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Interpretation of fracture spacing and intensity

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ABSTRACT: A class of fracture intensity measures in one, two, and three dimensions has been defined that allows for the definition of fracture frequency without requiring reference to specific sets or orientations. Measure $\mathbf{P_{32}}$ (fracture area per unit volume) was found to be the most useful measure for fracture intensity in three dimensions. Relationships between intensity measures are described based on solutions from the field of stochastic geometry. The use of these measures significantly improves the consistency of discrete fracture analysis and modelling for mechanical and hydrologic applications.

1. INTRODUCTION

Fracture patterns are described in terms of distributions for orientation, size, shape, spatial location, and intensity (Dershowitz and Einstein, 1988). Of these, intensity is one of the most important, but the least well characterized. Fracture intensity is generally noted in terms of fracture spacing $\mathbf{S_f}$, the mean distance between fractures within a given set, as measured along a particular line such as a borehole or scanline (Figure 1). This measure is relatively easy to determine in

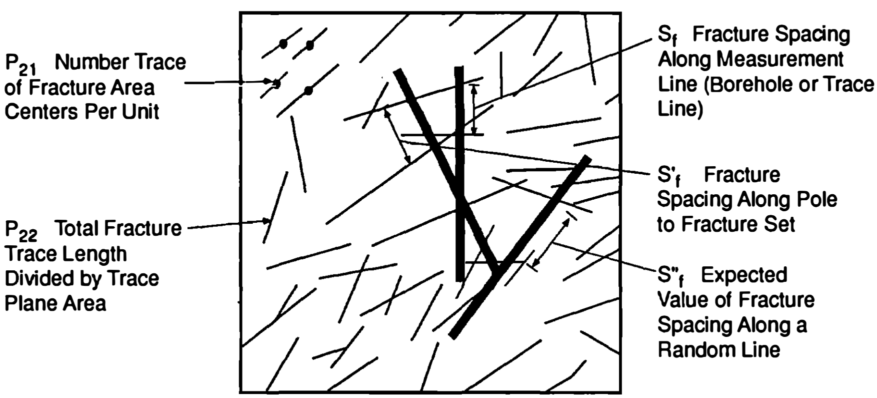

Figure 1. One and Two Dimensional Fracture Intensity Measures

the field, but is not useful for describing fractures in two or three dimensions. It is also dependent upon the subjective definition of fracture sets in the field.

In three dimensional analysis, fracture intensity is generally defined as the number of fracture centers per unit volume, $\mathbf{P_{31}}$. This measure is useful only where fractures are much smaller than the region being analyzed, such that fracture centers represent individual fractures, rather than fragments of fractures that may be within or outside the region. In addition, $\mathbf{P_{31}}$ can only be related to fracture spacing $\mathbf{S_f}$ through the fracture size and orientation distributions.

The goal of this paper is to define a consistent set of fracture intensity measures in one, two, and three dimensions, together with the relationships between them.

2. FRACTURE INTENSITY MEASURES

Fracture intensity measures are classified based upon the dimension of the measurement region, and the dimension of the fracture (Table 1). The formats for fracture intensity measures are based upon the designation **P** ("persistence"), followed by subscripts designating the dimension of the measurement region and the fracture attribute (Dershowitz, 1984). The measurement region dimension is specified as $\mathbf{P_1}$ (one dimensional line), $\mathbf{P_2}$ (two dimensional trace plane), or $\mathbf{P_3}$

Table 1: Measures for Fracture Intensity

Dimension of Measurement Region --> / Dimension of Fracture Measure	1: Line Measure (Borehole or Scan line)	2: Area Measure (Traceplane)	3: Volume (Rock Mass)
1: Number of Fractures	$\mathbf{P_{11}}$: Number of fractures per unit length of scan line or borehole (inverse spacing) $[L^{-1}]$	$\mathbf{P_{21}}$: Number of fractures per unit area of trace plane $[L^{-2}]$	$\mathbf{P_{31}}$: Number of fractures per unit volume of rock $[L^{-3}]$
2: Dimension one less than that of measurement region		$\mathbf{P_{22}}$: Length of fracture traces per unit area of trace plane $[L^{-1}]$	$\mathbf{P_{32}}$: Area of Fractures per unit volume of rock $[L^{-1}]$
3: Dimension equal to that of measurement region			$\mathbf{P_{33}}$: Volume of Fractures per unit volume of rock [-]

(three dimensional volume). The fracture dimension is specified by the relationship between the dimension used for describing fractures and the dimension used for the region.

2.1 *One Dimensional Intensity Measures*

Fracture spacing $\mathbf{S_f}$ is the most commonly used measure for fracture intensity (Figure 1). Unfortunately, fracture spacing is a very difficult measure to interpret because it is closely coupled with the definition of sub-parallel fracture sets, and is dependent upon the relationship between the orientation of fractures and the orientation of the line along which spacing is measured.

For sets of parallel fractures, attempts have been made (Einstein et al., 1980) to define a corrected mean spacing $\mathbf{S_f}'$ as a measure along the line perpendicular to the fractures. The spacing measured in the field is then corrected based on the angle θ between the mean pole of the fractures and the line along which the fracture spacing was measured,

$$S_f' = S_f \cos\theta \tag{1}$$

However, for sub-parallel fractures or fractures with significant orientation dispersion, the angle θ becomes increasingly difficult to define. Further, the definition of sets is frequently subjective and irreproducible, further diminishing the usefulness of $\mathbf{S_f}$ and $\mathbf{S_f}'$.

Intensity measure $\mathbf{P_{11}}$ is directly related to fracture spacing $\mathbf{S_f}$ by,

$$P_{11} = \frac{1}{S_f'} \tag{2}$$

Like $\mathbf{S_f}$, $\mathbf{P_{11}}$ is directly dependent upon the relative orientations of the fractures and the line along which orientation is measured. Another approach to normalizing $\mathbf{S_f}$ is to specify $\mathbf{S_f}''$ not for a given orientation, but as the expected value of spacing for a randomly oriented measurement line. In this case,

$$S_f'' = S_f \int_{-\pi/2}^{\pi/2} f_\theta(\theta)\cos\theta\, d\theta \tag{3}$$

where $f_\theta(\theta)$ is the distribution of the fracture orientations. For a Fisher orientation distribution,

$$S_f'' = S_f \int_{-\pi/2}^{\pi/2} e^{\kappa\cos(\theta-\bar{\theta})}\sin(\theta-\bar{\theta})\cos(\theta)\, d\theta \tag{4}$$

Intensity measure P_{11} has the same characteristics as S_f, and must also be defined in terms of the relative orientations of the fractures and the line along which P_{11} is defined. In general, however, when dealing with anything except parallel fracture sets, the definition of P_{11} should be based upon a random line orientation as in equation 4.

It is important to note that while P_{11} and S_f are dependent upon the orientation of fractures, they are independent of fracture size (Figure 2). For a given fracture orientation, P_{11} and S_f do not change when the fracture size is doubled and the number of fractures is cut in half. As a result, P_{11} and S_f are scale independent — they do not depend on the region in which they are defined.

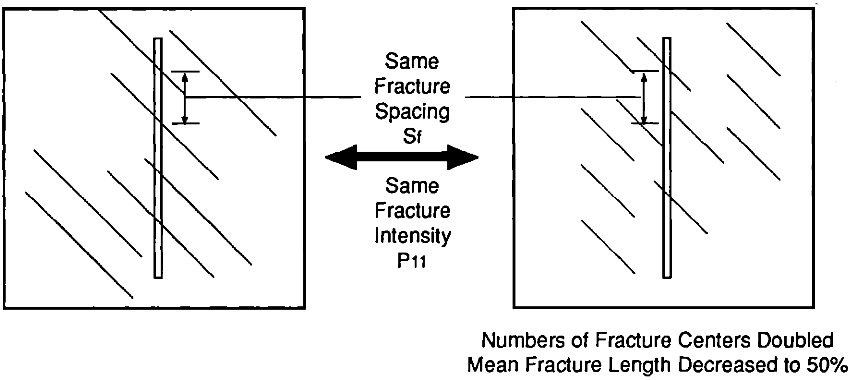

Figure 2. Scale Independent Fracture Intensity Measures S_f and P_{11}

2.2 *Two Dimensional Fracture Intensity Measures*

Fracture intensity on a two dimensional surface is measured by either P_{21}, the number of traces per unit area or P_{22}, the length of traces per unit area (Figure 1). Intensity measure P_{21} depends upon the distribution of fracture size, and as a result is scale dependent at scales smaller than the maximum fracture trace length. Figure 3 illustrates the change in intensity measure P_{21} for decreasing analysis regions.

Intensity measure P_{22} is not scale dependent, since it directly incorporates fracture size.

$$P_{22} = \frac{\int_A L f_L(L) dL}{A} \tag{5}$$

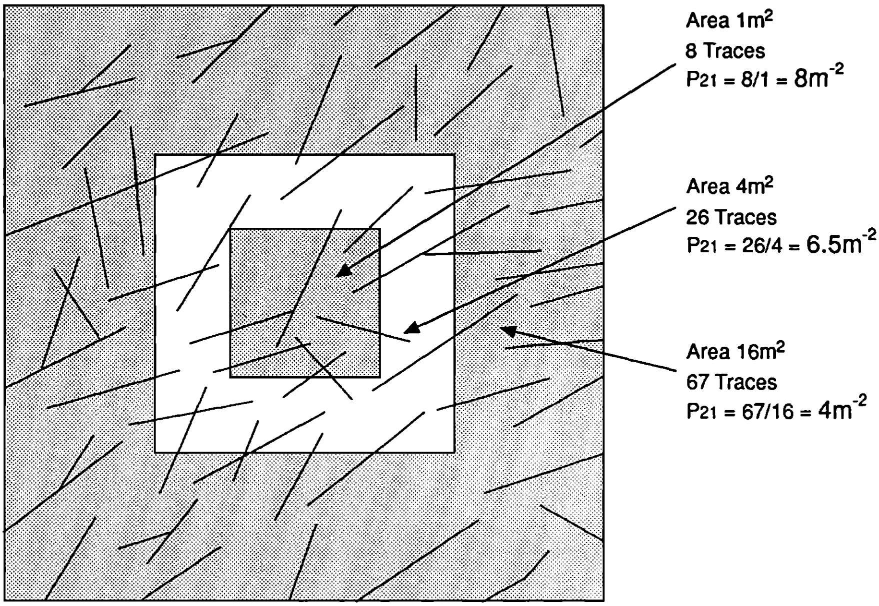

Figure 3. Scale Dependent Fracture Intensity Measure P_{21}

Both $\mathbf{P_{21}}$ and $\mathbf{P_{22}}$ are dependent upon the relative orientation of the fractures and the trace plane on which the measures are defined.

Intensity measure $\mathbf{P_{21}}$ can be related to $\mathbf{P_{22}}$ through the distribution of fracture size,

$$P_{22}=P_{21}\int_0^{\infty} Lf_L(L)dL \qquad (6)$$

$$P_{22}=P_{21}\bar{L}$$

2.3 *Three Dimensional Fracture Intensity Measures*

In three dimensions, fracture intensity can be defined in terms of $\mathbf{P_{31}}$, the number of fractures in a volume, $\mathbf{P_{32}}$, the area of fractures in a volume, or $\mathbf{P_{33}}$, the volume of fractures in a volume (Figure 4). Like $\mathbf{P_{21}}$, $\mathbf{P_{31}}$ is scale dependent, and changes with the size of the region being analyzed for regions smaller than the maximum fracture size. $\mathbf{P_{32}}$ and $\mathbf{P_{33}}$ are not scale dependent, since they directly incorporate fracture size,

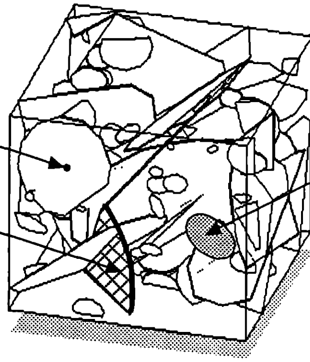

Figure 4. Three Dimensional Fracture Intensity Measures

$$P_{32}=\frac{\int_V A f_A(A)\,dA}{V} \tag{7}$$

$$P_{33}=\frac{\int_V A t f_A(A) f_t(t)\,dA\,dt}{V} \tag{8}$$

Intensity measure $\mathbf{P_{31}}$ can be related to $\mathbf{P_{32}}$ through the distribution of fracture size,

$$P_{32}=P_{31}\int_0^\infty A f_A(A)\,dA \tag{9}$$

$$P_{32}=P_{31}\overline{A}$$

Intensity measure $\mathbf{P_{33}}$ is equivalent to fracture porosity and requires information on fracture thickness. Like $\mathbf{P_{32}}$, it is not scale dependent, and can be defined without respect to fracture orientation. However, it requires additional information on fracture thickness, and is therefore most useful for features in which thickness is significant. Note that $\mathbf{P_{32}}$ is scale dependent if the maximum fracture thickness is comparable to the size of the region being analyzed.

$\mathbf{P_{33}}$ can be related to $\mathbf{P_{32}}$ by the equation,

$$P_{33}=P_{32}\frac{\int_0^\infty A t f_A(A) f_t(t)\,dA\,dt}{\overline{A}} \tag{10}$$

3. RELATIONSHIPS AMONG INTENSITY MEASURES OF DIFFERENT DIMENSIONS

Intensity measures with the dimension $[L^{-1}]$ are scale independent, and can be interrelated by removing the effect of the orientation of the scan line ($\mathbf{P_{11}}$) or trace plane ($\mathbf{P_{22}}$), without additional information. The distribution of fracture size does not affect this transformation unless there is a correlation between orientation and size. As a result, the most useful measure of intensity for three dimensional fracture modelling is $\mathbf{P_{32}}$, since it does not reflect any orientation effects and can be directly related to fracture spacing $\mathbf{S_f}$ and $\mathbf{P_{11}}$ without specification of fracture size. When $\mathbf{P_{32}}$ is used to define intensity, size can be defined independently. The transformation between fracture spacing $\mathbf{S_f}$ and $\mathbf{P_{32}}$ is,

$$P_{32}=\frac{C_{P_3}}{S_f} \tag{11}$$

$\mathbf{C_P}$ is a constant dependent upon the distribution of the orientation of features relative to the line along which the spacing $\mathbf{S_f}$ was measured.

A similar relationship can be defined between $\mathbf{S_f}$ and $\mathbf{P_{22}}$,

$$P_{22}=\frac{C_{P_2}}{S_f} \tag{12}$$

For simple fracture geometries, $\mathbf{C_{P2}}$ and $\mathbf{C_{P3}}$ can be found by the methods of stochastic geometry (Santalo, 1976). For a uniform distribution of fracture orientation, Dershowitz (1984) found a value of $\mathbf{C_{P3}}$ of 2.0. For most fracture geometries, $\mathbf{C_{P3}}$ will vary between 1.0 and 3.0. Values of $\mathbf{C_{P3}}$ and $\mathbf{C_{P2}}$ for different fracture geometries are shown in Figures 5 and 6. These values were obtained by simulation of boreholes with different orientations within fracture networks simulated with the FracMan discrete fracture model (Dershowitz et al., 1991). Fracture networks were defined with a vertical mean pole, a constant fracture radius of 5 meters, and location defined by a Poisson Point Process of fracture centers (Baecher and Lanney, 1978). The trace plane for calculation of $\mathbf{C_{P2}}$ was defined by a vertical plane containing the 6 boreholes with trends from 0° to 90°.

Intensity measures $\mathbf{P_{21}}$ and $\mathbf{P_{31}}$ are dependent upon the distribution of fracture size. As a result, conversions between these measures and scale independent intensity measures such as fracture spacing $\mathbf{S_f}$, $\mathbf{P_{22}}$, and $\mathbf{P_{32}}$ can only be made by specifying the fracture size distribution. Conversions between $\mathbf{P_{21}}$ and $\mathbf{P_{22}}$, and between $\mathbf{P_{31}}$ and $\mathbf{P_{32}}$ are given in equations 6 and 9.

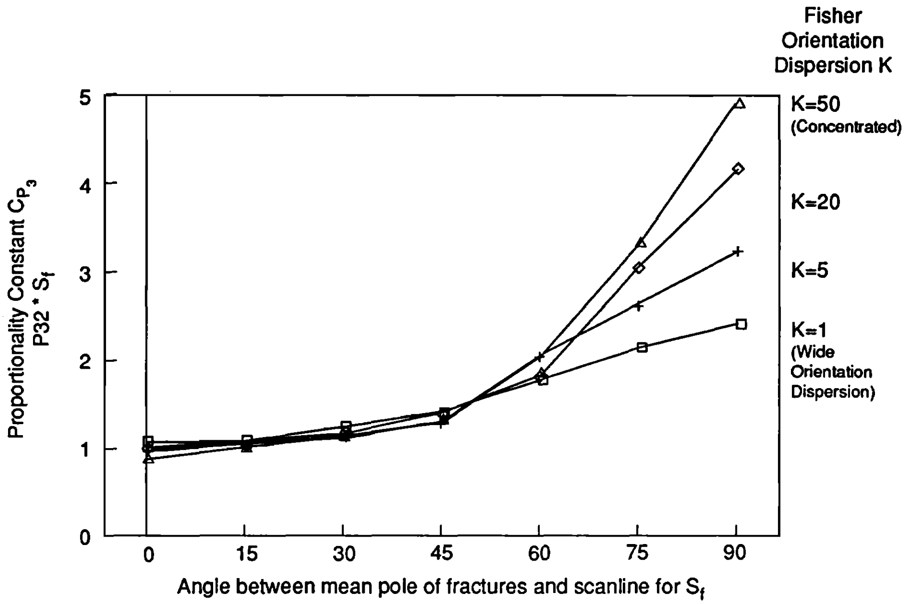

Figure 5. Proportionality Constant C_{P_3} (from FracMan Simulation)

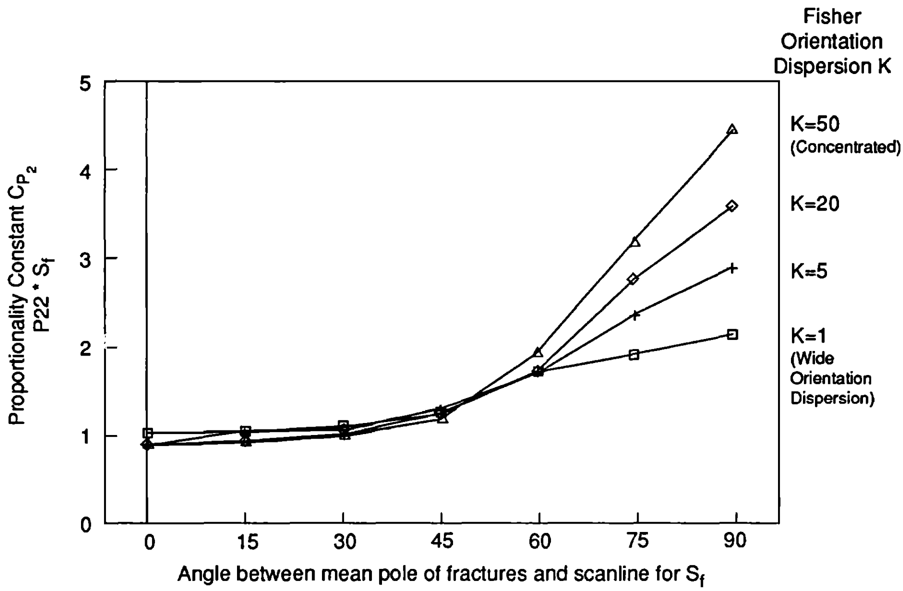

Figure 6. Proportionality Constant C_{P_2} (from FracMan Simulation)

4. APPLICATION OF INTENSITY MEASURES

The above theoretical development has demonstrated the utility of intensity measures $\mathbf{P}_{22}$ and $\mathbf{P}_{32}$ for two and three dimensional rock mechanics applications. In fracture mapping, collection of $\mathbf{P}_{22}$ has been shown to be a more versatile measure of fracture intensity than either $\mathbf{P}_{21}$ or $\mathbf{S}_f$.

Most three dimensional discrete fracture simulation published to date has been carried out based upon constant values of $\mathbf{P}_{31}$. However, when this procedure is used, fracture intensity as measured by mean spacing $\mathbf{S}_f'$, $\mathbf{P}_{22}$, or $\mathbf{P}_{32}$ will vary with the variance of fracture area. The second moment approximation for this is given by,

$$P_{32_\sigma} \approx [P_{31}{}^2 A_{\sigma^2} + A_\mu{}^2 P_{31_{\sigma^2}} + P_{31_{\sigma^2}} A_{\sigma^2}]^{1/2} \tag{13}$$

The coefficient of variation in $\mathbf{P}_{32}$ for a constant value of $\mathbf{P}_{31}$ is therefore given by,

$$\left(\frac{P_{32_\sigma}}{P_{32_\mu}}\right) \approx \left(\frac{A_\sigma}{A_\mu}\right) \tag{14}$$

This degree of variability is sufficient to cause the hydraulic properties of the networks to range from poorly connected to well beyond the percolation threshold. This demonstrates the danger of relying on $\mathbf{P}_{31}$ as a specification of fracture intensity.

5. CONCLUSIONS

Both fracture spacing ($\mathbf{S}_f$) and fracture length per unit area on a trace plane ($\mathbf{P}_{22}$) can be used to derive three dimensional measures for intensity. Intensity measure $\mathbf{P}_{32}$ provides a useful three dimensional measure of intensity which is independent of fracture size, and is not affected by the scale of the region analyzed. Definition of intensity in terms of fractures per unit area ($\mathbf{P}_{21}$) or fractures per unit volume ($\mathbf{P}_{31}$) can produce a variety of problems in interpretation and simulation. The collection of intensity measure $\mathbf{P}_{22}$ in the field provides a more reliable measure of intensity than fracture spacing $\mathbf{S}_f$.

The use of $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ as intensity measures in numerical modelling should be discouraged, due to the scale and fracture size dependence of these measures.

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SYMBOLS

P_{11}-P_{33}	Fracture Intensity Measures
S_f	Fracture spacing as measured on a borehole or scanline
S_f'	Fracture spacing as measured on the normal to a fracture set
S_f''	Expected value of fracture spacing as measured on a random line
θ	Angle between measurement line and mean orientation pole
$\bar{\theta}$	Mean angle between measurement line and mean orientation pole
C_{P2}	Constant of proportionality between spacing and intensity measure P_{22}
C_{P3}	Constant of proportionality between spacing and intensity measure P_{32}
A	Fracture Area
t	Fracture Thickness
$f_x(x)$	Probability Density Function ("distribution")
μ	Mean (as subscript)
σ	Standard deviation (as subscript)
σ^2	Variance (standard deviation squared), (as subscript)