1 Problem 1

1.1 a

The level $\alpha = P_{\eta=0}(S \geqslant 16)$. Since $S \sim Bin(25, 0.5)$, $\alpha = 0.115$

1.2 b

Since $X \sim N(0.5, 1)$ we can get that $s(X_i) \sim Bernoulli(p)$ with $p = P(X > 0) = 0.691$. Thus the power is 0.782

2 Problem 2

2.1 a

To use t-test, we assume that the samples are from a normal distribution. And we can use Q-Q plot to check this assumption.

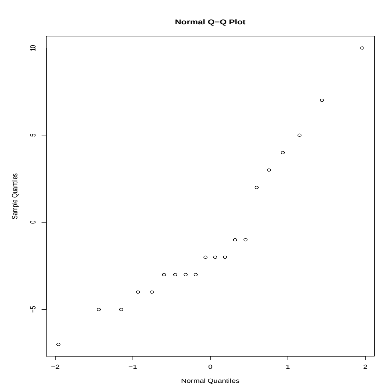

Figure 1: Q-Q Plot

From the plot, we can see the normality assumption is appropriate.

We get the test statistics

$$t = \frac{\bar{X}}{SE(\bar{X})}$$

where

$$SE(\bar{X}) = s/\sqrt{20}$$

is the standard error of $\bar{X}$

We reject H_0 if $|t| > t_{19}(0.025)$ and the p-value is $P_{\mu=0}(|t| > 0.705) = 0.4891$

2.2 b

A 95% confidence interval for μ is $[-2.78, 1.38]$

2.3 c

Let $T = \sum_{i=1}^{20} s(X_i) = 6$, $P\!-\!value = 2min(P(T \geqslant 6), P(T \leqslant 6)) = 2P(T \leqslant 6)$. Since $T \sim Bin(20, \frac{1}{2})$, we get P-value=0.116

2.4 d

Actually,$P(T \geqslant 14) + P(T \leqslant 5) \leqslant 0.05$ and we reject H_0 if $T \geqslant 14$ or $T \leqslant 5$. We want to find the confidence interval and we only need to find the value of η such that $T = \sum_{i=1}^{20} s(X_i - \eta)$ fall in the interval $[6, 13]$.We can sort the data from low to high and find the value. Thus the interval is $[-3, 2]$ Also, we can calculate the intervals in R

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                        Conf.Level  L.E.pt   U.E.pt
Lower Achieved CI           0.8847      -3  -1.0000
Interpolated CI             0.9500      -3   1.6506
Upper Achieved CI           0.9586      -3   2.0000
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3 Problem 3

3.1 Parametric

Using t-distribution for two samples, we assume the data are from normal distribution and here we can use unequal variances case.

The statistics

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

is approximately a t-distribution with degrees of freedom

$$d = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

We calculate $d = 9.976$ and $t_d(0.025) = 2.23$. Thus we reject H_0 if $|t| > 2.23$. The sample statistics is -1.84 and we can't reject H_0.

3.2 Nonparametric

Using WMW procedure, we assume the samples are independent from two distributions and the population have the same shapes and spreads. The statistics

$$W = \frac{T_X - n_1(n_1 + n_2 + 1)/2}{\sqrt{n_1 n_2 (n_1 + n_2 + 1)/12}}$$

where

$$T_X = \sum_{i=1}^{n1} \sum_{j=1}^{n2} I(X_i > Y_i) + \frac{n_1(n_1 + 1)}{2}$$

is approximately a normal distribution. Thus we reject H_0 if $|W| > 1.96$ and we calculate the sample statistics is -1.60 which means we can't reject H_0