

# Dual Learning Algorithm for Delayed Conversions

---

Yuta Saito<sup>1</sup>, Gota Morishita<sup>2</sup>, and Shota Yasui<sup>3</sup>

<sup>1</sup>Tokyo Institute of Technology

<sup>2</sup>Independent.

<sup>3</sup>CyberAgent, Inc.

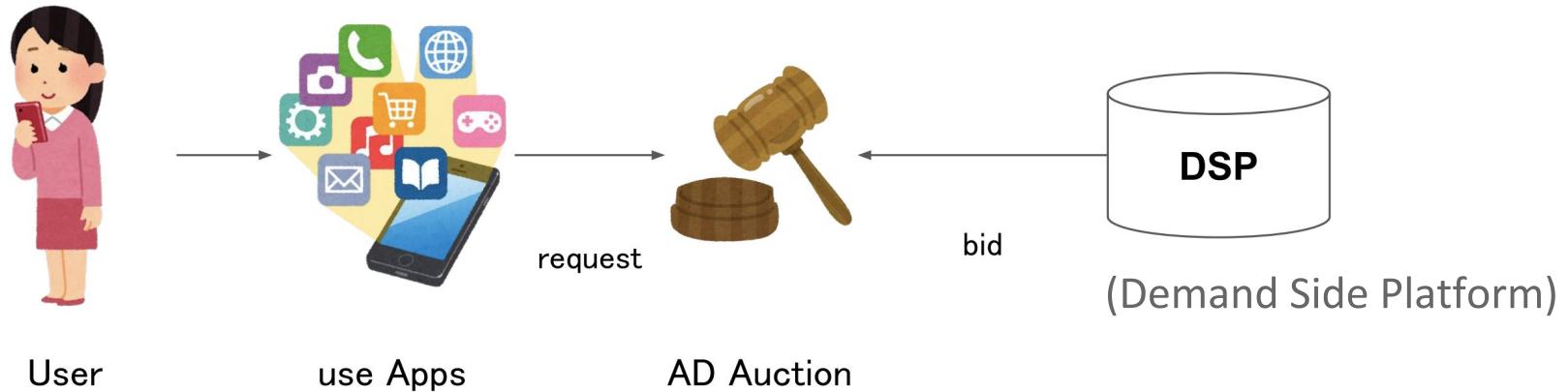


Tokyo Tech



# CVR prediction in Real-Time Bidding (RTB)

- In online advertising, DSP participates in ad auction to obtain ad impression
- **The optimal bid price in the auction is user's conversion rate  
(auction theory result)**



## The ideal loss function in predicting CVR

---

To predict CVR, one wants to minimize the following **ideal loss function**

$$\mathcal{L}_{ideal}^{CVR}(f) = \mathbb{E}_{(X,Y)} \left[ \underline{Y} \delta^{(1)}(f(X)) + (1 - \underline{Y}) \delta^{(0)}(f(X)) \right]$$



**True Conversion Label**

## The ideal loss function in predicting CVR

---

To predict CVR, one wants to minimize the following **ideal loss function**

$$\mathcal{L}_{ideal}^{CVR}(f) = \mathbb{E}_{(X,Y)} \left[ Y \delta^{(1)}(\underline{f(X)}) + (1 - Y) \delta^{(0)}(\underline{f(X)}) \right]$$


**CVR Predictor  
(machine learning)**

## The ideal loss function in predicting CVR

---

To predict CVR, one wants to minimize the following **ideal loss function**

$$\mathcal{L}_{ideal}^{CVR}(f) = \mathbb{E}_{(X,Y)} \left[ Y \underline{\delta^{(1)}}(f(X)) + (1 - Y) \underline{\delta^{(0)}}(f(X)) \right]$$

example) cross-entropy loss

**(local) loss functions**

$$\delta^{(1)}(f) = -\log(f(X)), \quad \delta^{(0)}(f) = -\log(1 - f(X))$$

## The delayed feedback issue in CVR prediction

---

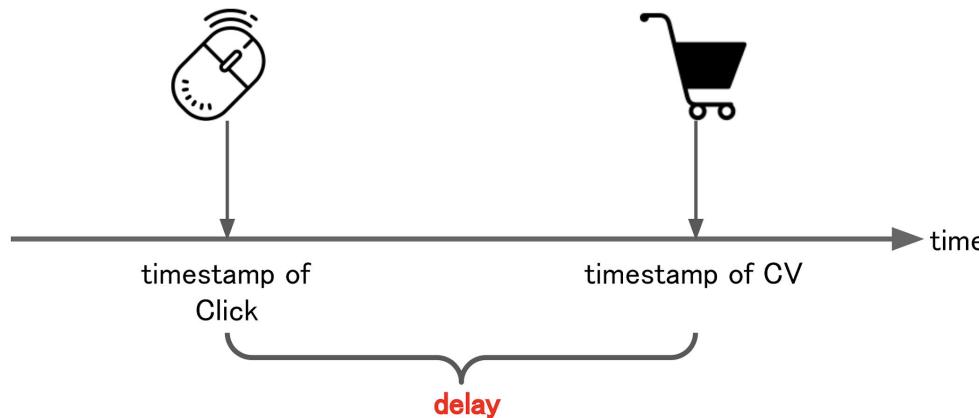
It is **desirable to optimize the ideal loss function to predict CVR (empirical risk minimization; ERM)**

## The delayed feedback issue in CVR prediction

---

It is desirable to optimize the ideal loss function to predict CVR

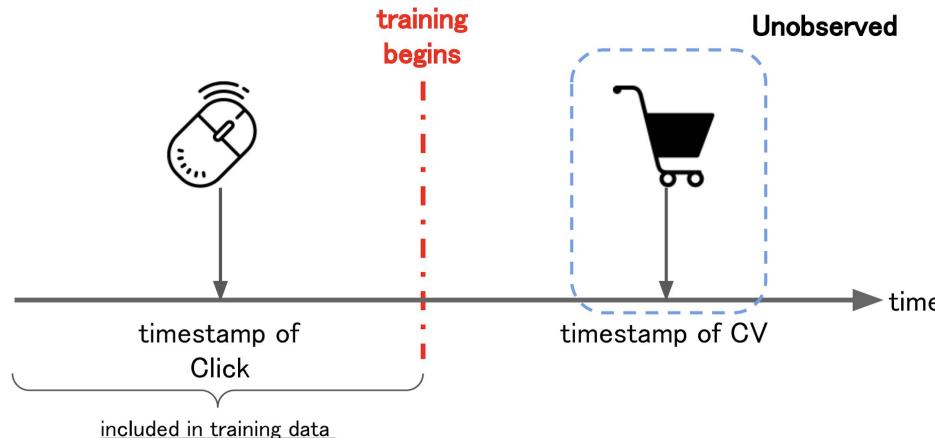
However, the **delayed feedback** issue emerges here



## The delayed feedback issue in CVR prediction

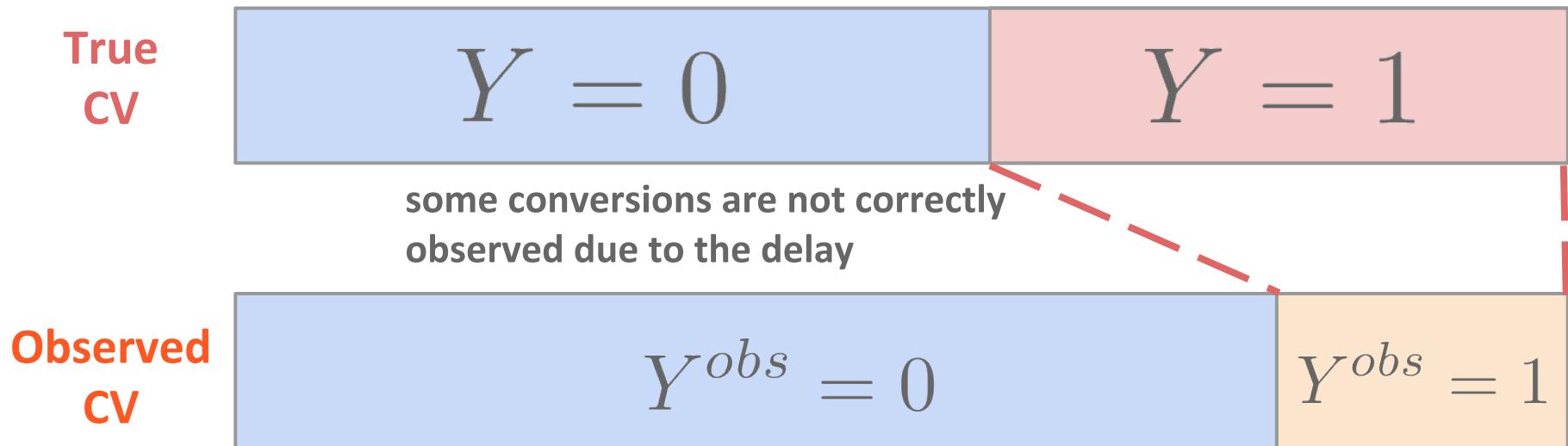
It is desirable to optimize the ideal loss function to predict CVR

However, the **delayed feedback** issue emerges here



## The delayed feedback issue in CVR prediction

As a result, there is a critical difference between  
the **true conversion label** and the **observed conversion label**



## Modeling Observed Conversions

---

To understand difficulties in modeling delayed feedback, we used the following probabilistic model

$$Y^{obs} = O_i \cdot Y_i$$

Observed  
Conversion Label

True Conversion Label  
(prediction target)

## Modeling Observed Conversions

---

To understand difficulties in modeling delayed feedback,  
we used the following probabilistic model

$$Y^{obs} = O_i \cdot Y_i$$



Observation indicator:

whether the true conversion is observed or not

$$Y^{obs} = O_i \cdot Y_i$$

## Challenge 1: positive-unlabeled (PU) problem

---

Only **positive-side feedback is observed**,  
and the **negative feedback is always unobserved**

$$\underline{Y^{obs} = 0} \cancel{\Rightarrow} \underline{Y = 0}$$

The unobservation  
of a conversion

doesn't  
imply

The user will not  
convert eventually

## Challenge 2: missing-not-at-random (MNAR) problem

---

Some positive conversions are much more frequently observed

$$P(Y_i^{obs} = 1 \mid X_i, E_i) = \theta(X_i, E_i) \cdot \gamma(X_i)$$

---

not uniform among ad requests

propensity score  $\theta(X_i, E_i) = P(O_i = 1 \mid X_i, E_i)$

CVR  $\gamma(X_i) = P(Y_i = 1 \mid X_i)$

## Naive Approach: Directly Imputing Observed Conversions

A simple way to predict CVR is naive direct imputation

ideal loss



$$\frac{1}{n} \sum_{i=1}^n \left[ \underline{Y_i} \cdot \delta_i^{(1)} + (1 - \underline{Y_i}) \cdot \delta_i^{(0)} \right]$$

naive loss

$$\frac{1}{n} \sum_{i=1}^n \left[ \underline{Y_i^{obs}} \cdot \delta_i^{(1)} + (1 - \underline{Y_i^{obs}}) \cdot \delta_i^{(0)} \right]$$

## Naive Approach: Directly Imputing Observed Conversions

---

Naive loss is biased because it ignores critical challenges

$$\mathbb{E} [\text{naive loss}] \neq \mathcal{L}_{\text{ideal}}(f)$$

---

The expectation of the naive loss

The ideal loss function

**Naive loss fails to approximate the ideal loss**

## Existing Methods

---

- **Delayed Feedback Model (Chapelle. 2014)**
  - addresses PU problem by EM-like procedure
  - based on parametric assumption on delay distribution
  - does not consider missing-not-at-random problem
- **Importance Weighting Methods (Ketena et al., 2019)**
  - addresses MNAR problem by importance weighting
  - does not tackle the positive-unlabeled problem

## Our Approach: Unbiased Estimation of Ideal Loss Function

We propose the **first unbiased estimator** combining  
inverse propensity weighting & positive-unlabeled learning

The diagram illustrates the construction of the IPS loss from the ideal loss. It shows two equations side-by-side. The top equation is the ideal loss, and the bottom equation is the IPS loss. Dashed arrows point from the terms in the ideal loss equation to the corresponding terms in the IPS loss equation, indicating their correspondence.

ideal loss

$$\frac{1}{n} \sum_{i=1}^n \left[ Y_i \cdot \delta_i^{(1)} + (1 - Y_i) \cdot \delta_i^{(0)} \right]$$

IPS loss

$$\frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i^{obs}}{\theta(X_i, E_i)} \delta_i^{(1)} + \left( 1 - \frac{Y_i^{obs}}{\theta(X_i, E_i)} \right) \delta_i^{(0)} \right]$$

## Our Approach: Unbiased Estimation of Ideal Loss Function

---

We propose the **first unbiased estimator** combining  
**inverse propensity weighting & positive-unlabeled learning**

$$\frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i^{obs}}{\underline{\theta(X_i, E_i)}} \delta_i^{(1)} + \left( 1 - \frac{Y_i^{obs}}{\underline{\theta(X_i, E_i)}} \right) \delta_i^{(0)} \right]$$

The basic idea:

upweight conversions having fewer chances to be observed

## Our Approach: Unbiased Estimation of Ideal Loss Function

---

This estimator is proven to be **theoretically unbiased for the ideal loss function**

$$\mathbb{E}[\text{IPS loss}] = \mathcal{L}_{ideal}(f)$$

---

The proposed loss function

The ideal loss function

**The IPS loss successfully approximates the ideal loss**

## Our Approach: Unbiased Estimation of Ideal Loss Function

---

This estimator is proven to be **theoretically unbiased for the ideal loss function**

$$\frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i^{obs}}{\theta(X_i, E_i)} \delta_i^{(1)} + \left( 1 - \frac{Y_i^{obs}}{\theta(X_i, E_i)} \right) \delta_i^{(0)} \right]$$

But, how to estimate the **propensity score** from data?

## Our Approach: Unbiased Estimation of Ideal Loss Function

We can follow the same logic to estimate propensity score with a theoretical guarantee

ideal loss for  
propensity estimation

$$\frac{1}{n} \sum_{i=1}^n \left[ \underline{O_i} \cdot \delta_i^{(1)} + (1 - \underline{O_i}) \cdot \delta_i^{(0)} \right]$$

↓                          ↓                          ↓

ICVR loss

$$\frac{1}{n} \sum_{i=1}^n \left[ \underline{\frac{Y_i^{obs}}{\gamma(X_i)}} \delta_i^{(1)} + \left( 1 - \underline{\frac{Y_i^{obs}}{\gamma(X_i)}} \right) \delta_i^{(0)} \right]$$

## Our algorithm: Dual Learning Algorithm for Delayed Feedback

---

Update CVR predictor ( $f$ ) based on the IPS loss

$$\frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i^{obs}}{g(X_i, E_i)} \delta_i^{(1)} + \left( 1 - \frac{Y_i^{obs}}{g(X_i, E_i)} \right) \delta_i^{(0)} \right]$$


Update propensity estimator ( $g$ ) based on the ICVR loss

$$\frac{1}{n} \sum_{i=1}^n \left[ \frac{Y_i^{obs}}{f(X_i)} \delta_i^{(1)} + \left( 1 - \frac{Y_i^{obs}}{f(X_i)} \right) \delta_i^{(0)} \right]$$

## Experiment: Setups

---

We generated a synthetic dataset:

- 100,000 samples and 30 features
- follows our probabilistic model on delayed feedback
- different delay distributions: exponential or normal

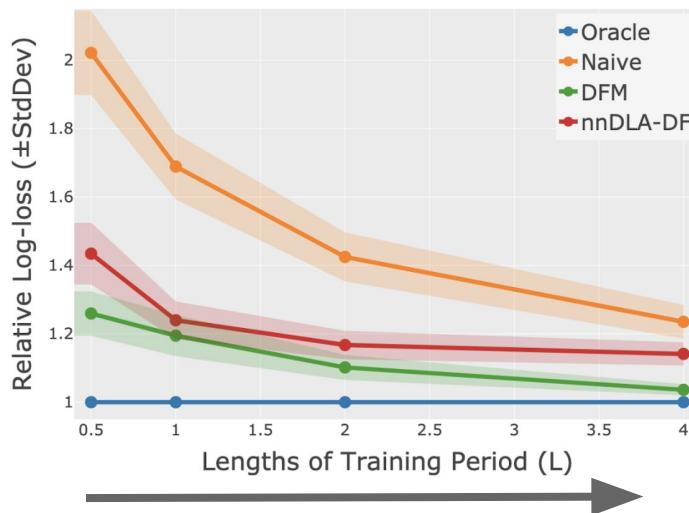
We tested the following methods:

Naive, DFM (Chappelle. 2014), DLA-DF (ours), and Oracle (reference)

## Experiment: Results

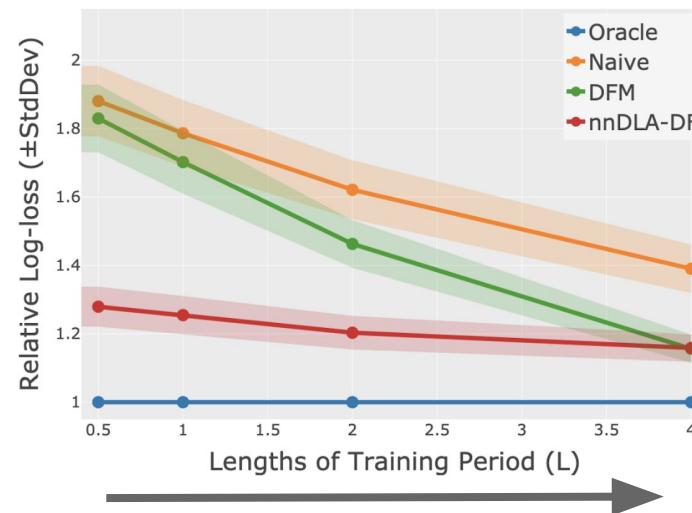
Our method (red) is robust to the delay distribution

delay distribution: exponential



smaller bias in log data

delay distribution: normal



smaller bias in log data

## Conclusions

---

- In predicting CVR, naively using observed conversions might lead to sub-optimal predictions due to the conversion delay
- It is essential to address both positive-unlabeled and missing-not-at-random problems
- We proposed dual learning algorithm that simultaneously addresses the challenges with theoretical guarantees

# Thank you for listening!

---

email: saito.y.bj at m.titech.ac.jp

preprint: <https://usaito.github.io/publications/>