table>
<tr><th>Day</th><th>6|2</th><th>7|2</th><th>8|3</th><th>9|3</th><th>10|3</th><th>11|4</th><th>12|4</th><th>13|4</th><th>14|5</th><th>15|5</th><th>16|5</th><th>17|6</th><th>18|6</th><th>Total boats</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>13</td></tr>
<tr><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>14</td></tr>
<tr><td>3</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>17</td></tr>
<tr><td>4</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>20</td></tr>
<tr><td>5</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>23</td></tr>
<tr><td>6</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>26</td></tr>
<tr><td>7</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>25</td></tr>
<tr><td>8</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>26</td></tr>
<tr><td>9</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>26</td></tr>
<tr><td>10</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>28</td></tr>
<tr><td>11</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>27</td></tr>
<tr><td>12</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>31</td></tr>
<tr><td>13</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
<tr><td>14</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
<tr><td>15</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>35</td></tr>
<tr><td>16</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>36</td></tr>
<tr><td>17</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>35</td></tr>
<tr><td>18</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>37</td></tr>
<tr><td>19</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>28</td></tr>
<tr><td>20</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>31</td></tr>
<tr><td>21</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
</table>

Fig.2 Table Schedule (Part)

The process of simulation

1. Build the list above to store the state of all boats on the river, the element in the list keep consistent with the boats on the river, it sands for every boat ,and store the total days and days the boat has already go. The front of the list stands for the upstream of the river, and the end of the list stands for the downstream of the river. Once it adds one day, the day for the boat has already gone will increase one day, when the time that a boat have already go on the river is equal to the time that it

should stay, this element will be deleted.

2. As the time goes on, all kinds of boats will get the corresponding right rely on their frequencies of set up. But the boat will not go when the speed of this boat is higher than it's on the upstream of the river. We use the number 1 stands for a boat will go. Then judgment and calculation are as the same way everyday.

3. If the boat can't set up during the whole season, and don't let it go.

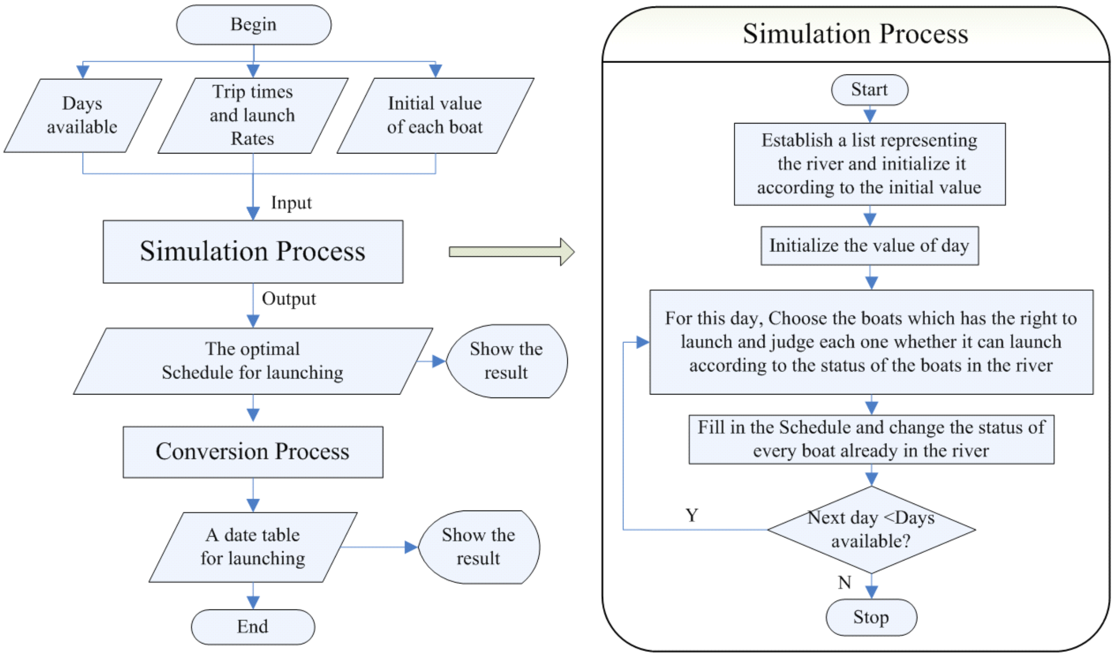

Fig.3 The flow diagram of the simulation

4.2.The result analyze

With minimum contact with other groups on the river, we simulate for the data, and find if we want to make the group number to a maximum, we have to simulate all possibilities. We get 8000 kinds of possible results through the calculation. But we can't simulate all possibles in a short time, we can choose some possible results and simulate, at last we get some right conclusions.

Only consider the total number of groups is not reasonable that will led the final results very unreasonable. If we assume the frequency of every group is 1. We can get a large total number of groups by considering the maximum carrying capacity. We can only let groups which the travel time more than 13 days go, so it is unreasonable.

In order to get a reasonable result, we need to build a criteria for judging to judge schedule. The standards should ensure when the number of staring groups as many as possible, the proportion of the travel program is appropriate.we get a approximate proportion rely on the data in Appendix 1. According to the part of the data that we simulate, we get the maximum carrying capacity of the river is about 35 when the number of starting groups is about $200 \sim 300$, the distribution of groups is reasonable basically.

Conclusion

According to the simulation, if we consider the starting group only, the number of the groups is over 900, and the maximum carrying capacity of the river over 100. We find when the group is maximum, the project is not reasonable, so it also take the resources allocation into consideration.

Synthesize the allocation of resources, resources can be allocated fully when the number of groups in every year is about $200 \sim 300$.

Strengths

1. In our model we consider the requirements in the title fully, and reference conditions in the title, we can list all possible results and get a optimal solution.
2. We optimized for the maximum rely on the original title, and arrange the resource and every travel reasonable.
3. After the reasonable arrangements ,we get several results,
4. The simulation program is not limited in one model. For different situation, we can adjust the parameter.

Weaknesses

1. The period of boats set up is biased away from the actual situation.
2. It's wasting time to get the optimal solution

Reference

[1]Randy Gimblett and Catherine A.Roberts,An Intelligent Agent Based Model for Simulatting and Evaluating River Trip Scenarios Along the Colorado River in Grand Canyon National Park,Chapter in Integrating GIS and Agent based modeling techniques for Understanding Social and Ecological Processes,ed.H.R.Gimblett,Oxford Press,2000,245-275

[2]Ronald E.Borkan and A.Heaton Underhill,Simulating the Effects of Glen Canyon Dam Releases on Grand Canyon River Trips

[3]http://canyonx.com/trip_lengths_logistics.php

[4]A.Heaton Underhill,A.Busa Xaba,Ronald E.Borkan,The Wilderness Use Simulation Model Applied to Colorado River Boating in Grand Canyon National Park,USA

Appendix

1. Source Code of the Simulation Program

```
process[ads_,input_,initial_]:=Module[{len,sch={},dur={} ,maxday ,flag={},river={},
ord,tran={} ,tlast ,ii ,jj      ,total ,ta={}                              },
If[Length[input]!=Length[initial],Return["errer!"]];
len=Length[input];
Do[If[initial[[i]]==1,AppendTo[flag,i]],{i,1,len}];
Do[AppendTo[tran,input[[flag[[i]],1]]],{i,1,Length[flag]}];
ord=Ordering[tran];flag=flag[[ord]];
Do[AppendTo[river,{1,input[[flag[[i]],1]]}],{i,1,Length[flag]}];
total=Length[river];
AppendTo[sch,Join[{1},
Table[If[initial[[i]]==1,Style[initial[[i]],Red,Background->Yellow],initial[[i]]],{i,1,len}] ,{total}]];
Do[AppendTo[sch,Join[{nday},Table[0,{len}],{Null}]],{nday,2,ads}];
Do[(
flag={};tran={};
If[ads-day>=Last[river][[2]]-Last[river][[1]],
Do[If[Mod[day,input[[i,2]]]==0,AppendTo[flag,i]],{i,1,len}];
Do[AppendTo[tran,input[[flag[[i]],1]]],{i,1,Length[flag]}];
ord=Ordering[tran];flag=flag[[ord]];
For[jj=1,jj<=Length[flag],jj++,
tlast=Last[river][[2]]-Last[river][[1]];
If[tlast>=input[[First[flag],1]],flag=Delete[flag,1],Break[]]];
Do[sch[[day,flag[[i]]+1]]=Style[1,Red,Background->Yellow];
AppendTo[river,{0,input[[flag[[i]],1]]}];total++,
{i,1,Length[flag]}]
];
sch[[day,len+2]]=Length[river];
Do[river[[i,1]]=river[[i,1]]+1,{i,1,Length[river]}];
For[ii=1,ii<=Length[river],ii++,
If[Rational@@First[river]>=1,river=Delete[river,1],Break[]]
];
),{day,2,ads}];
AppendTo[ta,"Day"];
Do[AppendTo[ta,StringForm["`1`|`2`",input[[i,1]],input[[i,2]]]],{i,len}];
AppendTo[ta,"Total\nboats"];
PrependTo[sch,ta];
Print[total];
Grid[sch,Frame->All,Alignment->Bottom]
]
```

```
todate[first_,data_]:=Module[{dl,dw,datesch={},zhan={},minmon=0,maxmon =0 ,datetable ,ta },
dl=Length[data];
dw=Length[data[[1]]];
Do[zhan={};AppendTo[zhan,data[[1,wi]]];
Do[If[data[[li,wi]]==1,AppendTo[zhan,DatePlus[first,{data[[li,1]],"Day"}]]],
{li,2,dl}];
AppendTo[datesch,zhan],
{wi,2,dw-1}];
minmon=first[[2]];
Do[If[Length[datesch]>1,
Do[If[datesch[[li,wi]][[2]]>=maxmon,maxmon=datesch[[li,wi]][[2]]],
{wi,2,Length[datesch[[li]]]}]],
{li,1,Length[datesch]}];
datetable=Table[Null,{Length[datesch]+1},{maxmon-minmon+2}];
ta=minmon;
Do[datetable[[1,wi]]=ta;ta++,{wi,2,maxmon-minmon+2}];
Do[datetable[[li,1]]=datesch[[li-1,1]],{li,2,Length[datetable]}];
Do[Do[
zhan={};
Do[If[datesch[[li,wi]][[2]]==mon,AppendTo[zhan,StringForm["`1`/`2`",datesch[[li,wi]][[2]],dates
ch[[li,wi]][[3]]]]],{wi,2,Length[datesch[[li]]]}];
datetable[[li+1,mon-minmon+2]]=Column[zhan],
{mon,minmon,maxmon}],
{li,1,Length[datesch]}];
Grid[datetable,Frame->All]
]
```

Mome

Dear managers:

In order to make the travel team to reach maximum each year, we simulate the objective condition. When we simulate, we assume the period of boats is given, and we get a basic schedule. Then we get a optimized schedule. This schedule will offer a better situation. Because the number of the camp sites over is 18, and the distance between two contiguous sites are not more than $12 miles$, the speed of oar-powered rubber rafts is $4mph$, and the speed of motorized boats is $8mph$, so you can adjust the time of the boat start to arrange the camp sites.

We find the travel resources will distribute unfairly if only consider the number of the travel groups, there will be few people join in the short time trip. In order to make the distribute reasonable, we reevaluate the optimization project. We find the best project, the groups of each year about $200 \sim 300$ is better, we get a schedule after optimization. The schedule is showed bellow:

	April	May	June	July	Aut	Sept
6|1	4/2 4/3 4/4 4/12 4/13	5/6 5/18 5/19 5/30 5/31	6/23 6/24	7/5 7/6 7/29	8/10 8/11 8/22 8/23	9/15 9/16
7|3	4/4 4/10 4/13 4/16 4/28	5/4 5/10 5/16 5/19 5/22 5/28 5/31	6/9 6/15 6/21 6/24 6/27	7/3 7/6 7/9 7/21 7/27	8/2 8/8 8/11 8/14 8/20 8/23	9/1 9/7 9/13 9/16
8|4	4/5 4/13 4/17 4/29	5/11 5/19 5/23 5/31	6/16 6/24 6/28	7/6 7/10 7/22	8/3 8/11 8/15 8/23	9/8 9/16
9|4	4/5 4/9 4/13 4/17 4/21 4/25 4/29	5/3 5/7 5/11 5/15 5/19 5/23 5/31	6/4 6/8 6/12 6/16 6/20 6/24 6/28	7/2 7/6 7/10 7/14 7/18 7/22 7/26 7/30	8/3 8/7 8/11 8/15 8/23 8/27 8/31	9/4 9/8 9/12 9/16
13|6	4/7 4/13 4/19 4/25	5/1 5/7 5/19 5/25 5/31	6/6 6/12 6/18 6/24 6/30	7/6 7/12 7/18 7/24 7/30	8/11 8/17 8/23 8/29	9/4 9/10 9/16
14|7	4/8 4/15 4/22 4/29	5/6 5/13 5/27	6/3 6/10 6/24	7/1 7/8 7/15 7/22 7/29	8/5 8/19 8/26	9/2 9/16
15|4	4/5 4/9 4/13 4/17 4/21 4/25 4/29	5/3 5/7 5/11 5/15 5/19 5/23 5/27 5/31	6/4 6/8 6/12 6/16 6/20 6/24 6/28	7/2 7/6 7/10 7/14 7/18 7/22 7/26 7/30	8/3 8/7 8/11 8/15 8/19 8/23 8/27 8/31	9/4 9/8 9/12 9/16
16|7	4/8 4/15 4/22 4/29	5/6 5/13 5/20 5/27	6/3 6/10 6/17 6/24	7/1 7/8 7/15 7/22 7/29	8/5 8/12 8/19 8/26	9/2 9/9 9/16