HW6- Advanced Data Analysis

1. (6pt) Suppose T is a life time and it satisfies

$$\log(T) = \mu + \sigma\epsilon$$

where $\epsilon \sim N(0, 1)$.

   (a) (2pt) Give the density of T. What is the name of this distribution?

   (b) (2pt) Find $E(T)$ and $Var(T)$ (hint: see HW 1)

   (c) (2pt) If $\mu = 4$ and $\sigma = 3$, find $P(T \leq 100)$.

2. (4pt) Suppose T is a life time and it satisfies

$$\log(T) = \mu + W/\alpha$$

where $\alpha > 0$ and

$$F_W(w) = 1 - e^{-e^w}$$

Show that T has Weibull distribution and specify it paramters.

3. (10pt) Suppose that T has a Weibull distribution with a survival function is given by

$$S(t) = e^{-(\alpha t)^\beta}$$

where $\alpha > 0$ and $\beta > 0$. (Hint: compute $P(T \leq t)$)

   (a) (2pt) Find the density, $f_T(t)$ of T

   (b) (2pt) Find the hazard function $\lambda(t)$ of T

   (c) (2pt) Show that

$$\log(-\log(S(t))) = \beta\log(\alpha) + \beta\log(t)$$

   Based on this, describe a graphical method for checking whether or not the data is from a Weibull distribution.

   (d) (2pt) Consider the following data

$$143, 164, 188, 188, 190, 192, 206, 209, 213, 216, 220, 227, 230, 234, 246, 265, 304$$

   and use as an estimate of $S(t_{(i)})$

$$\hat{S}(t_{(i)}) = 1 - (i - 0.5)/n$$

   were $t_{(i)}$ is the ith ordered value and n is the sample size. Use the graphical technique in the previous question to check if a Weibull distribution is appropriate for these data

(e) (2pt) Assume that the Weibull distribution is a good fit, use least squares approach to estimate its parameters.