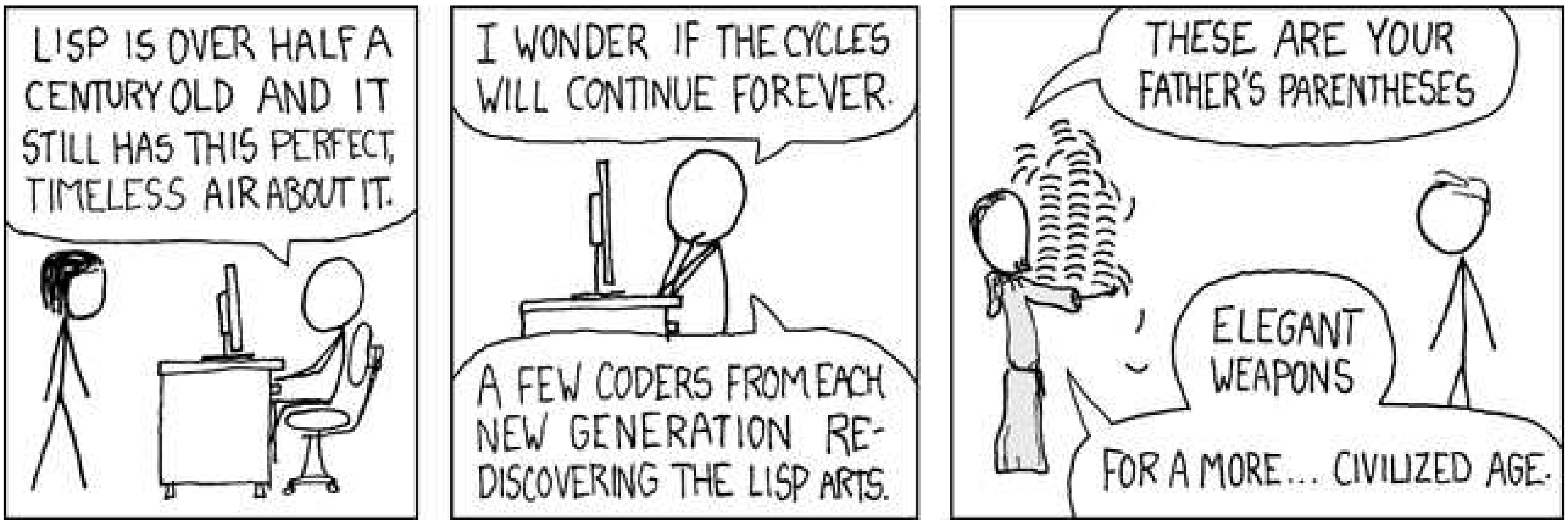
Lecture #24: The Scheme Language

Scheme is a dialect of Lisp:

- "The only programming language that is beautiful."
  —Neal Stephenson
- "The greatest single programming language ever designed"
  —Alan Kay



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Scheme Background

- The programming language Lisp is the second-oldest programming language still in use (introduced in 1958).
- Scheme is a Lisp dialect invented in the 1970s by Guy Steele ("The Great Quux"), who has also participated in the development of Emacs, Java, and Common Lisp.
- Designed to simplify and clean up certain irregularities in Lisp dialects at the time.
- Used in a fast Lisp compiler (Rabbit).
- Still maintained by a standards committee (although both Brian Harvey and I agree that recent versions have accumulated an unfortunate layer of cruft).

Our Subset

- In part, we'll use Scheme to illustrate the *applicative programming* paradigm—computing with no side-effects (save output of results), no assignments, and only non-destructive operations.
- Therefore, we'll leave out Scheme features such as assignment, as well as mutable data structures.
- What's so great about applicative programming?
  - Reasoning about programs can be easier without side-effects.
  - Side-effects and mutations make correct parallel programming more difficult.

Data Types

- We divide Scheme data into *atoms* and *pairs*.
- The classical atoms:
  - Numbers: integer, floating-point, complex, rational.
  - Symbols.
  - Booleans: #t, #f.
  - The empty list: ().
  - Procedures (functions).
- Pairs are like two-element Python tuples, where the elements are (recursively) Scheme values.

Symbols

- Lisp was originally designed to manipulate *symbolic data:* e.g., formulae as opposed merely to numbers.
- Typically, such data is recursively defined (e.g., "an expression consists of an operator and subexpressions").
- The "base cases" had to include numbers, but also variables or words.
- For this purpose, Lisp introduced the notion of a *symbol:*
  - Essentially a constant string.
  - Two symbols with the same "spelling" (string) are by default the same object (but usually, case is ignored).
- The main operation on symbols is *equality.*
- Examples:

```
a   bumblebee   numb3rs   *   +   /   wide-ranging !?@*!!
```

(As you can see, symbols can include non-alphanumeric characters.)

Pairs and Lists

- The Scheme notation for the pair of values V_1 and V_2 is

  $(V_1 \;\; . \;\; V_2)$

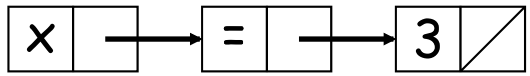
- As we've seen, one can build practically any data structure out of pairs.
- In Scheme, the main one is the (linked) *list*, defined recursively:
  - The empty list, written "()", is a list.
  - The pair consisting of a value V and a list L is a list that starts with V, and whose tail is L.
- Lists are so prevalent that there is a standard abbreviation:

Abbreviation	**Means**
(V)	$(V\ .\ ())$
$(V_1\ V_2 \cdots V_n)$	$(V_1\ .\ (V_2\ .\ (\cdots (V_n\ .\ ()))))$
$(V_1\ V_2 \cdots V_{n-1}\ .\ V_n)$	$(V_1\ .\ (V_2\ .\ (\cdots (V_{n-1}\ .\ V_n))))$

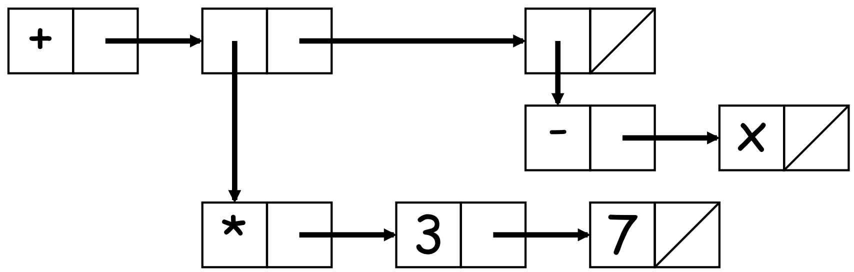
- For our purposes this semester, we'll use the abbreviation exclusively, and won't use structures that require dots.

Examples of Lists

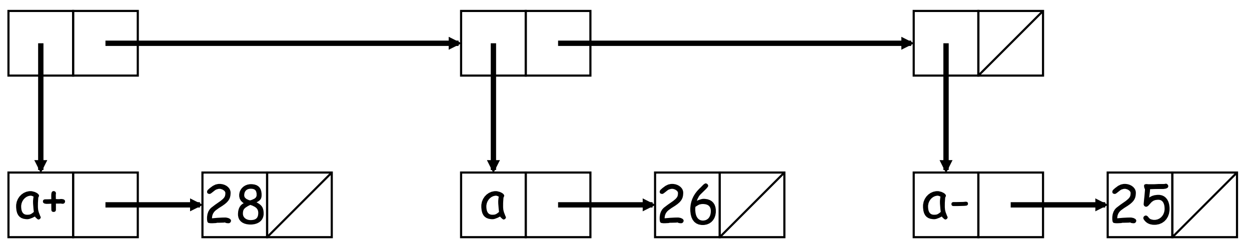
(x = 3)



(+ (* 3 7) (- x))



((a+ 28) (a 26) (a- 25))



Programs

- Scheme expressions and programs are *instances of Lisp data structures*

  "Scheme programs are Scheme data."

- At the bottom, numerals, booleans, characters, and strings are expressions that stand for themselves in a Scheme program: we say they are *self-evaluating.*

- Most lists (aka *forms*) stand for function calls in a Scheme program:

  (OP $E_1 \cdots E_n$)

  as a Scheme expression means "evaluate *OP* and the E_i (recursively), and then apply the value of *OP*, which must be a function, to the values of the arguments E_i." (Sound familiar? It's the same as in Python.)

- Examples:

```
(> 3 2)                     ; 3 > 2 ==> #t
(- (/ (* (+ 3 7 10) (- 1000 8)) 992) 17)
                            ; ((3 + 7 + 10) · (1000 − 8))/992 − 17
(pair? (list 1 2))          ; ==> #t
```

Quotation

- Since programs are data, we have a problem: How do we say, eg., "Set the variable x to the three-element list (+ 1 2)" without it meaning "Set the variable x to the value 3?"
- In English, we call this a *use vs. mention distinction*, and use quotation marks to distinguish mentions ("Copper" is a six-letter word, not a metal.) from uses (Copper is a metal, not a word.)
- In Scheme, we use a *special form*—a construct that does *not* simply evaluate its operands.
- (quote E) yields E itself as the value, *without* evaluating it as a Scheme expression:

```
scm> (+ 1 2)
3
scm> (quote (+ 1 2))
(+ 1 2)
scm> '(+ 1 2)      ; Shorthand. Converted to (quote (+ 1 2))
(+ 1 2)
```

- How about

```
scm> (quote (1 2 '(3 4)))      ;?
```

Special Forms

- (`quote` E) is a *special form*: an exception to the general rule for evaluting functional forms.
- A few other special forms—lists identified by their *OP*—also have meanings that generally do not involve simply evaluating their operands:

```
(if (> x y) x y)                  ; Like Python ... if ... else ...

(and (integer? x) (> x y) (< x z))       ; Like Python 'and'

(or (not (integer? x)) (< x L) (> x U)) ; Like Python 'or'

(lambda (x y) (/ (* x x) y))      ; Like Python lambda
                                  ; yields function

(define pi 3.14159265359)         ; Definition, like Python first assignment
(define (f x) (* x x))            ; Function Definition, like Python def
```

Traditional Conditionals

Also, the fancy traditional Lisp conditional form:

```
scm> (define x 5)
scm> (cond ((< x 1) 'small)
           ((< x 3) 'medium)
           ((< x 5) 'large)
           (#t      'big))
big
```

which is the Lisp version of Python's

```
"small" if x < 1 else "medium" if x < 3 else "large" if x < 5 else "big"
```

Symbols

- When evaluated as a program, a symbol acts like a variable name.
- Variables are bound in environments, just as in Python, although the syntax differs.
- To define a new symbol, either use it as a parameter name (later), or use the "define" special form:

```
(define pi 3.1415926)
(define pi**2 (* pi pi))
```

- This defines the symbols in the current environment. The last expression in each definition is evaluated first and then bound to the symbol.

Function Evaluation

- Function evaluation is just like Python: same environment frames, same rules for what it means to call a user-defined function.
- To create a new function, we use the `lambda` special form:

```
scm> ( (lambda (x y) (+ (* x x) (* y y))) 3 4)
25
scm> (define fib
         (lambda (n) (if (< n 2) n (+ (fib (- n 2)) (fib (- n 1))))))
scm> (fib 5)
5
```

- The last is so common, there's an abbreviation:

```
scm> (define (fib n)
         (if (< n 2) n (+ (fib (- n 2)) (fib (- n 1)))))
```

Numbers

- All the usual numeric operations and comparisons:

```
scm> (- (quotient (* (+ 3 7 10) (- 1000 8)) 992) 17)
3
scm> (/ 3 2)
1.5
scm> (quotient 3 2)
1
scm> (quotient -3 2) ; quotient rounds towards 0 (not like Python)
-1
scm> (> 7 2)
#t
scm> (= 3 (+ 1 2))
#t
scm> (integer? 5)
#t
scm> (integer? 'a)
#f
```

Lists and Pairs

- Pairs (and therefore lists) have a basic constructor (`cons`) and accessors (`car` and `cdr`):

```
scm> (cons 1 2)
(1 . 2)
scm> (cons 'a (cons 'b '()))  ; Like Link("a", Link("b", Link.empty))

(a b)
scm> (define L '(a b c))
scm> (car L)          ; Like L.first
a
scm> (cdr L)          ; Like L.rest (Pamela suggests cdr = see da rest)
(b c)
scm> (car (cdr L))
b
scm> (cdr (cdr (cdr L)))
()
```

- And one that is especially for lists:

```
scm> (list (+ 1 2) 'a 4)
(3 a 4)
scm> ; Why not just write ((+ 1 2) a 4)?
scm> ; Or '((+ 1 2) a 4)?
```

Equivalence Operations

```
scm> (= 1 (- 2 1))  ; Works for numbers only
#t
scm> (eqv? 1 2)   ; Works for numbers, empty list, booleans, symbols
#f
scm> (eqv? 1 (- 2 1))
#t
scm> (define L '(1 2 3))
scm> (eqv? L L) ; Like Python's "is" elsewhere
#t
scm> (eqv? L '(1 2 3))
#f
scm> (eq? L '(1 2 3))  ; eq? is Python's "is" (might not work for numbers)
#f
scm> ;  equal? is like eqv?, but also does deep equality for pairs (and
scm> ;  therefore also for lists.
scm> (equal? '((1 2) 3 (4)) (list (list 1 2) 3 (list 4)))
#t
scm> (eqv? '((1 2) 3 (4)) (list (list 1 2) 3 (list 4)))
#f
```

Binding Constructs: Let

- Sometimes, you'd like to introduce local variables or named constants.
- The `let` special form does this:

```
scm> (define x 17)
scm> (let ((x 5)
           (y (+ x 2)))
         (+ x y))
24
```

- This is a *derived form,* equivalent to:

```
scm> ((lambda (x y) (+ x y)) 5 (+ x 2))
```

Loops and Tail Recursion

- With just the functions and special forms so far, can write anything.
- But there is one problem: how to *get* an arbitrary iteration that doesn't overflow the execution stack because recursion gets too deep?
- In a correct Scheme implementation, *tail-recursive functions work like iterations.*

Loops and Tail Recursion (II)

- So for this program:

Scheme	Python
see below	see below

Scheme

```scheme
(define (sumsq n)
  (define (sumsq1 s n)
    (if (<= n 0) s
        (sumsq1 (+ s (* n n))
                (- n 1))))
  (sumsq1 0 n))
(sumsq 1000)
```

Python

```python
def sumsq(n):
    def sumsq1(s, n):
        if n <= 0:
            return s
        return sumsq1(s + n * n,
                      n - 1)
    return sumsq1(0, n)
sumsq(1000)
```

The typical Python implementation of `sumsq1` will execute `return s` at the time when there are 1000 other calls on `sumsq1` that have not yet returned. This often results in an exception.

- But in a correct Scheme implementation, each recursive call of `sumsq1` *replaces* the call from which it occurs.
- At each inner tail call, in other words, we forget the sequence of calls that got us there, so the system need not use more memory to go deeper.

Tail Recursion: A Simple Example

- We can think of the execution of `(sumsq1 1000)` as a sequence of steps in which one call is replaced by another:

```
    (sumsq 1000)
==> (sumsq1 0 1000)
==> (if (<= 1000 0) 0 (sumsq1 (+ 0 (* 1000 1000)) (- 1000 1)))
==> (sumsq1 (+ 0 (* 1000 1000)) (- 1000 1))
==> (sumsq1 1000000 999)
==> (if (<= 999 0) 1000000 (sumsq1 (+ 1000000 (* 999 999)) (- 999 1)))
==> (sumsq1 (+ 1000000 (* 999 999)) (- 999 1))
==> (sumsq1 1998001 998)
==> ...
==> (sumsq1 333833500 0)
==> (if (<= 0 0) 333833500 (sumsq1 (+ 333833500 (* 0 0)) (- 0 1)))
==> 333833500
```