Assessment of Nine Large Market Capitalization Securities

STAT4290 Final Project

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Summary

Nine publicly traded companies were selected from from the largest holdings of Berkshire Hathaway as of September 30, 2015 [1]. Five year asset and T-Bill returns from the United States Federal Reserve Bank were examined and combined into portfolios that were assessed on the basis of annualized returns, risk and monthly Value at Risk (VaR) and Expected Shortfall (ES) [2]. A time series analysis was performed on the S&P 500 in order to provide an outlook for the assets and portfolios examined. Portfolios generated from the assets selected have high annualized returns ($7\% < R < 21\%$) and can be constructed in a manner to substantially limit risk. Time series analysis of the S&P 500 suggests a positive outcome for short term investment while additional caution is required for longer term investment decisions.

Asset Analysis

Monthly price and return data (adjusted for splits and dividends) were analyzed for nine assets. Company characteristics are summarized and univariate distributions fit to individual asset returns, see Table 1.

Asset Summary					
Company	Ticker Symbol	Sector	Market Capitalization	β	Best Fit Distribution
American Express	AXP	Services	69.4B	0.91	t
International Business Machine	IBM	Technology	135.4B	0.59	ged
Coca-Cola	KO	Consumer Goods	187.7B	0.49	skewed ged
Moody's Corporation	MCO	Services	20.6B	1.23	ged
Wells Fargo & Company	WFC	Financial	282.7B	0.94	ged
DaVita HealthCare Partners	DVA	Healthcare	15.1B	0.99	t
Procter & Gamble Company	PG	Consumer Goods	212.7B	0.57	ged
U.S. Bancorp	USB	Financial	76.9B	0.79	skewed t
Wal-Mart Stores	WMT	Services	193.4B	0.27	t

Table 1. Summary of nine large market capitalization assets. Several sectors are represented. Best fitted distribution was determined by AIC.

General statistics were calculated for all assets, see Appendix A, Figure 7. Average monthly returns for all assets were found to range between 0.0 and 0.03. Additionally, the monthly returns of each of the nine assets were found to fluctuate around 0. Augmented Dickey-Fuller Tests were run on all assets and all returns were found to be stationary.

Monthly asset prices were examined and equity curves were generated, see Figure 1. Approximately half of the assets outperform the S&P 500 during this time interval, see Appendix A, Figure 6. There are clear upward trends in the prices of KO and DVA, MCO, WFC, and USB. Notice that WMT has a gradual price drop starting throughout 2015 and IBM has been trending downward since 2013. We believe that the price drop in WMT is caused by two main reasons. One is the announcement that it planned to hike the minimum wage for its U.S. workforce, which would cost an additional \$1.2 billion in 2015 and \$1.5 billion in 2016. The other reason is that as Wal-Mart continues to invest in online grocery service, which shows the potential for massive growth, the upfront cost remains an issue for investors. For IBM, the stock appears to have been oversold since the summer of 2013, which leads to the downward trend of the stock price.

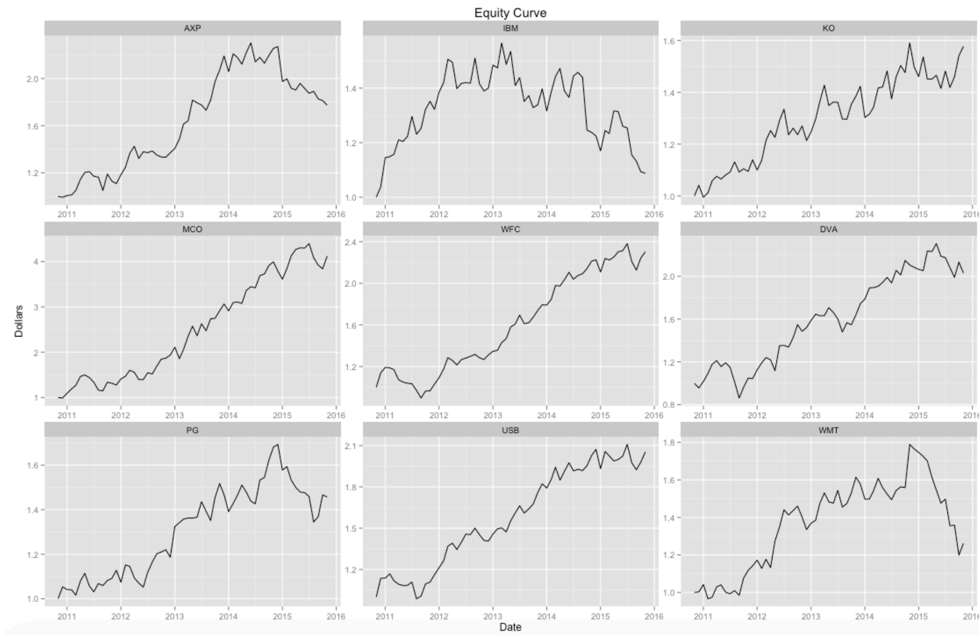
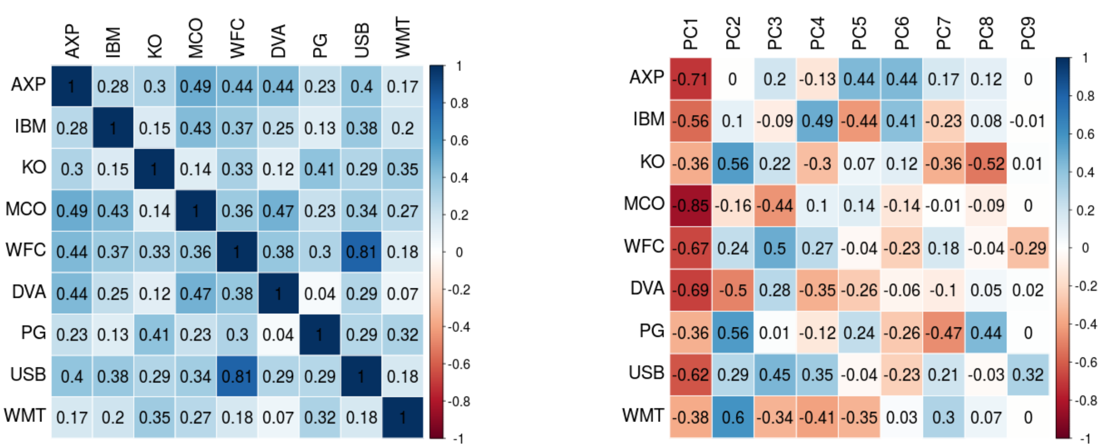


Figure 1: Equity curves for all assets examined.

Principle component analysis (PCA) was performed on the returns of assets, see Figure 2. All assets have negative correlation with the first principal component (PC1) indicating a positive movement would result in losses for all stocks. However, it is also noted that the the the correlation between assets and the remaining PCs is much more varied. Also note the first PC accounts for 42.4% of the total variance and 8 of 9 PCs are required to account for $> 95\%$ of the total variance, see Appendix A, Figure 14. The data suggest diverse portfolios can be constructed from these assets.



(a) Correlation matrix of asset returns, note all correlations are positive.

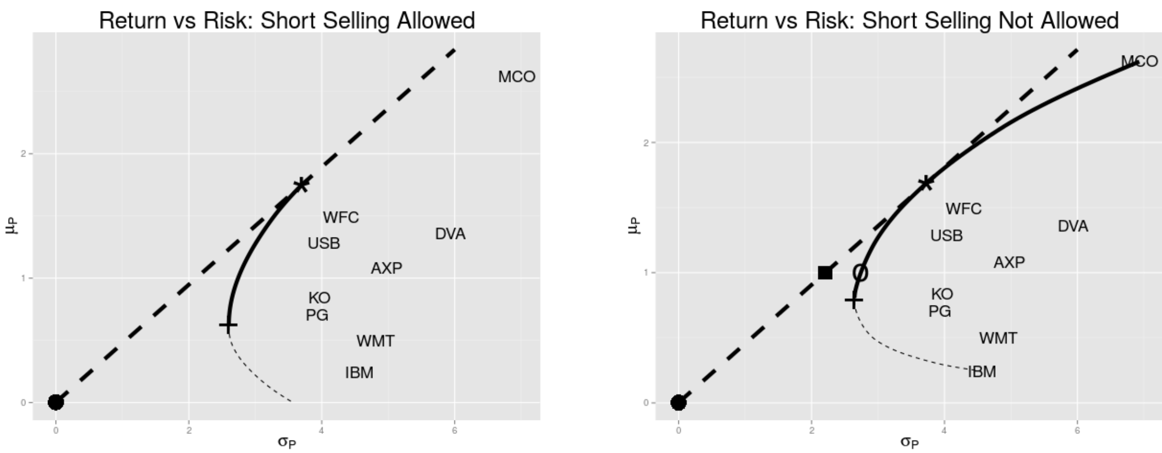
(b) Correlation of assets returns with asset return principal components

Figure 2: Asset and PC correlations, note for PCA analysis some companies within the same sector appear to have similar relations to first and second principle components, for example KO and PG or WFC and UBS where as other companies in the same sectors do not ie APX, WM and MC.

Portfolio Analysis

In order to determine an optimal investment strategy, six portfolios were constructed from the nine companies and risk free asset determined from the monthly T-Bill returns: Minimum Variance Portfolios without/with Short Selling allowed, Tangency Portfolios without/with Short selling Allowed, and Two Portfolios with a targeted annual return of 12%, see Figure 3.

The portfolios were constructed in such a manner that their asset allocation was unconstrained except for the case of the portfolios with shorting allowed. Here asset weights were constrained to $-10\% < w_i < 80\%$ to prevent potentially risky large short positions. However, some portfolios still had unbalanced asset allocations relying heavily on companies in the finance/financial services sectors, see Appendix B, Table 9.



(a) Short selling allowed with individual assets weights constrained to $-10\% < w_i < 80\%$.

(b) Portfolio with short selling disallowed. No constraints imposed on asset weights.

Figure 3: Efficient Fronter (Solid Line), Line of Efficient Portfolios (Heavy Dashed Line), Inefficient Portfolios (Light Dashed Line), Minimum Variance Portfolio (+), Risk Free (·), Tangency Portfolio (*), Efficient Portfolio with 1.0% Monthly Return (o), Combination Tangency and Risk Free with 1.0% Monthly Return (■).

Portfolio viability was assessed on the basis of expected annualized risk and return along with copula-based portfolio VaR assessments. Annualized portfolio characteristics were generated for the Minimum Variance Portfolios without/with Short Selling allowed and Tangency Portfolios without/with Short selling Allowed, see Table 2.

Portfolio Characteristics (Annualized)				
	Min Variance (No Shorts)	Min Variance	Tangency (No Shorts)	Tangency
Return	9.53	7.49	20.27	21.00
Risk	9.16	9.01	9.67	12.81
σ^2	83.97	81.23	93.45	164.08
Sharpe Ratio	1.04	0.83	2.09	1.64

Table 2. Measures of portfolio risk and return. Note the minimum variance portfolios have Sharpe ratios similar to the individual assets while tangency portfolios have higher Sharpe ratios than any individual asset.

The two tangency portfolios have higher expected returns than the minimum variance portfolios while only having higher variances than the observed minimum values. The tangency portfolio with short selling has only a slightly higher return than that of the tangency portfolio that does not allow for shorting. As there is significantly more risk in the tangency portfolio that allows for shorting, the slightly higher return does not justify potential investment.

To further quantify risk, individual asset monthly ES and VaR values were obtained and confidence intervals were established for both parametric and nonparametric bootstraps, see Appendix B, Table II. While these values provide approximate bounds to potential losses, a higher fidelity measure of risk was computed using a t-copula, see Appendix C. 5% VaR and ES monthly estimates were computed for all portfolios, see Tables 3 and 4.

Monthly VaR and ES by Portfolio				
	Min Variance (No Shorts)	Min Variance	Tangency (No Shorts)	Tangency
$\widehat{VaR}^t(0.05)$	3537	3635	2879	4303
$\widehat{ES}^t(0.05)$	4773	4851	4183	6031

Table 3. Monthly VaR and ES estimates assuming $100,000.00 invested in each portfolio.

Monthly Risk for 12%/year Return Portfolios		
	Efficient Portfolio	Tangency w/ Risk Free
Risk	2.74	2.21
$\widehat{VaR}^t(0.05)$	3484	1545
$\widehat{ES}^t(0.05)$	4764	2155

Table 4. Risk metrics for portfolios with 12% targeted annualized return; VaR and ES are calculated for a portfolio of $100,000.00. Note the tangency portfolio with risk free asset achieves a moderate return with extremely low VaR and ES values.

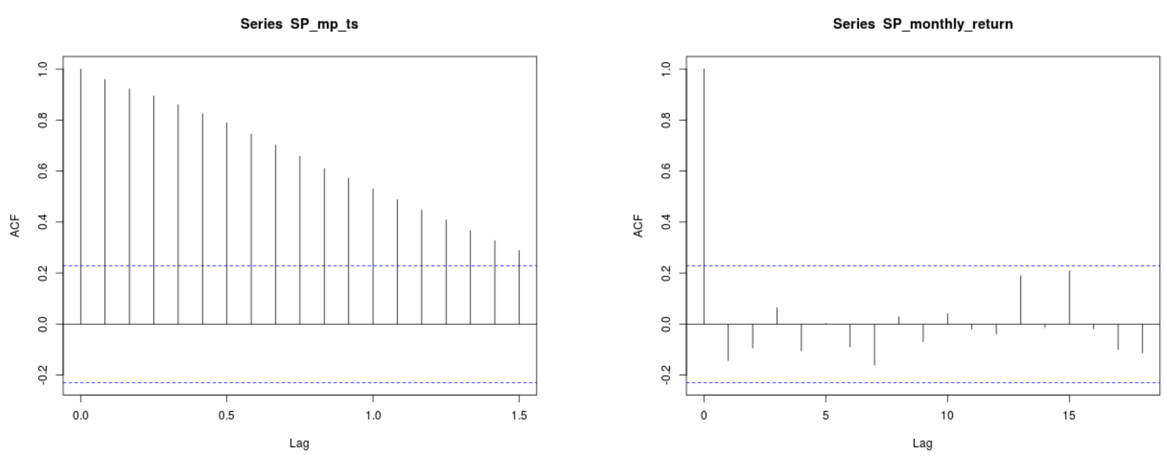
Generally, VaR and ES values for portfolios were smaller than for individual assets given equivalent investment. Given all measures of risk and return, several of the derived portfolios would likely be attractive for investors depending on individual objectives.

Time Series Analysis on S&P500

A straightforward investment strategy is to invest in an index fund. Accordingly, a time series analysis was performed on the S&P500 index. This analysis gives an expected investment outcome based on past market performance, see Appendix D, Figure 18. Given all assets used in the generation of portfolios have positive correlations with the S&P 500, this estimate of future returns can be used to determine when market conditions are favorable.

Stationariness

Autocorrelation Function (ACF) plots of the returns and the prices were generated, see Figure 4.



(a) The ACF of prices.

(b) The ACF of Returns.

Figure 4: Autocorrelation Function Plots for the S&P 500.

Given the results of the Box test, returns of the index are stationary while the price is not. Therefore, predictions cannot be made for returns using historical data, but predictive methods can be applied to prices.

ARIMA on Prices

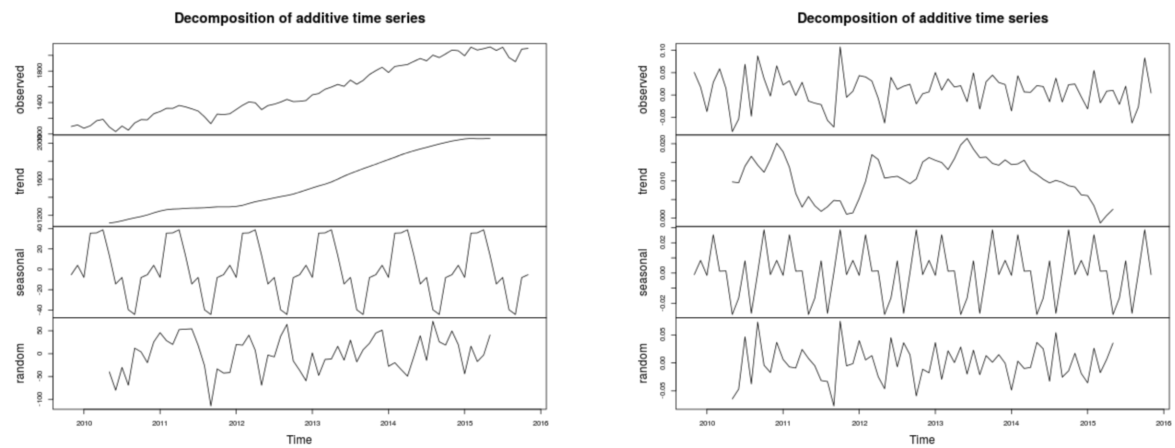
An Autoregressive Integrated Moving Average (ARIMA) was applied to the prices of the index and the the best model was determined to be ARIMA(1,1,0), see Table 1, Appendix D. The fitted ARIMA model was used for to predict the price of the market over the next two months, see Table 5.

Prediction Result of ARIMA Mode		
Month	2015 Dec	2016 Jan
Predicted Price	2087.87	2087.98
SE	55.48	74.45

Table 5. ARIMA predictions. Note the market is expected to remain stationary.

Decomposition

The decomposition method is used to extract additional information on the trend and seasonality of the S&P 500, see Figure 5.



(a) Decomposition of the Price.

(b) Decomposition of the Return.

Figure 5: Decomposition Plots for the S&P 500.

The price of the S&P 500 is increasing, however the rate of increase is slowing. In addition, January and September appear to be favorable times to increase holdings while holdings should be reduced in February or March. Given this information, investors stand to gain in the short term. However, given historically poor market performance in February or March and the slowing rate of gains, caution should be used in the long term.

Appendix

A. Assets

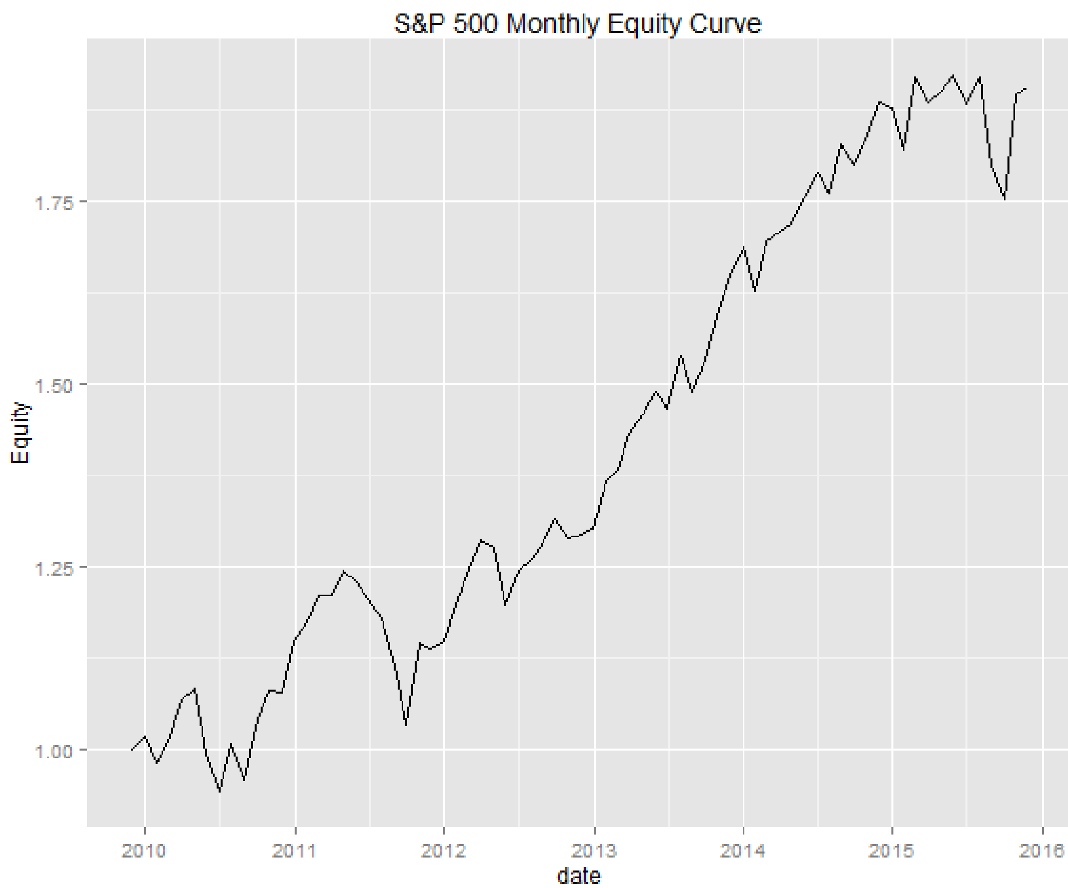


Figure 6: Equity Curve for the S&P 500.

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
AXP	1	73	0.01	0.06	0.01	0.01	0.05	-0.14	0.17	0.31	0.01	0.33	0.01
IBM	2	73	0.00	0.05	0.01	0.01	0.05	-0.13	0.10	0.24	-0.33	-0.06	0.01
KO	3	73	0.01	0.04	0.02	0.01	0.04	-0.08	0.10	0.18	-0.34	-0.71	0.00
MCO	4	73	0.03	0.08	0.02	0.03	0.08	-0.17	0.18	0.35	-0.21	-0.38	0.01
WFC	5	73	0.01	0.05	0.02	0.02	0.04	-0.15	0.14	0.29	-0.54	0.90	0.01
DVA	6	73	0.02	0.06	0.02	0.02	0.06	-0.15	0.21	0.36	0.11	0.98	0.01
PG	7	73	0.01	0.04	0.00	0.01	0.04	-0.08	0.12	0.19	0.32	-0.18	0.00
USB	8	73	0.01	0.05	0.02	0.01	0.04	-0.13	0.14	0.27	-0.32	0.76	0.01
WMT	9	73	0.01	0.05	0.00	0.00	0.05	-0.12	0.15	0.26	0.28	0.52	0.01

Figure 7: Summary of sample statistics of nine large capitalization assets

We can see that six of the assets have mean return of 0.01. IBM has the lowest mean return of 0.00 with standard deviations of 0.05. Even though MCO has the highest mean return of 0.03, it also has the highest standard deviation of 0.08. AXO has the least skewed because it has the lowest absolute value of skewness coefficient. WFC has both a high skewness coefficient and a high kurtosis coefficient.

Figure 8: Adjusted monthly prices of each of the nine large capitalization assets.

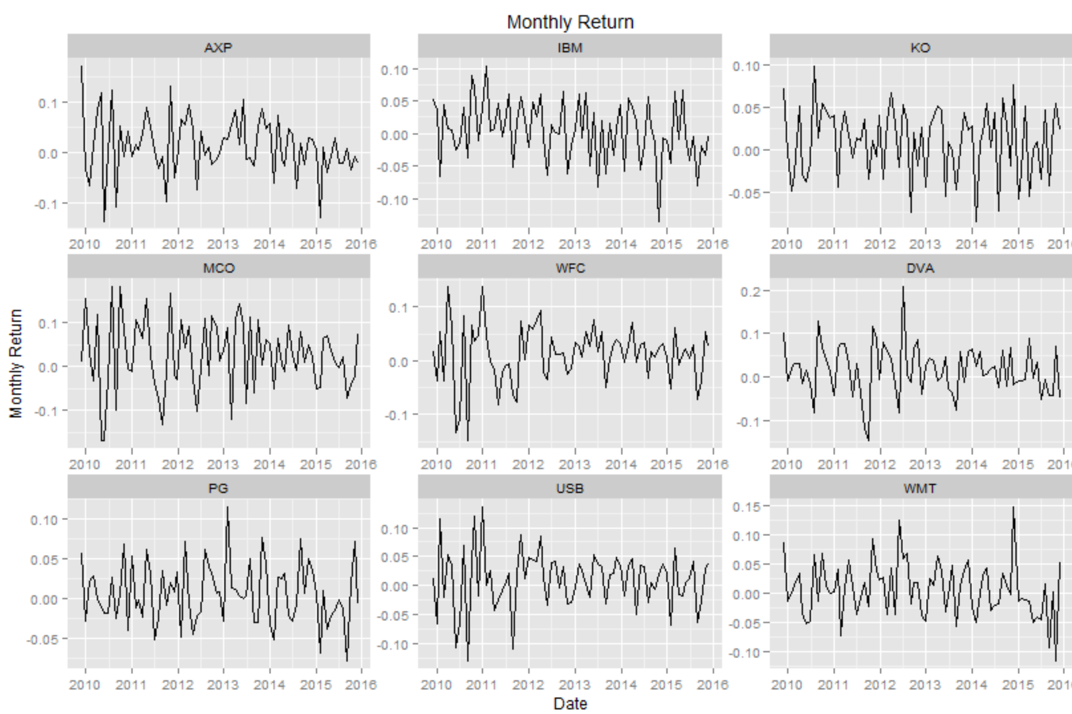


Figure 9: Monthly returns of each of the nine large capitalization assets.

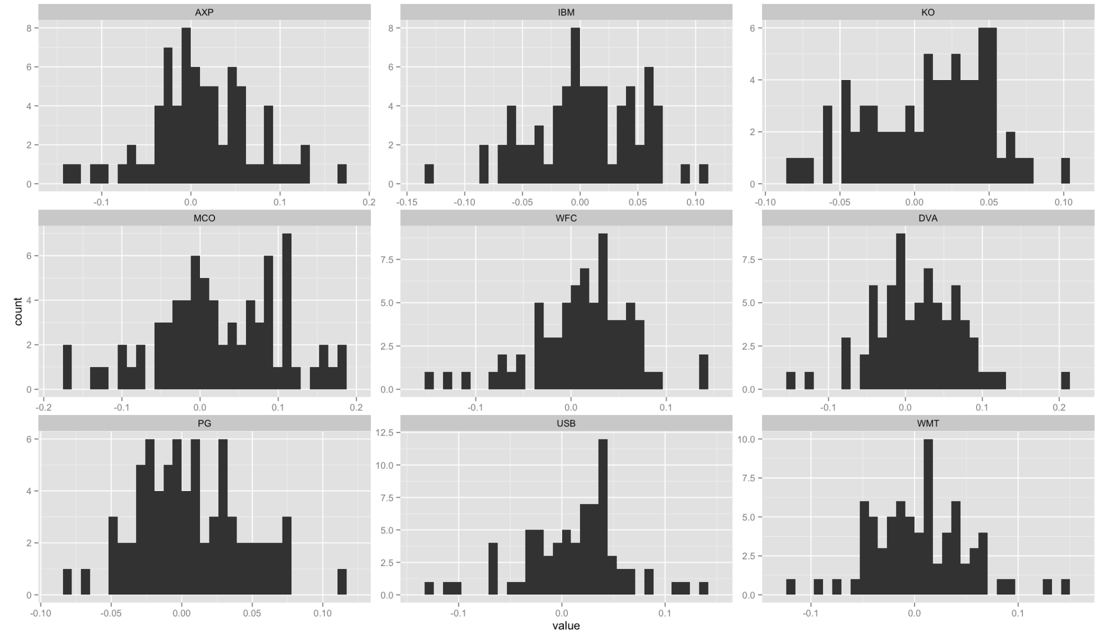


Figure 10: Histograms of assets returns.

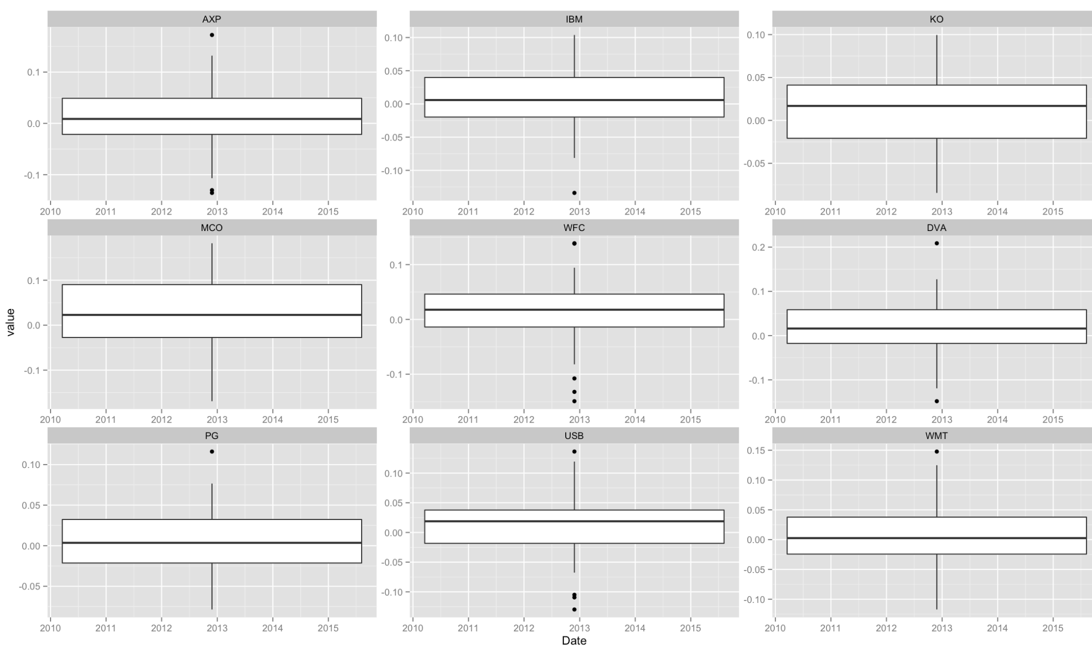


Figure 11: Boxplots of assets returns.

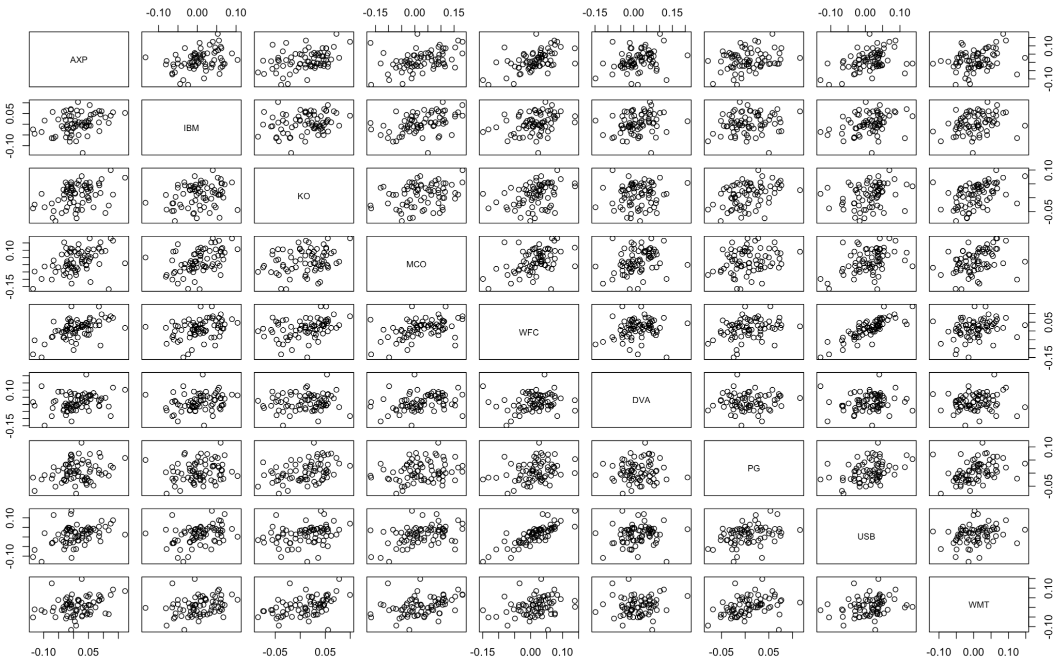


Figure 12: Pairwise scatter plots between assets returns.

Note that no outliers are seen in the scatterplots. Changes in return in WFC show a positive relationship with changes in return in USB. So they are useful for predicting changes in the return of each other. The correlations between changes in return in other variables are not as strong. Next, we look at the sample covariance matrix, see Figure 13.

	AXP	IBM	KO	MCO	WFC	DVA	PG	USB	WMT
AXP	0.0034467444	0.0008962769	0.0009494842	0.0019940326	0.0017032799	0.0009355794	0.0005592464	0.0012145366	0.0007864860
IBM	0.0008962769	0.0020663537	0.0004546459	0.0016149851	0.0008030064	0.0005932900	0.0003141326	0.0007403191	0.0005803757
KO	0.0009494842	0.0004546459	0.0016689834	0.0009041414	0.0007850985	0.0002737735	0.0006109145	0.0005757083	0.0008311674
MCO	0.0019940326	0.0016149851	0.0009041414	0.0063664496	0.0018926067	0.0013737008	0.0007062582	0.0015762539	0.0013353294
WFC	0.0017032799	0.0008030064	0.0007850985	0.0018926067	0.0028508994	0.0005563895	0.0005920648	0.0021531062	0.0006848081
DVA	0.0009355794	0.0005932900	0.0002737735	0.0013737008	0.0005563895	0.0033859430	0.0001048996	0.0003850430	0.0002554052
PG	0.0005592464	0.0003141326	0.0006109145	0.0007062582	0.0005920648	0.0001048996	0.0014482707	0.0006702827	0.0006351888
USB	0.0012145366	0.0007403191	0.0005757083	0.0015762539	0.0021531062	0.0003850430	0.0006702827	0.0023870163	0.0005708331
WMT	0.0007864860	0.0005803757	0.0008311674	0.0013353294	0.0006848081	0.0002554052	0.0006351888	0.0005708331	0.0022212994

Figure 13: Sample covariance matrix of the returns on the nine assets.

Notice that WFC and USB have the strongest correlation. It makes sense because WFC (Wells Fargo & Co) and USB (U.S. Bancorp) are both financial services company, so their returns tend to move together.

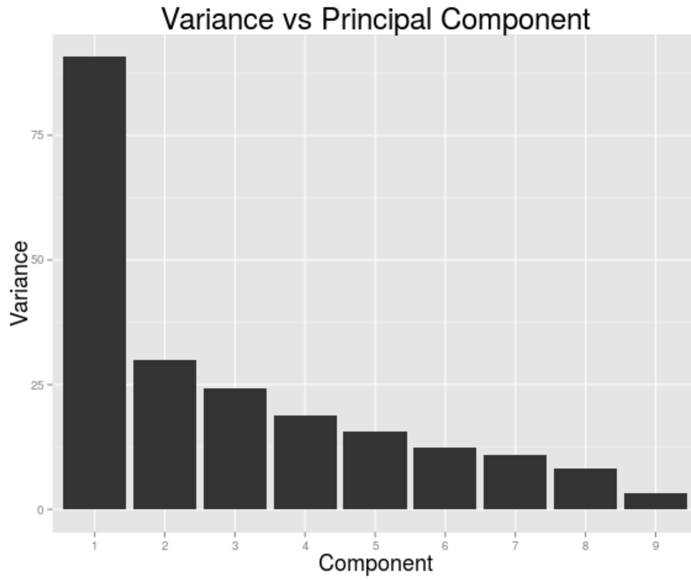


Figure 14: Scree Plot of the Variance of the Principle Components. The first component accounts for 42% of the variance whereas 8 components are required to account for > 95% of the total variance.

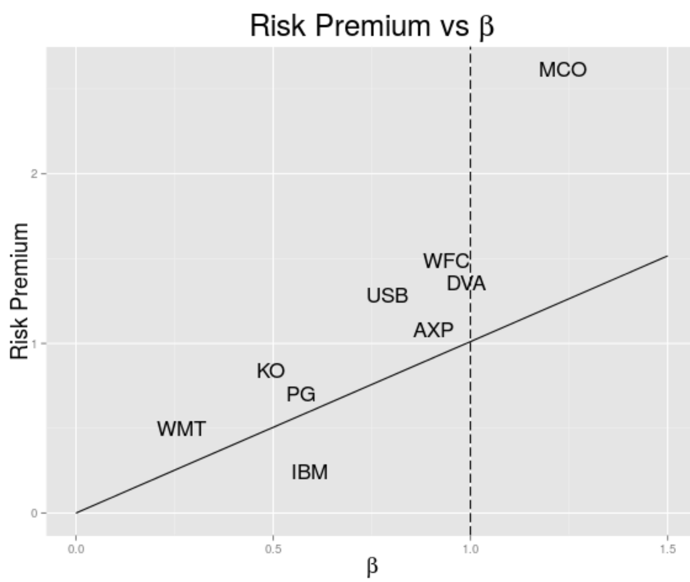


Figure 15: Asset Risk Premiums Plotted with the security Market Line (SML) and dashed line indicating the market. All assets were found to obey the Capital Asset Pricing Model.

Annualized Asset Sharpe Ratio								
AXP	IBM	KO	MCO	WFC	DVA	PG	USB	WMT
0.75	0.18	0.73	1.31	1.20	0.79	0.62	1.10	0.36

Table 6. Asset Sharpe Ratios, note financial and financial services companies tend to have higher values.

B. Asset Allocations

% Asset Weight by Portfolio Type				
Ticker Symbol	Min Variance (No Shorts)	Min Variance	Tangency (No Shorts)	Tangency
AXP	2.8	6.3	0.0	-9.8
IBM	16.6	19.4	0.0	-10.0
KO	19.5	17.2	21.0	28.1
MCO	0.0	-8.7	33.4	32.4
WFC	0.0	-5.2	30.3	26.7
DVA	9.7	12.9	0.7	5.8
PG	24.5	25.8	2.0	10.3
USB	15.3	19.4	12.6	20.6
WMT	11.5	13.0	0.00	-4.2

Table 7. Asset allocation, note both tangency portfolios depend heavily on companies in the finance/financial services sectors (MCO, WFC, and UBS).

% Asset Weights for 12%/year Return Portfolios		
Ticker Symbol	Efficient Portfolio	Tangency w/ Risk Free
Risk Free	0.0	40.9
AXP	1.8	0.0
IBM	6.5	0.0
KO	20.8	12.4
MCO	3.9	19.7
WFC	4.3	17.9
DVA	11.1	0.4
PG	21.6	1.17
USB	21.2	7.42
WMT	8.7	0.0

Table 8. Asset allocation for portfolios targeting 12% annualized returns. Note the portfolio that combines the tangency portfolio and T-Bills was constructed from a previously derived tangency portfolio but does not suffer as much from the same potential lack of asset diversity due to the large allocation of funds to the risk free asset.

Bootstrap Estimates of ES and VaR

Parametric VaR Estimates				
Ticker Symbol	$\widehat{VaR}^{par}(0.05)(\$)$	95% CI ($)	BIAS_{boot}	SE_{boot}
AXP	7101	(5401, 9784)	1.24e-3	1.08e-2
IBM	7265	(5610, 9972)	1.01e-3	1.03e-2
KO	5675	(4082, 7371)	7.71e-4	8.55e-3
MCO	8787	(6460, 11910)	1.64e-3	1.34e-2
WFC	5569	(4003, 7590)	7.85e-4	9.13e-3
DVA	8405	(6257, 11722)	1.35e-3	1.32e-2
PG	5754	(4610, 7300)	8.71e-4	6.68e-3
USB	5350	(3767, 7771)	9.64e-3	9.57e-3
WMT	7410	(5773, 9631)	8.00e-4	9.63e-3

Parametric Expected Shortfall Estimates				
Ticker Symbol	$\widehat{ES}^{par}(0.05)(\$)$	95% CI ($)	BIAS_{boot}	SE_{boot}
AXP	9180	(7202, 12318)	-1.13e-3	1.22e-2
IBM	9173	(7300, 11957)	-7.87e-4	1.16e-2
KO	7331	(5571, 9492)	-1.36e-3	9.97e-3
MCO	11685	(9299, 15126)	-2.17e-3	1.46e-2
WFC	7362	(5678, 9709)	-1.01e-3	1.02e-2
DVA	10884	(8364, 14643)	-1.50e-3	1.54e-2
PG	7395	(6063, 9210)	-9.08e-4	7.89e-3
USB	7036	(5148, 9760)	-1.18e-3	1.12e-2
WMT	9419	(7544, 12148)	-1.82e-3	1.15e-2

Nonparametric VaR Estimates				
Ticker Symbol	$\widehat{VaR}^{np}(0.05)(\$)$	95% CI ($)	BIAS_{boot}	SE_{boot}
AXP	7000	(4086, 13019)	3.18e-3	1.85e-2
IBM	6532	(5913, 13396)	-3.82e-3	1.30e-2
KO	5893	(4780, 8448)	-2.48e-3	1.03e-2
MCO	8392	(7069, 13034)	-1.32e-3	2.04e-2
WFC	6225	(3792, 8199)	3.04e-3	1.36e-2
DVA	7677	(4614, 14827)	3.59e-3	2.54e-2
PG	5122	(3785, 7862)	7.10e-4	9.62e-3
USB	4927	(3241, 10936)	-5.47e-4	1.54e-2
WMT	5870	(4755, 11721)	-4.23e-3	1.67e-2

Nonparametric Expected Shortfall Estimates				
Ticker Symbol	$\widehat{ES}^{np}(0.05)(\$)$	95% CI ($)	BIAS_{boot}	SE_{boot}
AXP	9990	(5674, 13019)	2.54e-2	3.39e-2
IBM	9824	(6698, 13396)	2.46e-2	3.26e-2
KO	7707	(4224, 8448)	2.01e-2	2.36e-2
MCO	11750	(6247, 13034)	3.15e-2	3.66e-2
WFC	7676	(4100, 8199)	2.02e-2	2.31e-2
DVA	11675	(6300, 14827)	2.97e-2	3.89e-2
PG	6614	(3667, 7862)	1.64e-2	2.10e-2
USB	8008	(5286, 10936)	2.09e-2	2.66e-2
WMT	9491	(5861, 11721)	2.48e-2	3.06e-2

Table 9. Nonparametric and Parametric monthly ES and VaR estimates assuming $100,000.00 invested in each asset. Confidence intervals, bias and standard error estimated by bootstrap with B = 2000 resamplings.

C. Copulas

Five parametric copulas were fit to the returns: Gaussian, t, Gumbel, Frank, and Clayton.

Since the marginal distributions of the nine assets are unknown, to minimize biases in the estimated parameters of the copula, we applied pseudo-maximum likelihood estimation, where the marginal distributions are estimated nonparametrically. The results are in Table 10.

By comparing the five copulas AIC, we see that t-copula fits best since it minimizes AIC. Its also noted that Gaussian Copulas AIC is only slightly larger than t-copulas, which means the joint distribution of the returns is approximately normally distributed. This conclusion can be verified by Figure 16. The Q-Q plots of the nine assets are all approximately linear. More precisely, from the S shape of the Q-Q plots, we can conclude that the marginal distribution of the nine assets have lighter tail than normal distribution.

Copula Fit			
Copula Family	Estimates	Maximized log-likelihood	AIC
Gaussian	$\hat{\rho}$: between each pair of assets, values omitted	119.325	-220.650
t	$\hat{\rho}$: between each pair of assets, values omitted $\hat{\nu} = 21.613$	120.477	-222.955
Gumbel	$\hat{\theta} = 1.239$	48.452	-78.904
Frank	$\hat{\theta} = 1.707$	49.355	-80.709
Clayton	$\hat{\theta} = 0.377$	54.982	-91.964

Table 10. Summary of estimates of copula parameters, nine assets.

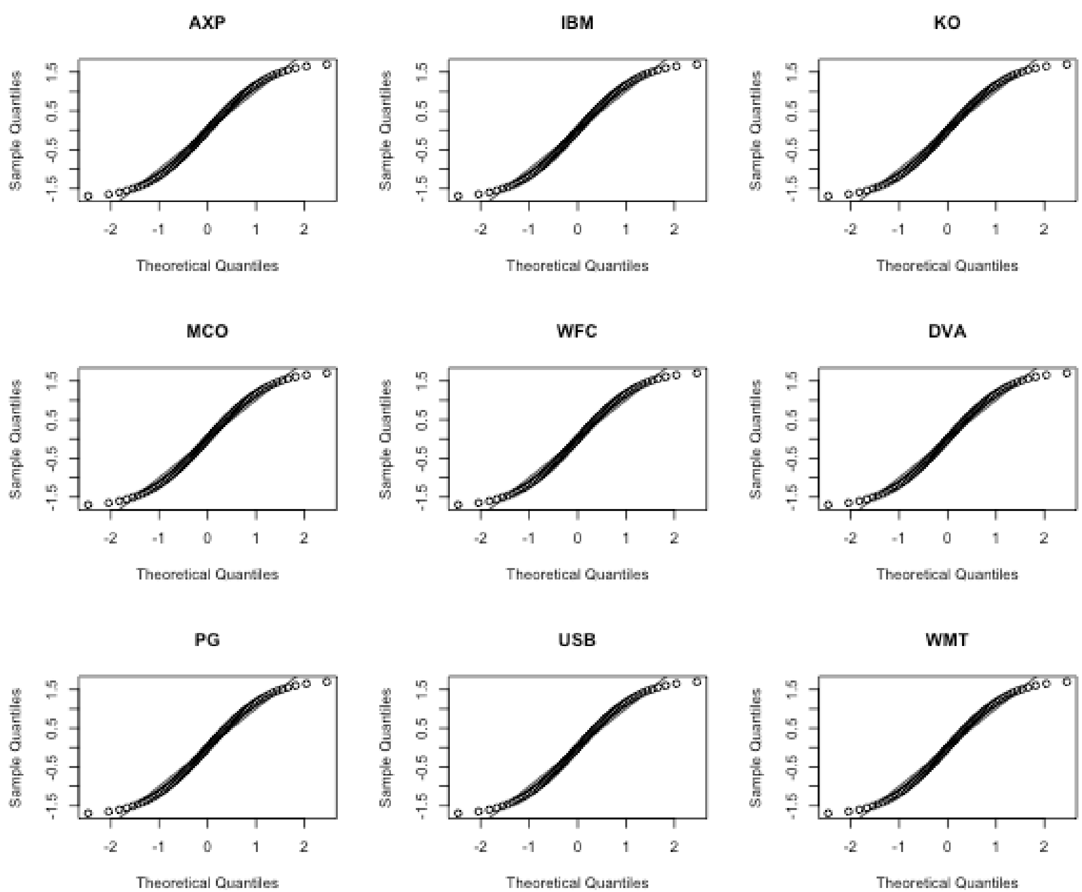


Figure 16: Q-Q plots of the nine large market capitalization assets.

Although t-Copula fits best the joint distribution of the nine assets, it may not fit best the joint distribution of subsets of the nine assets. Take PG and WMT as an example. Table 2 shows the summary of estimates of their copula parameters.

PG and WMT Copula Fit			
Copula Family	Estimates	Maximized log-likelihood	AIC
Gaussian	$\hat{\rho} = 0.442$	6.778	-9.556
t	$\hat{\rho} = 0.468$ $\hat{\nu} = 7.375$	7.635	-11.269
Gumbel	$\hat{\theta} = 1.366$	5.316	-6.633
Frank	$\hat{\theta} = 3.137$	8.366	-12.731
Clayton	$\hat{\theta} = 0.782$	7.802	-11.605

Table 11. Summary of estimates of copula parameters, PG and WMT.

We can see that instead of t-copula, Frank Copula minimizes AIC of PG and WMT. Figure 17 plots contours of the distribution functions of six copulas: the empirical copula and five estimated parametric copulas. Since the five estimated parametric copulas AIC are quite similar, their contours of the distribution functions are also similar to each other.

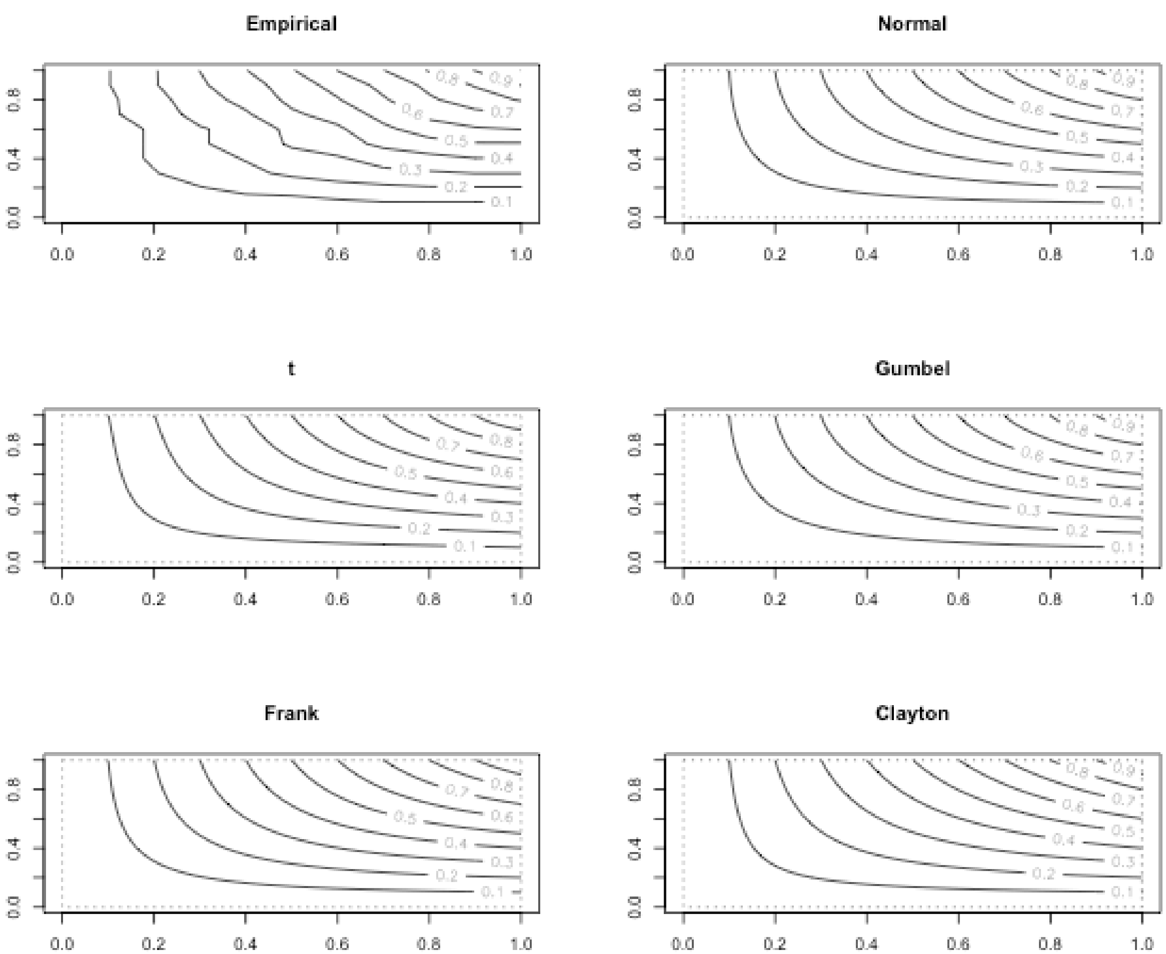
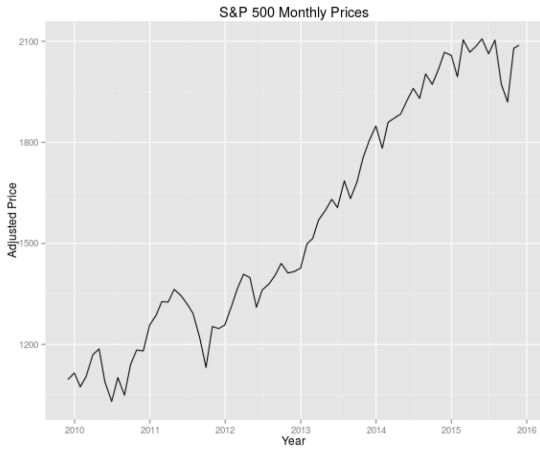
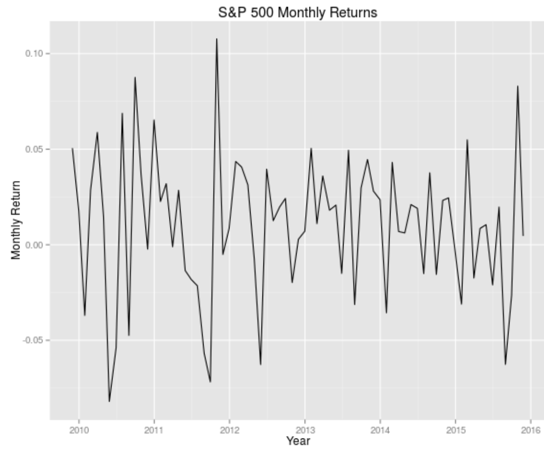


Figure 17: Empirical copula, and fitted copulas using five parametric models.

D. Time Series



(a) The Price of S&P500 Index



(b) The Return of S&P500 Index.

Figure 18: S&P Price and Returns.

Table S12: ARIMA Model

Call:

arima(x = SP_mp_ts, order = c(1, 1, 0))

Coefficients:

ar1
-0.105
s.e. 0.1165

sigma^2 estimated as 3078: log likelihood = -391.32, aic = 786.64

E. Univariate Fitting of Individual Assets

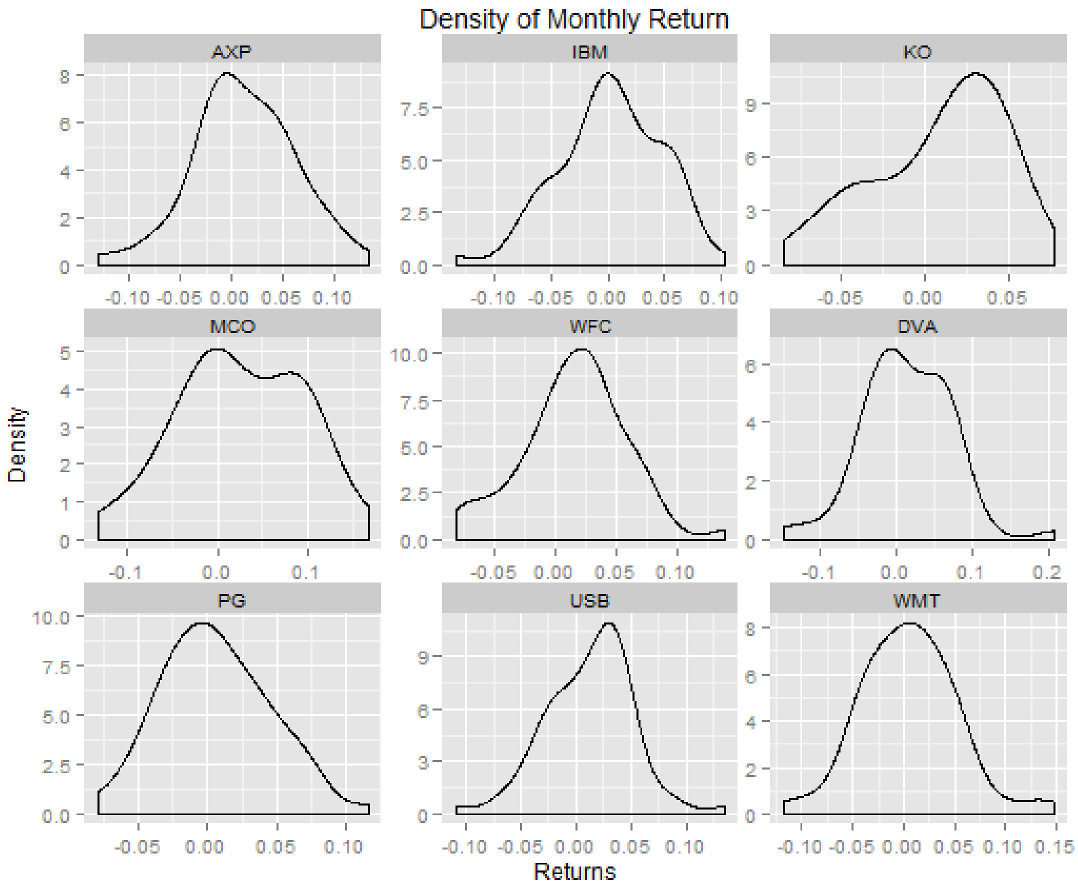


Figure 19: Density plots for all assets.

Univariate Fitting of Individual Assets					
Criteria	T-distribution	Skewed T-distribution	Generalized Error Distribution	Skewed Generalized Error Distribution	Best Fit
AXP.AIC	-189.00	-187.01	-188.92	-186.93	t
AXP.BIC	-182.67	-178.57	-182.59	-178.49	t
IBM.AIC	-199.48	-198.41	-199.51	-198.50	ged
IBM.BIC	-193.14	-189.97	-193.18	-190.05	ged
KO.AIC	-216.27	-223.46	-220.26	-223.47	skewed ged
KO.BIC	-209.94	-215.02	-213.93	-215.02	skewed ged
MCO.AIC	-148.28	-146.57	-150.27	-149.07	ged
MCO.BIC	-141.94	-138.12	-143.94	-140.62	ged
WFC.AIC	-206.99	-205.36	-207.42	-205.78	ged
WFC.BIC	-200.66	-196.92	-201.09	-197.33	ged
DVA.AIC	-169.52	-167.54	-168.24	-166.44	t
DVA.BIC	-163.19	-159.09	-161.90	-158.00	t
PG.AIC	-216.48	-215.88	-216.54	-215.91	ged
PG.BIC	-210.15	-207.43	-210.21	-207.47	ged
USB.AIC	-216.51	-216.61	-215.43	-216.27	skewed t
USB.BIC	-210.17	-208.17	-209.10	-207.83	t
WMT.AIC	-194.74	-192.95	-194.26	-192.54	t
WMT.BIC	-188.41	-184.50	-187.93	-184.10	t

Table 13. Univatiate model fits.

References

[1] "Berkshire Hathaway Portfolio Tracker." CNBC. Web. 7 Dec. 2015. http://www.cnbc.com/berkshire-hathaway-portfolio/.

[2] "Selected Interest Rates (Daily) - H.15." FRB: H.15 Release–Selected Interest Rates–Historical Data. Web. 7 Dec. 2015. http://www.federalreserve.gov/releases/h15/data.htm.

[3] Ruppert, D. (2011). Statistics and data analysis for financial engineering. New York, NY: Springer.