Chapter 1

FDTD scheme of electromagnetic field and cold plasma current.

1.1 Update Equaitions

1.1.1 Maxwell's equations

$$\partial_t \boldsymbol{E} = \frac{1}{c^2}\nabla \times \boldsymbol{B} - \frac{1}{\varepsilon_0}\sum_s \boldsymbol{J}_s - \boldsymbol{J}_{ext} \quad (1.1)$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E} \quad (1.2)$$

1.1.2 Cold plasma

The update equation of each linear current are

$$\partial_t \boldsymbol{J}_{s1} = \frac{Z_s}{m_s}\left(Z_s n_{s0} \boldsymbol{E}_1 + \boldsymbol{J}_{s1} \times \boldsymbol{B}_0\right), \quad (1.3)$$

where$\boldsymbol{B}_0$andn_{s0} are background magnetic field and density,$\boldsymbol{E}_1$ and $\boldsymbol{J}_{s1}$ are linear electrotic field and current density, andν is an artificial damping coefficient.

1.2 Time domain difference scheme

The time difference equation of Ampere's law, Eq.1.1, is given by

$$\varepsilon_0 \frac{E^1 - E^0}{\Delta t} = \frac{1}{\mu_0}\nabla \times B_1^{1/2} - J_{ext}^{1/2} - \sum_s \frac{J_s^1 + J_s^0}{2} \quad (1.4)$$

$$\varepsilon_0 \frac{E^+ - E^-}{\Delta t} = -\sum_s \frac{J_s^1 + J_s^0}{2}$$

where

$$E^- = E^0 + \frac{\Delta t}{2\varepsilon_0}\left(\frac{1}{\mu_0}\nabla \times B_1^{1/2} - J_{ext}^{1/2}\right) \quad (1.5)$$

$$E^+ = E^1 - \frac{\Delta t}{2\varepsilon_0}\left(\frac{1}{\mu_0}\nabla \times B_1^{1/2} - J_{ext}^{1/2}\right) \quad (1.6)$$

The linear cold plasma currents$\boldsymbol{J}_{s1}$ and electric field$\boldsymbol{E}_1$ are put at full time steps, the hot plasma currentJ_p, which are obtained from the kinetic simulation, and linear magnetic field B_1 are put at he half time steps. The time difference equation is given by

$$\frac{J_s^1 - J_s^0}{\Delta t} = \frac{Z_s}{m_s}\left(Z_s n_{0s}\frac{E^1 + E^0}{2} + \frac{J_s^1 + J_s^0}{2} \times \boldsymbol{B}_0\right) \quad (1.7)$$

$$J_s^1 - J_s^0 = \underbrace{\frac{\Delta t Z_s}{2m_s}}_{\alpha_s}\left(Z_s n_{0s}\left(E^+ + E^-\right) + \left(J_s^1 + J_s^0\right) \times \boldsymbol{B}_0\right)$$

$$\alpha_s \equiv \frac{\Delta t Z_s}{2m_s}$$

$$J_s^1 - \alpha_s J_s^1 \times \boldsymbol{B} = \underbrace{J_s^0 + \alpha_s J_s^0 \times \boldsymbol{B} + \alpha_s Z_s n_s E^-}_{K_s^0} + \alpha_s Z_s n_s E^+$$

Eq.1.7 can be wrote as a linear equation ofJ_s^1

$$J_s^1 - \alpha_s J_s^1 \times \boldsymbol{B} = K_s^0 + \alpha_s Z_s n_{0s} E^+ \quad (1.8)$$

where

$$K_s^0 \equiv J_s^0 + \alpha_s J_s^0 \times \boldsymbol{B} + \alpha_s Z_s n_s E^-$$

Solve it we get,

$$J_s^1 = \overleftrightarrow{M_s} \cdot \left(K_s^0 + \alpha_s Z_s n_s E^+\right) \quad (1.9)$$

where

$$\overleftrightarrow{M_s} \equiv \frac{\overleftrightarrow{I} + \alpha \overleftrightarrow{I} \times \boldsymbol{B} + \alpha^2 \overleftrightarrow{I} \cdot \boldsymbol{BB}}{1 + \alpha^2 \boldsymbol{B} \cdot \boldsymbol{B}}$$

Substituting Eq.1.9 into Eq.1.4 yields

$$\varepsilon_0 \frac{E^+ - E^-}{\Delta t} = -\sum_s \frac{J_s^1 + J_s^0}{2}$$

$$
\begin{aligned}
E^{+}-E^{-} &= -\frac{\Delta t}{2\varepsilon_0}\sum_s\left[\overleftrightarrow{M_s}\cdot\left(K_s^0+\alpha_s Z_s n_s E^{+}\right)+J_s^0\right]\\
E^{+}+\frac{\Delta t}{2\varepsilon_0}\sum_s\overleftrightarrow{M_s}\left(\alpha_s Z_s n_s\right)\cdot E^{+} &= \underbrace{E^{-}-\frac{\Delta t}{2\varepsilon_0}\sum_s\left(\overleftrightarrow{M_s}\cdot K_s^0+J_s^0\right)}_{Q}
\end{aligned}
$$

where

$$
aE^{+}+bE^{+}\times B+cE^{+}\cdot BB=Q
$$

$$
\begin{aligned}
a &\equiv \frac{\Delta t}{2\varepsilon_0}\sum_s\frac{Z_s n_s\alpha_s}{\alpha_s^2 B\cdot B+1}+1\\
b &\equiv \frac{\Delta t}{2\varepsilon_0}\sum_s\frac{Z_s n_s\alpha_s^2}{\alpha_s^2 B\cdot B+1}\\
c &\equiv \frac{\Delta t}{2\varepsilon_0}\sum_s\frac{Z_s n_s\alpha_s^3}{\alpha_s^2 B\cdot B+1}
\end{aligned}
$$

$$
E^{+} = \left(aQ-bQ\times B+\frac{b^2-ca}{a+cB^2}Q\cdot BB\right)/\left(a^a+b^2B^2\right)
$$

Then,$E_1^1$is solved from Eq.**??** asSubstituting Eq. **??** back to Eq.**??**, we getJ_{s1}^1