Equations of IAPWS-IF97

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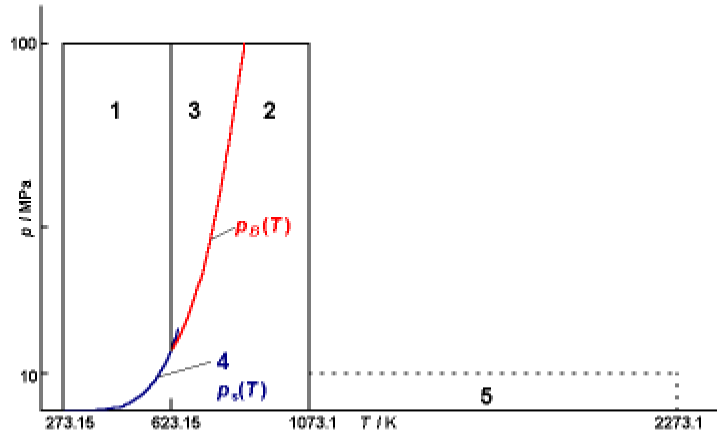
Symbols

c_v	specific isochoric heat capacity
c_p	specific isobaric heat capacity
f	specific Helmholtz free energy
g	specific Gibbs free energy
h	specific enthalpy
I, J	exponents
n	coefficient
p	pressure
p_S	saturation pressure
R	specific gas constant, R = 0.461 526 kJ/(kg K)
s	specific entropy
T	temperature
T_S	saturation temperature
u	specific internal energy
v	specific volume
w	speed of sound
β	transformed pressure
δ	reduced density
ϕ	dimensionless Helmholtz free energy
γ	dimensionless Gibbs free energy
π	reduced pressure
θ	reduced temperature
ϑ	transformed temperature
ρ	density
τ	inverse reduced temperature

1. Regions



In the following equations for regions 1 to 4 are given. For high-temperature region 5 see references.

2. Equations for Region 1

Boundaries: 273.15 K ≤ T ≤ 623.15 K and $p_S(T) \le p \le$ 100 MPa

For saturation pressure $p_S(T)$ see section 5 "Equations for Region 4"

Specific volume:

$$v(T,p)\frac{p}{RT} = pg_p$$

Specific internal energy:

$$\frac{u(T,p)}{RT} = tg_t - pg_p$$

Specific entropy:

$$\frac{s(T,p)}{R} = tg_t - g$$

Specific enthalpy:

$$\frac{h(T,p)}{RT} = tg_t$$

Specific isobaric heat capacity:

$$\frac{c_p(T,p)}{R} = -t^2 g_{tt}$$

Specific isochoric heat capacity:

$$\frac{c_v(T,p)}{R} = -t^2 g_{tt} + \frac{(g_p - tg_{pt})^2}{g_{pp}}$$

Speed of sound:

$$\frac{w^2(T,p)}{RT} = \frac{g_p^2}{\frac{(g_p - tg_{pt})^2}{t^2 g_{tt}} - g_{pp}}$$

Fundamental equation for g:

$$g(T,p) = \frac{g(T,p)}{RT} = \sum_{i=1}^{34} n_i \left(7.1 - \boldsymbol{p}\right)^{I_i} \left(\boldsymbol{t} - 1.222\right)^{J_i}$$

Derivatives of g:

$$g_p = \left(\frac{\partial g}{\partial \boldsymbol{p}}\right)_t = \sum_{i=1}^{34} - n_i I_i \left(7.1 - \boldsymbol{p}\right)^{(I_i - 1)} \left(\boldsymbol{t} - 1.222\right)^{J_i}$$

$$g_{pp} = \left(\frac{\partial^2 g}{\partial \boldsymbol{p}^2}\right)_t = \sum_{i=1}^{34} n_i I_i \left(I_i - 1\right)\left(7.1 - \boldsymbol{p}\right)^{(I_i - 2)} \left(\boldsymbol{t} - 1.222\right)^{J_i}$$

$$g_t = \left(\frac{\partial g}{\partial \boldsymbol{t}}\right)_p = \sum_{i=1}^{34} n_i \left(7.1 - \boldsymbol{p}\right)^{I_i} J_i \left(\boldsymbol{t} - 1.222\right)^{(J_i - 1)}$$

$$g_{tt} = \left(\frac{\partial^2 g}{\partial \boldsymbol{t}^2}\right)_p = \sum_{i=1}^{34} n_i \left(7.1 - \boldsymbol{p}\right)^{I_i} J_i \left(J_i - 1\right)\left(\boldsymbol{t} - 1.222\right)^{(J_i - 2)}$$

$$g_{pt} = \left(\frac{\partial^2 g}{\partial \boldsymbol{p} \partial \boldsymbol{t}}\right) = \sum_{i=1}^{34} - n_i I_i \left(7.1 - \boldsymbol{p}\right)^{(I_i - 1)} J_i \left(\boldsymbol{t} - 1.222\right)^{(J_i - 1)}$$

Inverse reduced temperature:

$$\boldsymbol{t} = \frac{1386\,\mathrm{K}}{T}$$

Reduced pressure:

$$\boldsymbol{p} = \frac{p}{16.53\,\mathrm{MPa}}$$

Table 1: Coefficients and exponents of the fundamental equation and its derivatives

i	I_i	J_i	n_i
1	0	-2	0.146 329 712 131 67 E+00
2	0	-1	-0.845 481 871 691 14 E+00
3	0	0	-0.375 636 036 720 40 E+01
4	0	1	0.338 551 691 683 85 E+01
5	0	2	-0.957 919 633 878 72 E+00
6	0	3	0.157 720 385 132 28 E+00
7	0	4	-0.166 164 171 995 01 E-01
8	0	5	0.812 146 299 835 68 E-03
9	1	-9	0.283 190 801 238 04 E-03
10	1	-7	-0.607 063 015 658 74 E-03
11	1	-1	-0.189 900 682 184 19 E-01
12	1	0	-0.325 297 487 705 05 E-01
13	1	1	-0.218 417 171 754 14 E-01
14	1	3	-0.528 383 579 699 30 E-04
15	2	-3	-0.471 843 210 732 67 E-03
16	2	0	-0.300 017 807 930 26 E-03
17	2	1	0.476 613 939 069 87 E-04
18	2	3	-0.441 418 453 308 46 E-05
19	2	17	-0.726 949 962 975 94 E-15
20	3	-4	-0.316 796 448 450 54 E-04
21	3	0	-0.282 707 979 853 12 E-05
22	3	6	-0.852 051 281 201 03 E-09
23	4	-5	-0.224 252 819 080 00 E-05
24	4	-2	-0.651 712 228 956 01 E-06
25	4	10	-0.143 417 299 379 24 E-12
26	5	-8	-0.405 169 968 601 17 E-06
27	8	-11	-0.127 343 017 416 41 E-08
28	8	-6	-0.174 248 712 306 34 E-09
29	21	-29	-0.687 621 312 955 31 E-18
30	23	-31	0.144 783 078 285 21 E-19
31	29	-38	0.263 357 816 627 95 E-22
32	30	-39	-0.119 476 226 400 71 E-22
33	31	-40	0.182 280 945 814 04 E-23
34	32	-41	-0.935 370 872 924 58 E-25

3. Equations for Region 2

Boundaries:

273.15 K ≤ T ≤ 623.15 K and 0 < p ≤ $p_S(T)$
623.15 K ≤ T ≤ 863.15 K and 0 < p ≤ $p_B(T)$
863.15 K ≤ T ≤ 1073.15 K and 0 < p ≤ 100 MPa

For saturation pressure $p_S(T)$ see section 5 "Equations for Region 4"

The boundary $p_B(T)$ between regions 2 und 3 (see figure above) is defined by a pressure-temperature relation which covers the range from 623.15 K at 16.5292 MPa up to 863.15 K at 100 MPa and which can be either expressed explicitly in pressure

$$\pi = n_1 + n_2\theta + n_3\theta^2$$

or in temperature

$$\theta = n_4 + \left(\frac{\pi - n_5}{n_3}\right)^{0.5}$$

$$\pi = \frac{p_B}{1\text{MPa}}$$

$$\theta = \frac{T_B}{1\text{K}}$$

Table 2: Coefficients of the boundary equation

i	n_i
1	0.348 051 856 289 69 E+03
2	-0.116 718 598 799 75 E+01
3	0.101 929 700 393 26 E-02
4	0.572 544 598 627 46 E+03
5	0.139 188 397 788 70 E+02

Specific volume:

$$v(T,p)\frac{p}{RT} = \pi\left(\gamma_\pi^0 + \gamma_\pi^r\right)$$

Specific internal energy:

$$\frac{u(T,p)}{RT} = \tau\left(\gamma_\tau^0 + \gamma_\tau^r\right) - \pi\left(\gamma_\pi^0 + \gamma_\pi^r\right)$$

Specific entropy:

$$\frac{s(T,p)}{R} = \tau\left(\gamma_\tau^0 + \gamma_\tau^r\right) - \left(\gamma^0 + \gamma^r\right)$$

Specific enthalpy:

$$\frac{h(T,p)}{RT} = \tau\left(\gamma_\tau^0 + \gamma_\tau^r\right)$$

Specific isobaric heat capacity: $$\frac{c_p(T,p)}{R} = -\boldsymbol{t}^2\left(g^0_{tt} + g^{\mathrm{r}}_{tt}\right)$$

Specific isochoric heat capacity: $$\frac{c_v(T,p)}{R} = -\boldsymbol{t}^2\left(g^0_{tt} + g^{\mathrm{r}}_{tt}\right) - \frac{\left(1+\boldsymbol{p}g^{\mathrm{r}}_p - \boldsymbol{tp}g^{\mathrm{r}}_{pt}\right)^2}{1-\boldsymbol{p}^2 g^{\mathrm{r}}_{pp}}$$

Speed of sound: $$\frac{w^2(T,p)}{RT} = \frac{1+2\boldsymbol{p}g^{\mathrm{r}}_p + \boldsymbol{p}^2\left(g^{\mathrm{r}}_p\right)^2}{\left(1-\boldsymbol{p}^2 g^{\mathrm{r}}_{pp}\right) + \dfrac{\left(1+\boldsymbol{p}g^{\mathrm{r}}_p - \boldsymbol{tp}g^{\mathrm{r}}_{pt}\right)^2}{\boldsymbol{t}^2\left(g^0_{tt} + g^{\mathrm{r}}_{tt}\right)}}$$

Fundamental equation for g: $$g(T,p) = \frac{g(T,p)}{RT} = g^0(T,p) + g^{\mathrm{r}}(T,p)$$

$$g^0 = \ln \boldsymbol{p} + \sum_{i=1}^{9} n_i^0 \boldsymbol{t}^{J_i^0}$$

$$g^{\mathrm{r}} = \sum_{i=1}^{43} n_i \boldsymbol{p}^{I_i} (\boldsymbol{t} - 0.5)^{J_i}$$

Derivatives of the ideal-gas part $\mathbf{g^0}$:

$$g^0_p = \left(\frac{\partial g^0}{\partial \boldsymbol{p}}\right)_t = \frac{1}{\boldsymbol{p}}$$

$$g^0_{pp} = \left(\frac{\partial^2 g^0}{\partial \boldsymbol{p}^2}\right)_t = -\frac{1}{\boldsymbol{p}^2}$$

$$g^0_t = \left(\frac{\partial g^0}{\partial \boldsymbol{t}}\right)_p = \sum_{i=1}^{9} n_i^0 J_i^0 \boldsymbol{t}^{J_i^0-1}$$

$$g^0_{tt} = \left(\frac{\partial^2 g^0}{\partial \boldsymbol{t}^2}\right)_p = \sum_{i=1}^{9} n_i^0 J_i^0 \left(J_i^0 - 1\right) \boldsymbol{t}^{J_i^0-2}$$

$$g^0_{pt} = \left(\frac{\partial^2 g^0}{\partial \boldsymbol{p} \partial \boldsymbol{t}}\right) = 0$$

Derivatives of the residual part $\mathbf{g^r}$: $$g^{\mathrm{r}}_p = \left(\frac{\partial g^{\mathrm{r}}}{\partial \boldsymbol{p}}\right)_t = \sum_{i=1}^{43} n_i I_i \boldsymbol{p}^{I_i-1} (\boldsymbol{t} - 0.5)^{J_i}$$

$$g^{\mathrm{r}}_{pp} = \left(\frac{\partial^2 g^{\mathrm{r}}}{\partial \boldsymbol{p}^2}\right)_t = \sum_{i=1}^{43} n_i I_i \left(I_i - 1\right) \boldsymbol{p}^{I_i-2} (\boldsymbol{t} - 0.5)^{J_i}$$

$$g_{t}^{\mathrm{r}} = \left(\frac{\partial g^{\mathrm{r}}}{\partial \boldsymbol{t}}\right)_{p} = \sum_{i=1}^{43} n_i \boldsymbol{p}^{I_i} J_i (\boldsymbol{t} - 0.5)^{J_i - 1}$$

$$g_{tt}^{\mathrm{r}} = \left(\frac{\partial^2 g^{\mathrm{r}}}{\partial \boldsymbol{t}^2}\right)_{p} = \sum_{i=1}^{43} n_i \boldsymbol{p}^{I_i} J_i (J_i - 1)(\boldsymbol{t} - 0.5)^{J_i - 2}$$

$$g_{pt}^{\mathrm{r}} = \left(\frac{\partial^2 g^{\mathrm{r}}}{\partial \boldsymbol{p} \partial \boldsymbol{t}}\right) = \sum_{i=1}^{43} n_i I_i \boldsymbol{p}^{I_i - 1} J_i (\boldsymbol{t} - 0.5)^{J_i - 1}$$

Inverse reduced temperature: $\boldsymbol{t} = \frac{540\,\mathrm{K}}{T}$

Reduced pressure: $\boldsymbol{p} = \frac{p}{1\,\mathrm{MPa}}$

Table 3: Coefficients and exponents of the ideal-gas part of the fundamental equation and its derivatives

i	J_i^0	n_i^0
1	0	-0.969 276 865 002 17 E+01
2	1	0.100 866 559 680 18 E+02
3	-5	-0.560 879 112 830 20 E-02
4	-4	0.714 527 380 814 55 E-01
5	-3	-0.407 104 982 239 28 E+00
6	-2	0.142 408 191 714 44 E+01
7	-1	-0.438 395 113 194 50 E+01
8	2	-0.284 086 324 607 72 E+00
9	3	0.212 684 637 533 07 E-01

Table 4: Coefficients and exponents of the residual part of the fundamental equation and its derivatives

i	I_i	J_i	n_i
1	1	0	-0.177 317 424 732 13 E-02
2	1	1	-0.178 348 622 923 58 E-01
3	1	2	-0.459 960 136 963 65 E-01
4	1	3	-0.575 812 590 834 32 E-01
5	1	6	-0.503 252 787 279 30 E-01
6	2	1	-0.330 326 416 702 03 E-04
7	2	2	-0.189 489 875 163 15 E-03
8	2	4	-0.393 927 772 433 55 E-02
9	2	7	-0.437 972 956 505 73 E-01
10	2	36	-0.266 745 479 140 87 E-04
11	3	0	0.204 817 376 923 09 E-07
12	3	1	0.438 706 672 844 35 E-06
13	3	3	-0.322 776 772 385 70 E-04
14	3	6	-0.150 339 245 421 48 E-02
15	3	35	-0.406 682 535 626 49 E-01
16	4	1	-0.788 473 095 593 67 E-09
17	4	2	0.127 907 178 522 85 E-07
18	4	3	0.482 253 727 185 07 E-06
19	5	7	0.229 220 763 376 61 E-05
20	6	3	-0.167 147 664 510 61 E-10
21	6	16	-0.211 714 723 213 55 E-02
22	6	35	-0.238 957 419 341 04 E+02
23	7	0	-0.590 595 643 242 70 E-17
24	7	11	-0.126 218 088 991 01 E-05
25	7	25	-0.389 468 424 357 39 E-01
26	8	8	0.112 562 113 604 59 E-10
27	8	36	-0.823 113 408 979 98 E+01
28	9	13	0.198 097 128 020 88 E-07
29	10	4	0.104 069 652 101 74 E-18
30	10	10	-0.102 347 470 959 29 E-12
31	10	14	-0.100 181 793 795 11 E-08
32	16	29	-0.808 829 086 469 85 E-10
33	16	50	0.106 930 318 794 09 E+00
34	18	57	-0.336 622 505 741 71 E+00
35	20	20	0.891 858 453 554 21 E-24
36	20	35	0.306 293 168 762 32 E-12
37	20	48	-0.420 024 676 982 08 E-05
38	21	21	-0.590 560 296 856 39 E-25
39	22	53	0.378 269 476 134 57 E-05
40	23	39	-0.127 686 089 346 81 E-14
41	24	26	0.730 876 105 950 61 E-28
42	24	40	0.554 147 153 507 78 E-16
43	24	58	-0.943 697 072 412 10 E-06

4. Equations for Region 3

Boundaries: 623.15 K $\le T \le T_B(p)$ and $p_B(T) \le p \le$ 100 MPa

For $T_B(p)$ and $p_B(T)$ see section 3 "Equations for Region 2"

Pressure:

$$\frac{p(T,\, r)}{rRT} = d f_d$$

Specific internal energy:

$$\frac{u(T,\, r)}{RT} = t f_t$$

Specific entropy:

$$\frac{s(T,\, r)}{R} = t f_t - f$$

Specific enthalpy:

$$\frac{h(T,\, r)}{RT} = t f_t + d f_d$$

Specific isobaric heat capacity:

$$\frac{c_p(T,\, r)}{R} = -t^2 f_{tt} + \frac{\left(d f_d - d t f_{dt}\right)^2}{2 d f_d + d^2 f_{dd}}$$

Specific isochoric heat capacity:

$$\frac{c_v(T,\, r)}{R} = -t^2 f_{tt}$$

Speed of sound:

$$\frac{w^2(T,\, r)}{RT} = 2 d f_d + d^2 f_{dd} - \frac{\left(d f_d - d t f_{dt}\right)^2}{t^2 f_{tt}}$$

Fundamental equation for f:

$$f(T,\, r) = \frac{f(T,\, r)}{RT} = n_1 \ln d + \sum_{i=2}^{40} n_i d^{I_i} t^{J_i}$$

Derivatives of f :

$$f_d = \left(\frac{\partial f}{\partial d}\right)_t = \frac{n_1}{d} + \sum_{i=2}^{40} n_i I_i d^{I_i - 1} t^{J_i}$$

$$f_{dd} = \left(\frac{\partial^2 f}{\partial d^2}\right)_t = -\frac{n_1}{d^2} + \sum_{i=2}^{40} n_i I_i \left(I_i - 1\right) d^{I_i - 2} t^{J_i}$$

$$f_t = \left(\frac{\partial f}{\partial t}\right)_d = \sum_{i=2}^{40} n_i d^{I_i} J_i t^{J_i - 1}$$

$$f_{tt} = \left(\frac{\partial^2 f}{\partial t^2}\right)_d = \sum_{i=2}^{40} n_i d^{I_i} J_i \left(J_i - 1\right) t^{J_i - 2}$$

$$\phi_{\delta\tau} = \left(\frac{\partial^2 \phi}{\partial\delta\partial\tau}\right) = \sum_{i=2}^{40} n_i I_i \delta^{I_i-1} J_i \tau^{J_i-1}$$

Inverse reduced temperature: $\tau = \frac{T_c}{T} = \frac{647.096\,\mathrm{K}}{T}$

Reduced density: $\delta = \frac{\rho}{\rho_c} = \frac{\rho}{322\,\mathrm{kg/m}^3}$

Table 5: Coefficients and exponents of the fundamental equation and its derivatives

i	I_i	J_i	n_i
1	0	0	0.106 580 700 285 13 E+01
2	0	0	-0.157 328 452 902 39 E+02
3	0	1	0.209 443 969 743 07 E+02
4	0	2	-0.768 677 078 787 16 E+01
5	0	7	0.261 859 477 879 54 E+01
6	0	10	-0.280 807 811 486 20 E+01
7	0	12	0.120 533 696 965 17 E+01
8	0	23	-0.845 668 128 125 02 E-02
9	1	2	-0.126 543 154 777 14 E+01
10	1	6	-0.115 244 078 066 81 E+01
11	1	15	0.885 210 439 843 18 E+00
12	1	17	-0.642 077 651 816 07 E+00
13	2	0	0.384 934 601 866 71 E+00
14	2	2	-0.852 147 088 242 06 E+00
15	2	6	0.489 722 815 418 77 E+01
16	2	7	-0.305 026 172 569 65 E+01
17	2	22	0.394 205 368 791 54 E-01
18	2	26	0.125 584 084 243 08 E+00
19	3	0	-0.279 993 296 987 10 E+00
20	3	2	0.138 997 995 694 60 E+01
21	3	4	-0.201 899 150 235 70 E+01
22	3	16	-0.821 476 371 739 63 E-02
23	3	26	-0.475 960 357 349 23 E+00
24	4	0	0.439 840 744 735 00 E-01
25	4	2	-0.444 764 354 287 39 E+00
26	4	4	0.905 720 707 197 33 E+00
27	4	26	0.705 224 500 879 67 E+00
28	5	1	0.107 705 126 263 32 E+00
29	5	3	-0.329 136 232 589 54 E+00
30	5	26	-0.508 710 620 411 58 E+00
31	6	0	-0.221 754 008 730 96 E-01
32	6	2	0.942 607 516 650 92 E-01
33	6	26	0.164 362 784 479 61 E+00
34	7	2	-0.135 033 722 413 48 E-01
35	8	26	-0.148 343 453 524 72 E-01
36	9	2	0.579 229 536 280 84 E-03
37	9	26	0.323 089 047 037 11 E-02
38	10	0	0.809 648 029 962 15 E-04
39	10	1	-0.165 576 797 950 37 E-03
40	11	26	-0.449 238 990 618 15 E-04

5. Equations for Region 4

Range of validity: Vapor-liquid saturation curve 273.15 K ≤ T ≤ 647.096 K or 611.213 Pa ≤ p ≤ 22.064 MPa

Saturation pressure:

$$\frac{p_S(T)}{1\,\mathrm{MPa}} = \left[\frac{2C}{-B + \left(B^2 - 4AC\right)^{0.5}}\right]^4$$

$$A = \boldsymbol{J}^2 + n_1 \boldsymbol{J} + n_2$$

$$B = n_3 \boldsymbol{J}^2 + n_4 \boldsymbol{J} + n_5$$

$$C = n_6 \boldsymbol{J}^2 + n_7 \boldsymbol{J} + n_8$$

$$\boldsymbol{J} = \frac{T}{1\,\mathrm{K}} + \frac{n_9}{\dfrac{T}{1\,\mathrm{K}} - n_{10}}$$

Saturation temperature:

$$\frac{T_S(p)}{1\,\mathrm{K}} = \frac{n_{10} + D - \left[\left(n_{10} + D\right)^2 - 4\left(n_9 + n_{10} D\right)\right]^{0.5}}{2}$$

$$D = \frac{2G}{-F - \left(F^2 - 4EG\right)^{0.5}}$$

$$E = \boldsymbol{b}^2 + n_3 \boldsymbol{b} + n_6$$

$$F = n_1 \boldsymbol{b}^2 + n_4 \boldsymbol{b} + n_7$$

$$G = n_2 \boldsymbol{b}^2 + n_5 \boldsymbol{b} + n_8$$

$$\boldsymbol{b} = \left(\frac{p}{1\,\mathrm{MPa}}\right)^{0.25}$$

Table 6: Coefficients of the saturation pressure and temperature equations

i	n_i
1	0.116 705 214 527 67 E+04
2	-0.724 213 167 032 06 E+06
3	-0.170 738 469 400 92 E+02
4	0.120 208 247 024 70 E+05
5	-0.323 255 503 223 33 E+07
6	0.149 151 086 135 30 E+02
7	-0.482 326 573 615 91 E+04
8	0.405 113 405 420 57 E+06
9	-0.238 555 575 678 49 E+00
10	0.650 175 348 447 98 E+03

References

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