Understanding Factorial Effects

<u>Def</u>: An effect is a linear combination of all cell means

Suppose we have two factors A and B and each one has two levels

		B: 1	B: 2	
A	1	μ_{11}	μ_{12}	$\mu_{1\cdot}$
	2	μ_{21}	μ_{22}	$\mu_{2\cdot}$
		$\mu_{\cdot 1}$	$\mu_{\cdot 2}$	

A main effect: $A(1) = \frac{1}{2}\Big[\underbrace{(\mu_{11}-\mu_{21})}_{\text{A effect at } B=1} + \underbrace{(\mu_{12}-\mu_{22})}_{\text{A effect at } B=2}\Big]$

$= \mu_{1\cdot} - \mu_{2\cdot}$

B main effect: $B(1) = \frac{1}{2}\Big(\underbrace{(\mu_{11}-\mu_{12})}_{\text{B effect at } A=1} + \underbrace{(\mu_{21}-\mu_{22})}_{\text{B effect at } A=2}\Big)$

$= \mu_{\cdot 1} - \mu_{\cdot 2}$

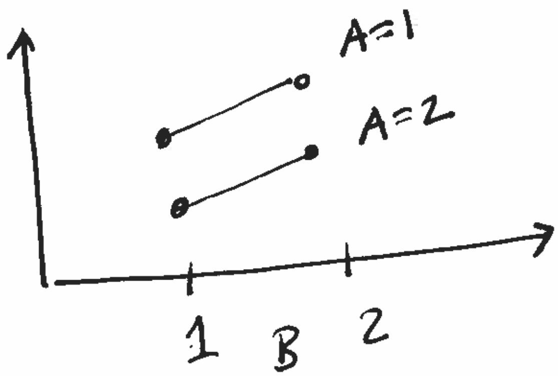
Observe that A main effect is obtained by averaging B-specific effects of A over the levels of B

B main effects are obtained by averaging A specific effects of B over the levels of A

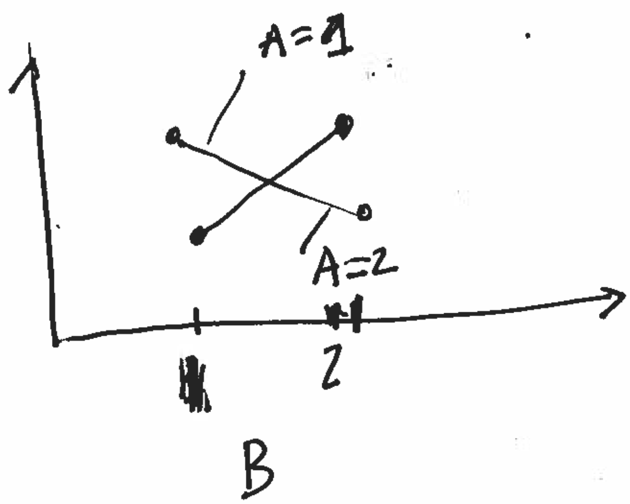
A×B interaction effect

$$AB(1) = (\mu_{11}-\mu_{21}) - (\mu_{12}-\mu_{22})$$

observe that A×B interaction effect is obtained by subracting B speific effect of A.



$A \times B(1) = 0$.



$AB(1) \neq 0$.

when $AB(1) \neq 0$, then the main effect A is meaningless in the following sense.

$$A(1) = \frac{1}{2}\left[(\mu_{11}-\mu_{21}) + (\mu_{12}-\mu_{22})\right]$$

= is the average of 2 unequal quantities (the B-speific effects of A), and thefore

[illegible] information that theyare

unequal if it is presented by itself. Thus we do not report the A main effect (or even test it equal to zero) if the $A \times B$ interaction is not zero. Similarly for B main effects.

General rule: Main effect may be misleading if $A \times B$ Interaction is not zero