BitTiger DS501 Week 1 HW Meina Wang

Question 1

You're about to get on a plane to Seattle. You want to know if you should bring an umbrella. You call 3 random friends of yours who live there and ask each independently if it's raining. Each of your friends has a 2/3 chance of telling you the truth and a 1/3 chance of messing with you by lying. All 3 friends tell you that "Yes" it is raining. What is the probability that it's actually raining in Seattle.

Answer 1:

Base on the Bayes' theorem.

$$P(\text{raining}|\text{all say raining}) = \frac{P(\text{all say raining}|\text{raining})P(\text{raining})}{P(\text{all say raining})}$$

P(all say raining | raining) is the probability that all 3 friends are telling the truth. Therefore,

P(all say raining raining) = $(\frac{2}{3})^3$

P(all say raining) = P(all say raining|raining)P(raining) + P(all say raining|not raining)P(not raining)We assume the P(raining) = p, which we can get calculate from past observation or experience. Then $P(\text{all say raining}) = \left(\frac{2}{9}\right)^3 p + \left(\frac{1}{9}\right)^3 (1-p)$

Substitute all the above values into the Bayes' theorem, we get
$$P(\text{raining|all say raining}) = \frac{\frac{2}{3} p}{\frac{2}{3} p + \left(\frac{1}{3}\right)^3 (1-p)} = \frac{8p}{7p+1}.$$

Once we can get the P(raining), we can plug in the above equation and calculate the probability it's actually raining given all friends saying it's raining.

Question 2

You have two coins. One of is fair and the other is biased and comes up heads with probability 3/4. You randomly pick coin and flip it twice" and get heads both times. What is the probability that you picked the fair coin?

Answer 2:

Base on the Bayes' theorem.

$$P(\text{fair coin}|\text{two heads}) = \frac{P(\text{two heads}|\text{fair coin})P(\text{fair coin})}{P(\text{two heads})}$$

Given the description, $\mathbb{P}(\text{two heads}|\text{fair coin}) = (\frac{1}{2})^2$, and $\mathbb{P}(\text{two heads}|\text{biased coin}) = (\frac{3}{4})^2$. Assume that the chance of picking up either fair coin or biased coin is the same. So $\mathbb{P}(\text{fair coin}) = \frac{1}{4}$.

P(two heads) = P(two heads|faircoin)P(fair coin) + P(two heads|biased coin)P(biased coin) =
$$(\frac{1}{2})^2 \frac{1}{2} + (\frac{3}{4})^2 \frac{1}{2} = \frac{13}{32}$$

With Bayes' theorem, substitute all the above values, P(fair coin two head) =

P(fair coin | two head) =
$$\frac{(\frac{1}{2})^2 \frac{1}{2}}{\frac{13}{22}} = \frac{4}{13}$$

Therefore, the probability that one picked the fair coin is $\frac{4}{13}$

Question 3

Provide a simple example of how an experimental design can help answer a question about behavior. How does experimental data contrast with observational data?

Answer 3:

Taking the example of the relationship between chocolate consumption with the number of Nobel prize winner. Observational data might suggest a country with higher chocolate consumption has more Nobel prize winners. We can design an experiment to test this statement. First, we would randomly split counties into two groups, and only provide one group with chocolate. Then we can use hypothesis testing and come to the conclusion that the number of Nobel prize winners do not show significant difference between the two groups, which proves the previous statement wrong.

The experimental data applies a treatment to a group, and it attempts to isolate the effects of the treatment. Therefore, we can have conclusion of causation from experimental data. Whereas the observational data does not attempt to influence the variable of interest, so the results can only be associations, not causation.

Question 4

In a study of emergency room waiting times, investigators consider a new and the standard triage systems. To test the systems, administrators selected 20 nights and randomly assigned the new triage system to be used on 10 nights and the standard system on the remaining 10 nights. They calculated the nightly median waiting time (MWT) to see a physician. The average MWT for the new system was 3 hours with a variance of 0.60 while the average MWT for the old system was 5 hours with a variance of 0.68. Consider the 95% confidence interval estimate for the differences of the mean MWT associated with the new system. Assume a constant variance. What is the interval? Subtract in this order (New System - Old System).

Answer 4:

The emergency room waiting time should follow a normal distribution. The Z value for 95% CI is 1.960,

Using the two sample t-test, the new system X1 follows the distribution of N(3, 0.60), and the old system X2 follows the distribution of N(5, 0.68). The pooled sample standard deviation is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{9 * 0.60 + 9 * 0.68}{10 + 10 - 2}} = 0.8$$

The standard error is

SE =
$$s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.8 * \sqrt{\frac{1}{10} + \frac{1}{10}} \approx 0.3578$$

From looking up (here: https://www.medcalc.org/manual/t-distribution.php) the Z-value associated with 95% confidence level with 2.5% on each tail, and 18 degree of freedom, Z = 2.101

```
Using the Z-value, the confidence interval is
```

```
CI = (X1 - X2) \pm Z * SE = (3 - 5) \pm 2.101 * 0.3578 = -2 \pm 0.7516 \approx -2.75, -1.25
```

Therefore, the CI is [-2.75, -1.25]

Question 5

Using lending club dataset examine relationship between each feature and response (interest rate). Pick 5 categorical and 5 numeric features which you think are the most predictive with reasoning.

Answer 5 (font color in blue is R code):

Numerical features:

I first selected column with only numerical values into another df called loan_nums.

```
nums <- sapply(loan, is.numeric)
loan_nums <- loan[, nums]</pre>
```

Similarly, columns with only characters were selected into another df called loan_categories

```
categories <- sapply(loan, is.character)
loan_categories <- loan[, categories]</pre>
```

First, check the numeric features and see if any of the features have constant values.

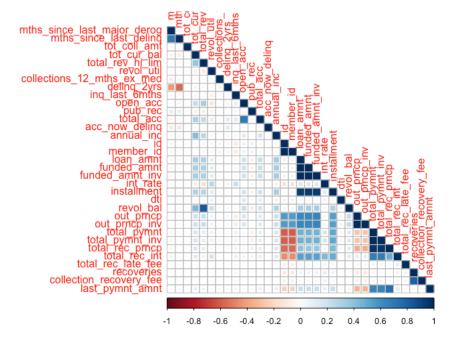
```
sapply(loan nums, function(col) length(unique(col)))
```

From there it showed the variable 'policy_code' has only 1 unique level. So, delete this variable since it does not provide any predictive power.

```
loan_nums_no_constant <- loan_nums[, sapply(loan_nums, function(x) {length(unique(x)) > 1})]
```

Then compute the Pearson correlation between each variable and 'int_rate', and plot the values.

```
nums_corr <- cor(loan_nums_no_constant,use = 'pairwise.complete.obs')
nums_corr
corrplot(nums_corr, method = 'square', tl.cex = 1, type = 'lower')</pre>
```



From this plot we can see the correlations associated with 'int_rate'. However, it is difficult to tell the exact value from this plot.

Next, build a simple linear model with all the features and predict the 'int_rate'.

```
ols = lm(int_rate~.,loan_nums_no_constant) summary(ols)
```

The summary of the model is:

Call

lm(formula = int_rate ~ ., data = loan_nums_no_constant)

Residuals:

Min 1Q Median 3Q Max -59.483 -2.203 -0.329 1.802 54.873

Coefficients:

Estimate Std. Error t value Pr(>ltl)

(Intercept) 1.309e+01 4.356e-02 300.426 < 2e-16 *** mths since last major derog 3.932e-03 4.501e-04 8.734 < 2e-16 *** mths since last deling -5.487e-03 4.969e-04 -11.043 < 2e-16 *** tot_coll_amt 4.423e-07 3.500e-07 1.264 0.20626 tot_cur_bal -1.438e-06 5.347e-08 -26.899 < 2e-16 *** total rev hi lim -4.696e-05 6.842e-07 -68.630 < 2e-16 *** revol util 7.702e-03 3.888e-04 19.809 < 2e-16 *** collections 12 mths ex med 2.376e-01 3.600e-02 6.599 4.15e-11 *** delinq_2yrs 4.766e-02 6.462e-03 7.375 1.65e-13 *** 8.549e-01 7.210e-03 118.570 < 2e-16 *** inq_last_6mths 6.111e-02 1.899e-03 32.178 < 2e-16 *** open acc pub rec 1.833e-01 1.069e-02 17.146 < 2e-16 *** -2.517e-02 8.335e-04 -30.202 < 2e-16 *** total acc acc_now_deling 1.061e+00 6.409e-02 16.558 < 2e-16 *** -5.029e-06 1.267e-07 -39.682 < 2e-16 *** annual_inc

```
id
              2.520e-08 9.625e-09 2.618 0.00884 **
             -4.308e-08 9.146e-09 -4.710 2.48e-06 ***
member id
                -1.641e-04 3.380e-04 -0.486 0.62724
loan amnt
                 1.262e-02 1.966e-03 6.418 1.38e-10 ***
funded_amnt
funded_amnt_inv
                   -1.247e-02 1.927e-03 -6.472 9.72e-11 ***
               4.023e-03 1.049e-04 38.370 < 2e-16 ***
installment
             5.919e-03 3.069e-04 19.287 < 2e-16 ***
dti
total_rec_late_fee
                -2.012e+00 9.348e+00 -0.215 0.82959
recoveries -2.021e+00 9.348e+00 -0.216 0.82881
collection recovery fee -1.348e-03 2.613e-04 -5.158 2.49e-07 ***
last pymnt amnt 5.166e-04 3.150e-06 164.006 < 2e-16 ***
Signif. codes: 0 "*** 0.001 "** 0.01 " 0.05 ". 0.1 " 1
Residual standard error: 3.245 on 212343 degrees of freedom
 (675003 observations deleted due to missingness)
Multiple R-squared: 0.4247,
                         Adjusted R-squared: 0.4246
F-statistic: 4898 on 32 and 212343 DF, p-value: < 2.2e-16
```

From comparing summary output, the top 5 numeric features with the highest t values and lowest p

values are: last_pymnt_amnt inq_last_6mths total_rev_hi_lim revol_bal installment

Categorical features:

To select the top 5 categorical variables most predictive towards 'int_rate', first, select all the categorical variables and store into a new dataframe.

```
categories <- sapply(loan, is.character)
loan_categories <- loan[, categories]
summary(loan_categories)
length(loan_categories)
```

Since the int rate is numerical values, we need to append it to the above dataframe 'categories'.

```
loan_categories['int_rate'] <-loan_nums['int_rate']</pre>
```

Then, check how many levels are in each variable. Delete any variables that only has one level (constant values), since it does not have any predictive power toward 'int' rate'.

```
sapply(loan_categories, function(col) length(unique(col)))
```

The output from the above sapply function showing number of levels in each variables is:

```
term
                                         sub_grade
                          grade
           2
                                          35
                           7
      emp title
                        emp_length
                                           home_ownership
       299273
                             12
                                               6
verification_status
                             issue_d
                                             loan_status
          3
                          103
                                            10
     pymnt_plan
                              url
                                              desc
                        887379
                                           124471
           2
       purpose
                           title
                                          zip code
                         63146
                                             935
          14
     addr state
                     earliest_cr_line
                                        initial_list_status
                          698
          51
                                             2
                         next_pymnt_d
                                            last_credit_pull_d
    last_pymnt_d
          99
                          101
                                            104
  application_type verification_status_joint
                                                  issue_year
                                           9
          2
                           4
      int rate
         542
```

As can see, some of the variables have too many levels (e.g. 'url', 'emp_title', 'title', etc.), and I realized that I need to preprocess the data.

Next, I tried to one-hot encoding to preprocess the data. However, RStudio showed error every time and session then got aborted. It was probably due to the laptop ran out of RAM.

To try another way, a another linear model was built with categorical variables that don't have too many levels (levels < 99), and the t values and p values associated with each variables towards predicting 'int_rate' were outputted below.

```
ols_cate = Im(int_rate~term + grade + sub_grade + emp_length + home_ownership + verification_status + loan_status + pymnt_plan + purpose + addr_state + initial_list_status + application_type + issue_year + verification_status_joint,loan_categories_no_constant) summary(ols_cate)
```

The summary of the model is:

Call:

```
Im(formula = int_rate ~ term + grade + sub_grade + emp_length +
home_ownership + verification_status + loan_status + pymnt_plan +
purpose + addr_state + initial_list_status + application_type +
issue_year + verification_status_joint, data = loan_categories_no_constant)
```

Residuals:

```
Min 1Q Median 3Q Max
-20.1891 -0.2329 -0.0426 0.2618 5.9391
```

Coefficients: (7 not defined because of singularities)

Estimate Std. Error t value Pr(>ltl)

(Intercept) term 60 months gradeB gradeC gradeD	2.799535 0.302543 9.253 < 2e-16 *** -0.010003 0.001441 -6.940 3.92e-12 *** 6.516217 0.004219 1544.686 < 2e-16 *** 9.504763 0.004394 2163.110 < 2e-16 *** 12.626303 0.005096 2477.511 < 2e-16 ***
gradeE	15.911980 0.006515 2442.233 < 2e-16 ***
gradeF	19.123404 0.010950 1746.436 < 2e-16 ***
gradeG	20.160835 0.022130 911.017 < 2e-16 ***
sub_gradeA2	0.708019 0.004902 144.438 < 2e-16 ***
sub_gradeA3	1.372714 0.004854 282.778 < 2e-16 ***
sub_gradeA4	1.757090 0.004457 394.215 < 2e-16 ***
sub_gradeA5	2.554843
sub_gradeB1	-3.368163 0.003422 -984.285 < 2e-16 ***
sub_gradeB2	-2.384828 0.003349 -712.085 < 2e-16 ***
sub_gradeB3	-1.494138
sub_gradeB4	-0.627663 0.003241 -193.664 < 2e-16 ***
sub_gradeB5	NA NA NA NA
sub_gradeC1	-2.388301 0.003436 -695.179 < 2e-16 ***
sub_gradeC2	-1.856051 0.003446 -538.538 < 2e-16 ***
sub_gradeC3	-1.259382 0.003472 -362.676 < 2e-16 ***
sub_gradeC4	-0.671803
sub_gradeC5	NA NA NA NA
sub_gradeD1	-2.350548 0.004509 -521.261 < 2e-16 ***
sub_gradeD2	-1.595050 0.004685 -340.457 < 2e-16 ***
sub_gradeD3	-1.082602
sub_gradeD4	-0.518531
sub_gradeD5 sub_gradeE1	-2.776285 0.006592 -421.180 < 2e-16 ***
sub_gradeE1	-2.276885 0.006672 -341.242 < 2e-16 ***
sub_gradeE3	-1.661149 0.006911 -240.362 < 2e-16 ***
sub_gradeE4	-0.885423
sub_gradeE5	NA NA NA NA
sub_gradeF1	-2.326271 0.011954 -194.604 < 2e-16 ***
sub_gradeF2	-1.641633 0.012476 -131.583 < 2e-16 ***
sub_gradeF3	-1.043825 0.012907 -80.874 < 2e-16 ***
sub_gradeF4	-0.513765
sub gradeF5	NA NA NA
sub_gradeG1	-0.523912 0.024892 -21.047 < 2e-16 ***
sub_gradeG2	-0.271966 0.025863 -10.516 < 2e-16 ***
sub_gradeG3	-0.072319
sub_gradeG4	-0.120578
sub_gradeG5	NA NA NA NA
emp_length1 year	0.001970 0.002939 0.670 0.50274
emp_length10+ years	0.013443
emp_length2 years	0.006701 0.002707 2.476 0.01330 *
emp_length3 years	0.008284 0.002788 2.972 0.00296 **
emp_length4 years	0.001797 0.003013 0.596 0.55087
emp_length5 years	0.015925
emp_length6 years	0.021649 0.003205 6.755 1.43e-11 ***
emp_length7 years	0.019138
emp_length8 years	0.014842
emp_length9 years	0.013832
emp_lengthn/a	0.008435 0.003223 2.617 0.00888 **

```
home ownershipMORTGAGE
                                       0.084142  0.301424  0.279  0.78013
home_ownershipNONE
                                    0.676544  0.310356  2.180  0.02927 *
                                    home ownershipOTHER
home_ownershipOWN
                                    0.093615  0.301428  0.311  0.75613
                                    0.091318  0.301425  0.303  0.76192
home_ownershipRENT
verification statusSource Verified
                                   verification statusVerified
                                 0.022754  0.001488  15.296  < 2e-16 ***
                                loan_statusCurrent
loan_statusDefault
                                -0.006983 0.015171 -0.460 0.64532
loan_statusDoes not meet the credit policy. Status:Charged Off -2.078593 0.019974 -104.067 < 2e-16
loan statusDoes not meet the credit policy. Status:Fully Paid -1.559654 0.013090 -119.153 < 2e-16 ***
                                 -0.002893 0.002738 -1.057 0.29055
loan_statusFully Paid
                                   loan statusIn Grace Period
loan_statusIssued
                                -0.057002  0.006350  -8.977  < 2e-16 ***
loan_statusLate (16-30 days)
                                   -0.014581 0.005491 -2.655 0.00792 **
loan statusLate (31-120 days)
pymnt_plany
                               -0.043880 0.165116 -0.266 0.79043
                                 0.007051 0.005685 1.240 0.21489
purposecredit card
purposedebt consolidation
                                   -0.001811 0.005615 -0.322 0.74709
                                 0.285799  0.026350  10.846 < 2e-16 ***
purposeeducational
purposehome improvement
                                     purposehouse
                                   0.001147  0.006825  0.168  0.86649
purposemajor purchase
purposemedical
                                0.015108 0.007933 1.904 0.05686.
purposemoving
                                0.026314 0.009036 2.912 0.00359 **
purposeother
                               purposerenewable energy
                                    0.019797 0.022475 0.881 0.37839
purposesmall business
                                  -0.046565 0.007587 -6.137 8.40e-10 ***
purposevacation
                                0.009893 0.009416 1.051 0.29341
purposewedding
                                 0.013226  0.012161  1.088  0.27679
                               -0.010300 0.012166 -0.847 0.39718
addr stateAL
addr stateAR
                               -0.002079 0.012834 -0.162 0.87132
                               addr stateAZ
addr stateCA
                               -0.012264 0.011753 -1.044 0.29672
addr stateCO
                               -0.018925 0.011990 -1.578 0.11448
addr stateCT
                               addr stateDC
                               -0.011010 0.015237 -0.723 0.46994
addr stateDE
addr stateFL
                               -0.025284 0.011318 -2.234 0.02549 *
                               -0.021358 0.011533 -1.852 0.06404.
addr stateGA
addr stateHI
                               -0.251862 0.140262 -1.796 0.07255.
addr stateIA
                               addr stateID
addr stateIL
                               -0.020141 0.011459 -1.758 0.07880.
                               -0.012551 0.011978 -1.048 0.29470
addr stateIN
addr stateKS
                               -0.012030 0.012571 -0.957 0.33857
                               addr stateKY
                               -0.012825 0.012222 -1.049 0.29403
addr stateLA
addr stateMA
                               addr stateMD
                               addr stateME
addr stateMI
                               -0.016075 0.011641 -1.381 0.16730
```

```
-0.015831 0.011862 -1.335 0.18202
addr stateMN
                                -0.014620 0.011951 -1.223 0.22120
addr_stateMO
addr stateMS
                                -0.000344 0.015172 -0.023 0.98191
addr_stateMT
                                -0.016891 0.011605 -1.456 0.14552
addr stateNC
addr stateND
                               addr stateNE
                               -0.013104 0.013678 -0.958 0.33805
addr_stateNH
addr_stateNJ
                               addr_stateNM
addr_stateNV
                               addr stateNY
                                -0.015644 0.011525 -1.357 0.17467
addr stateOH
                               -0.012890 0.012544 -1.028 0.30416
addr stateOK
                                -0.019903 0.012192 -1.632 0.10259
addr_stateOR
addr_statePA
                               -0.018554 0.013915 -1.333 0.18242
addr stateRI
                               -0.005989 0.012218 -0.490 0.62402
addr stateSC
                                0.016869 0.016547 1.019 0.30798
addr stateSD
addr stateTN
                               -0.014627 0.011290 -1.296 0.19510
addr stateTX
addr stateUT
                               -0.019460 0.011576 -1.681 0.09274.
addr stateVA
                               -0.007491 0.016592 -0.451 0.65164
addr stateVT
addr stateWA
                               -0.017842 0.011732 -1.521 0.12832
                               addr stateWI
                               addr stateWV
                                0.001736 0.016063 0.108 0.91393
addr stateWY
                              -0.061073  0.001236  -49.415 < 2e-16 ***
initial list statusw
application typeJOINT
                                  -0.022247 0.040433 -0.550 0.58217
issue_year2008
                                 1.073269 0.024007 44.707 < 2e-16 ***
                                 2.677300 0.023150 115.650 < 2e-16 ***
issue year2009
issue year2010
                                 2.106747  0.022593  93.246  < 2e-16 ***
                                 issue year2011
issue year2012
                                issue_year2013
                                4.166249 0.022444 185.626 < 2e-16 ***
                                3.209039  0.022443  142.984  < 2e-16 ***
issue_year2014
                                2.470872  0.022456  110.033  < 2e-16 ***
issue_year2015
verification_status_jointNot Verified
                                   -0.047563 0.050957 -0.933 0.35062
verification status jointSource Verified
                                     0.078993 0.078112 1.011 0.31188
verification status jointVerified
                                     NA
                                           NA
                                                 NA
                                                      NA
Signif. codes: 0 "*** 0.001 "** 0.01 " 0.05 ". 0.1 " 1
Residual standard error: 0.522 on 887240 degrees of freedom
Multiple R-squared: 0.9858,
                      Adjusted R-squared: 0.9858
F-statistic: 4.466e+05 on 138 and 887240 DF, p-value: < 2.2e-16
```

From comparing summary output, the top 5 numeric features with the highest t values and lowest p values are:

```
grade
sub_grade
```

issue_year loan_status initial_list_status

So, 'last_pymnt_amnt', 'inq_last_6mths', 'total_rev_hi_lim', 'revol_bal', 'installment' are the 5 numerical, and 'grade', 'sub_grade', 'issue_year', 'loan_status', 'initial_list_status' are the 5 categorical variables that are most predictive to interest rate give the higher t values and lower p values from the regression models, which indicates these variables are meaning additions to the model in predicting the response variable 'int_rate'.