Challenge Problem I

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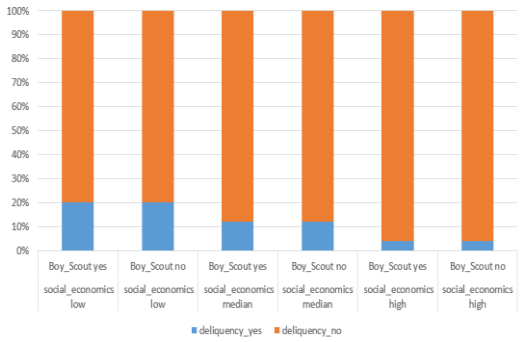
Abstract: In this report, I will use the method of logistic regression to make an analysis of the problem about whether a boy will be delinquent or not based on two potential factors (Social Economic Status and In Boy Scout or not). The analysis focus on five parts: exploratory data analysis, logistic regression, hypothesis tests, estimation and confidence interval, prediction.

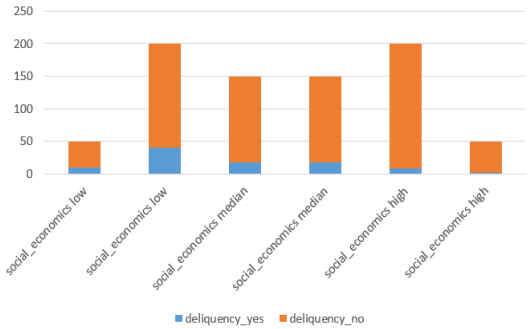
Key words: Logistic Regression; Likelihood Ratio Test; Prediction;

1 Exploratory Data Analysis

In this challenge problem, we are given the data with three variables (social economics status, whether in boy scout and delinquency). And for each situation, we are provided with the corresponding frequency.

I choose delinquency (Yes, No) to be the response variable. And the social economics status and whether in boy scout as the predictors.





As we can see from the plot, we may find some possible trends. First, relatively, boy from higher social economics status may have lower percentage of being delinquent. Second, the relative difference in delinquency for boy in scout and not in every social status are is very small. Third, for the boy in scout or not the difference of the number of being delinquent have relative small difference in median social economics status but high in other two status.

Based on these finding, I first build the following model, where I take Social Economics High and not in boy scout as the base category:

$$y = \begin{cases} 1, & being\ deliquent \\ 0, & not\ being\ deliquent \end{cases}, \quad x_{11} = Social - Economic_{low},\ x_{12} = Social - Economics_{median}, \quad x_2 = in\ boy\ socut$$

$$\mathbf{p(y = 1 \mid x_1, x_2) = \frac{e^{\beta_0+\beta_{11}x_{11}+\beta_{12}x_{12}+\beta_2x_2}}{1 + e^{\beta_0+\beta_{11}x_{11}+\beta_{12}x_{12}+\beta_2x_2}}} \qquad \boldsymbol{or} \qquad \boldsymbol{logit(\pi(y = 1)) = \beta_0 + \beta_{11}x_{11} + \beta_{12}x_{12} + \beta_2x_2}$$

2 Logistic Regression

Using R, we can get the estimated:

$$\boldsymbol{\beta_0 = -3.178054\ ;\ \beta_{11} = 1.791759\ ;\ \beta_{12} = 1.185624\ ;\ \beta_2 = 7.076634 \times 10^{-16} \approx 0}$$

Just looking at the result, we can find that the parameter for whether in boy scout is almost 0. Thus, we need to do the hypothesis test to find whether we need to keep this.

3 Hypothesis Tests

The result of the regression can be seen from the plot and I will do several hypothesis tests based on this:

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Coefficients:
                           Estimate Std. Error z value Pr(>|z|)
(Intercept)               -3.178e+00  3.802e-01  -8.360  < 2e-16 ***
factor(Boy_Scout)Yes       7.077e-16  2.511e-01   0.000  1.00000
factor(social_economics)low    1.792e+00  3.897e-01   4.598 4.27e-06 ***
factor(social_economics)Median  1.186e+00  3.760e-01   3.153  0.00162 **
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3.2752e+01  on 5  degrees of freedom
Residual deviance: 6.8834e-14  on 2  degrees of freedom
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- **Test One:** $\mathbf{H_0}: logit(\pi(y=1)) = \beta_0 + \beta_{11}x_{11} + \beta_{12}x_{12} + \beta_2 x_2$

The first test, I need to do is to ensure whether the form of current model is reliable or not. As we can see from the result, the residual deviance $6.8834 \times 10^{-14} \approx 0$. Thus, we fail to reject $\mathbf{H_0}$and conclude that: this is the suitable form of model.

- **Test Two:** $\mathbf{H_0}: \beta_{11} = \beta_{12} = \beta_2 = 0$

Based on likelihood ratio test and the result of the regression: the null deviance is 32.752 with 5 degrees of freedom while the residual deviance is $6.8834 \times 10^{-14} \approx 0$. Thus, the test statistics is 32.752, since the $\mathrm{p} = 3$. We can get that: $32.752 \geq \chi_2^2(0.95)$. Thus, we can reject $\mathbf{H_0}$and conclude that: at least one predictor would have significant inference.

- **Test Three:** $H_0: \beta_2 = 0$

We can see from the result of the table that the p-value of this test is 1. This means we can conclude that $\boldsymbol{\beta_2 = 0}$ and whether in boy scout would have no significant difference. Another way to do this test is use the idea of partial F test. The result would be the same. In the same way, we know that $\boldsymbol{\beta_{11} \neq 0}$ and $\boldsymbol{\beta_{12} \neq 0}$. Social economics status would have a significant inference.

Based on the result of these tests, now I update our model in a new form. Actually, the estimated value of parameters do not change while the degree of freedom changed.

$$\mathbf{p(y=1 \mid } \boldsymbol{social\_economics} \mathbf{)} = \frac{\boldsymbol{e^{-3.178+1.792x_{11}+1.186x_{11}}}}{\mathbf{1} + \boldsymbol{e^{-3.178+1.792x_{11}+1.186x_{11}}}} \quad \boldsymbol{or} \quad \boldsymbol{logit(\pi(y=1))} = \mathbf{-3.178} + \mathbf{1.792}\boldsymbol{x_{11}} + \mathbf{1.186}\boldsymbol{x_{11}}$$

4 Point Estimation and Confidence Interval

1. for $b_0 = -3.178$(intercept): for a boy who has high level social economics status, on average, we have 95% confidence estimate that the odd of this boy is delinquent is $e^{-3.178} = 0.04166891$. For confidence interval: on average, we have 95% confidence estimate that this multiplicative factor would between $e^{-3.9776309} = 0.01872996$ and $e^{-2.4766143} = 0.08402724$.

2. for $b_1 = 6.001443$ (social economics low): on average, we have 95% confidence estimate the odds odd of a boy has low level social economics status is delinquent is to be $e^{1.792} = 6.001443$ times the odds that boy who has high level social economics status (a 500% increase roughly). For confidence interval: on average, we have 95% confidence estimate that this multiplicative factor would between $e^{0.4832948} = 2.895996$ and $e^{2.6037768} = 13.51468$.

3. for $b_0 = -3.178$ social economics median): on average, we have 95% confidence estimate the odds odd of a boy has median level social economics status is delinquent is to be $e^{1.186} = 3.273959$ times the odds that boy who has high level social economics status (a 227% increase roughly). For confidence interval: on average, we have 95% confidence estimate that this multiplicative factor would between $e^{-0.4971858} = 0.60824$ and $e^{0.4897023} = 1.63183$.

5 Prediction

As a prediction based on the new model we can get that:

$$\frac{e^{-3.178}}{1+e^{-3.178}} = 4\% \ ; \ \frac{e^{-3.178+1.792\times1}}{1+e^{-3.178+1.792\times1}} = 12\% \ ; \ \frac{e^{-3.178+1.186\times1}}{1+e^{-3.178+1.186\times1}} = 20\%$$

1. for a boy who is from high level social economics background, the on average the probability for him to be delinquent is 4%.
2. for a boy who is from median level social economics background, the on average the probability for him to be delinquent is 12%.
3. for a boy who is from low level social economics background, the on average the probability for him to be delinquent is 20%.

In order to know how much the result of this finding is reliable, I do the risk analysis. This is shown that all of the predictions have the deviance residual equal to 0. This means that out prediction fit the real data.

Conclusion

The result shows that: only the factor of social economics status have significant inference. And we can see that a boy from higher social economics background would have less probability to be delinquent. At last, I make a prediction on average the probability for a boy from low, median and high social economics background would be delinquent is 4%, 12% and 20%.

- Extended thinking: how can we improve this research: 1. Involve more data (i.e. increase sample size); 2. Involve more predictor (there must have more factor would influence delinquency like education); 3. Multicategory model