HW0 Stat W4400

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1. $B_{2,1} = 3$
2. $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix}$
3. $A * B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 14 & 20 \end{bmatrix}$
4. rank(A) = number of leading 1s in rref(A)
   rref(A) = $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $\hat{}$rank(A) = 1
5. $\begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}$
   compute the determinant
   $det(\begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}) = (1-\lambda)(4-\lambda) = 0$
   $4 - \lambda - 4 * \lambda + \lambda^2 - 4 = 0$
   $\lambda^2 - 5 * \lambda = 0$
   $\lambda * (\lambda - 5) = 0$
   $\lambda_1 = 0$ and $\lambda_2 = 5$ Thus the greatest eigenvalue is 5
6. $ker(\begin{bmatrix} 1 & -1/2 \\ 2 & -1 \end{bmatrix})$ =¿ $\begin{bmatrix} 1 & -1/2 \\ 0 & 0 \end{bmatrix}$ The eigenvector is thus $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
7. $|B| = (4) - 6 = -2$
8. $x^T A x = \begin{bmatrix} 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix} * \begin{bmatrix} 4 \\ 8 \end{bmatrix} = 16$
9. $\begin{bmatrix} 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 2 & 1 \end{bmatrix} = 5$
10. $\begin{bmatrix} 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$
11. $\sqrt{4+1} = \sqrt{5}$

12. $\nabla_x(f(x)) = \nabla_x(x^T Ax)$
$= x^T * A^T + x^T A$
NOTE: $A^T = A$
$= 2X^T * A = 2 * (\begin{bmatrix} 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}) = \begin{bmatrix} 8 \\ 16 \end{bmatrix}$

13. $\nabla_x(2x^T A)$
$= 2A = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

14. 2

15. Var(Y) = E[Y^2]-$(E[X])^2$
Vary(Y) + $(E[X])^2 = E[Y^2]$

16. E(y+w) = E(y) + E(w) = 2.7 + 3.1 = 5.8
Var(y+w) = Var(w) + Var(Y) - 2Cov(w,y)
because w,y are independent cov(w,y) = 0
Var(y+w) = Var(w) + Var(Y) = 8 + 15 =23

17. The formula for the normalizing constant in this case is :$\frac{1}{(2\pi)^1 * |\sum|^{1/2}}$
$|\sum|$ is determined by taking the determinant of the covariance matrix:
$|\sum| = 12 - 3 = 9$
s$|\sum|^{1/2} = 3$
$\frac{1}{(2\pi)^1 * |\sum|^{1/2}} = \frac{1}{2\pi * 3} = \frac{1}{6\pi}$

18. support(z) = k ϵ 0,1

19. By the Binomial Theorem: $\binom{n}{k} p^k (1-p)^{n-k}$

20. $h(x_1) = 1/3 * x_1^3 - 1/2 * x_1^2 - 6x_1 + 27/2$
$h'(x_1) = x_1^2 - x_1 - 6$
$(x-3)(x+2) = 0$
$x = 3; x = -2$
$h(3) = 1/3 * 3^3 - 1/2 * 3^2 - 6 * 3 + 27/2 = 0$
$h(-2) = 1/3 * -2^3 - 1/2 * -2^2 - 6 * -2 + 27/2 = 20.833$
now check the boudnary: $h(-4) = 49/6$
$h(4) = 17/6$
Thus the max is when $x_1 = -2$

21. Looking at the values computed in the previous question the min occurs when $x_1 = 0$

22. $\int_0^1 1/z * h(x) = 1/z * \int_0^1 1/3h(x)^3 - 1/2(x)^3 - 6x + 27/2 = 1/z * [1/12 * x^4 - 1/6 * x^3 - 3x^2 + 27/2 * x]_0^1 = 1/z * 12/125$
$1/z * 125/12 = 1$
$z = 125/12$

23. $b(x_1 * x_2^3)$
$\int_A b(x)dx = \int_A x_1 * x_2^3 = [9/8 * x_2^4]_0^2 = 9/8 * 16 = 9 * 2 = 16$

24. $c(x) = x_1 + \sqrt{3} * x_2$
$x_1^2 + x_2^2 = 1$
$x_1 = \sqrt{1 - x_2^2}$ $c(x_2) = \sqrt{1 - x_2^2} + \sqrt{3} * x_2$
$\frac{dc(x_2}{dx_2} = \frac{-x}{\sqrt{1-x_2^2}} = 0$
$\sqrt{3} * \sqrt{1 - x_2^2} = x_2$
$3 * (1 - x_2^2) = x_2$
$x_2 = \sqrt{3/4}$
$x_1 = 1/2$

25. $g(x) = -x_1 * log(x_1) - x_2 * log(x_2)$
$x_1 = 1 - x_2$
$g(x_2) = (1 - x_2) * log(1 - x_2 - x_2 * log(x_2))$
$\frac{dg(x_2)}{dx_2} = -(1 - x_2 * -1/(1 - x_2) - log(1 - x_2)) - x_2 * 1/x_2 - log(x_2)$
$= log(1 - x_2) + 1 - log(x_2 - 1) = log(1 - x_2) - log(x_2)$
$1 - x_2 - x_2 = 0$
$x_2 = 1/2 => x_1 = 1/2$