Route-base Network and Fleet Planning Model – Lecture 3

$$Max\ Profit = \sum_{r\in \boldsymbol{R}}\sum_{i\in N}\sum_{j\in N}\left[Yield \times d_{ij} \times (x_{ij}^{r} + \sum_{n\in \boldsymbol{R}} w_{ij}^{rn})\right] - \sum_{r\in \boldsymbol{R}}\sum_{k\in \boldsymbol{K}}(CASK^{k} \times d_{r} \times s^{k} \times z_{r}^{k})$$

Demand constraints:

$$\sum_{r\in \boldsymbol{R}}\left(x_{ij}^{r} + \sum_{n\in \boldsymbol{R}} w_{ij}^{rn}\right) \le q_{ij} \quad ,\forall\, i\,, j \in \boldsymbol{N}$$

$$x_{ij}^{r} \le q_{ij} \times \delta_{ij}^{r} \quad ,\forall\, r \in \boldsymbol{R}, i, j \in \boldsymbol{N}$$

$$w_{ij}^{rn} \le q_{ij} \times \delta_{ih}^{r} \times \delta_{hj}^{n} \quad ,\forall\, r, n \in \boldsymbol{R}, \qquad i, j\ \in \boldsymbol{N}$$

Note: for a route, *r1*, composed by H-> A -> B -> H, the auxiliary parameters δ_{ij}^{r} will only be non-null for the following values:
$\delta_{HA}^{r1} = 1; \delta_{HB}^{r1} = 1; \delta_{AB}^{r1} = 1; \delta_{AH}^{r1} = 1; \delta_{BH}^{r1} = 1.$
Note: for a route, *r2*, composed by H-> C-> H, the auxiliary parameters δ_{ij}^{r} will only be non-null for the following values:
$\delta_{HC}^{r2} = 1; \delta_{CH}^{r2} = 1.$

Flow constraints:

from the hub node (H)

$$\sum_{m\in \boldsymbol{S}_{\boldsymbol{H}}^{r}} x_{Hm}^{r} + \sum_{n\in \boldsymbol{R}}\sum_{p\in \boldsymbol{N}}\sum_{m\in \boldsymbol{S}_{\boldsymbol{H}}^{r}} w_{pm}^{nr} \le \sum_{k\in \boldsymbol{K}} z_{r}^{k} \times s^{k} \times LF \qquad ,\forall\, r \in \boldsymbol{R}\ (with\ j = \boldsymbol{S}_{\boldsymbol{H}}^{r}(\boldsymbol{1})\ and\ i = H)$$

between the spokes

$$\sum_{m\in \boldsymbol{S}_{j}^{r}} x_{im}^{r} + \sum_{m\in \boldsymbol{P}_{i}^{r}} x_{mj}^{r} + \sum_{n\in \boldsymbol{R}}\sum_{p\in \boldsymbol{N}} w_{pj}^{nr} + \sum_{n\in \boldsymbol{R}}\sum_{p\in \boldsymbol{N}} w_{ip}^{rn} \le \sum_{k\in \boldsymbol{K}} z_{r}^{k} \times s^{k} \times LF\ ,\forall\, r \in \boldsymbol{R2}\ with\ i = \boldsymbol{S}_{\boldsymbol{H}}^{r}(\boldsymbol{1})\ and\ j = \boldsymbol{S}_{\boldsymbol{H}}^{r}(\boldsymbol{2})$$

to the hub node (H)

$$\sum_{m\in \boldsymbol{P}_{i}^{r}} x_{mH}^{r} + \sum_{n\in \boldsymbol{R}}\sum_{p\in \boldsymbol{N}}\sum_{m\in \boldsymbol{P}_{i}^{r}} w_{mp}^{rn} \le \sum_{k\in \boldsymbol{K}} z_{r}^{k} \times s^{k} \times LF \qquad ,\forall\, r \in \boldsymbol{R2}\ with\ i = \boldsymbol{S}_{\boldsymbol{H}}^{r}(\boldsymbol{2})\ \&\ \forall\, r \in \boldsymbol{R}\backslash\boldsymbol{R2}\ with\ i = \boldsymbol{S}_{\boldsymbol{H}}^{r}(\boldsymbol{1})\ \ (and, for\ both, j = H)$$

Note: ***R2*** is the set of routes that are composed by more than 2 nodes (e.g., *r1* in our examples)

Note: for a route, *r1*, composed by H-> A -> B -> H, we will have the following sets (dictionaries):
S^{r1} = { H: (A, B, H); A: (B, H); B: (H)} [Subsequent nodes]
P^{r1} = { H: (H); A: (A, H); B: (B, A, H)} [Precedent nodes, including the reference node]

Note: for a route, *r2*, composed by H-> C-> H, we will have the following sets (dictionaries):
S^{r2} = { H: (C, H); C: (H)} [Subsequent nodes]
P^{r2} = { H: (H); C: (C, H)} [Precedent nodes, including the reference node]

Aircraft utilization constraints:

$$\sum_{r\in\boldsymbol{R}}\left(\frac{d_r}{sp^k}+LTO^{rk}+Charge_r^k\right)\times z_r^k\leq BT^k\times AC^k\ ,\forall\ k\in\boldsymbol{K}$$

Aircraft allocation constraints:

$$z_{ij}^k\leq a_r^k\qquad\rightarrow\ a_r^k=\begin{cases}10000\ if\ d_r\leq Range^k\\ \quad 0\quad otherwise\end{cases}$$

NOTE: should be already considered in the generation of the routes set (removed)

Fleet budget constraint:

$$\sum_{k\in\boldsymbol{K}}C^k\times AC^k\leq B$$