## Tethys-Chloris energy and mass balance model

The physically-based model Tethys-Chloris (T and C) is used to simulate the mass, energy and water fluxes of the five Peruvian study sites. T and C is an ecohydrological model with an hourly time step that can resolve the atmospheric energy balance, soil and vegetation dynamics, processes of runoff generation and flow routing, and the evolution of snow and ice packs (see Fatichi *et al.* (2012ab) and Mastrotheodoros *et al.* (2020) for a description of the distributed model). However, for the purposes of this study we restrict its use to the point scale and to the computation of the surface energy fluxes and snow and ice mass balance. In T and C both ice and snow are not considered landcovers, instead they must be prescribed on top of another landcover (for ice this is usually over rock but it can also occur over frozen water). The fractional snowcover over a surface is also explicitly accounted for within T and C but in the current model set up ice can only be snow free or completely snow-covered.

The net input of energy to the snow/ice surface () is given by:

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where is net radiation, the heat flux due to precipitation, the heat released or gained within the snowpack due to melting or freezing of liquid water in the snowpack ( is not accounted for when the surface is ice), is the sensible heat flux, the latent heat flux and the ground heat flux. All fluxes are in W m-2 and we use the convention of positive fluxes being directed towards the surface (this differs from previous publications). The prognostic surface temperature of the snow/ice surface ( is required to calculate the energy fluxes. It is derived using an iterative numerical approach which solves for in a way that allows closure of the energy balance.

## Radiative fluxes

is given by:

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With incoming shortwave radiation at the measurement height, the snow or ice albedo, incoming longwave radiation and outgoing longwave radiation. Measured hourly albedo derived from is used for for both snow and ice surfaces, where measurements of (outgoing shortwave radiation) are available. Otherwise, is modelled based on the surface type: for ice a single value is used (derived from measurements at the appropriate site); and for snow the Brock *et al.* (2000) snow albedo parameterisation is applied, including both the deep and shallow snow albedo equations. Since the Brock *et al.* (2000) parameterisation was developed at a daily scale and the modelling approach employed is hourly, a threshold precipitation value is added , which must be exceeded by the sum of the precipitation over the previous 24 hours to refresh the snow albedo to that of new snow. We use the parameters as given in Brock *et al.* (2000) rather than modifying them for each site by calibration to measured albedo, since of the three sites to use modelled albedo (AG, CG and QIC) only AG had sufficient data for this process. However, the rarity of the formation of a snowpack for long periods at AG meant the calibration tended towards extreme values which reduced the fit of the modelled melt to the validation data. Comparisons of runs with modelled and measured albedo are discussed in section \*\*.

Values of are derived from measurements when available and otherwise derived from cloudiness using:

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Where is the attenuation cloud cover, the clear sky emissivity, the Stefan-Boltzmann constant (5.6704 10-8 W m-2 K-4) and the air temperature in Kelvins. is derived following Dilley and O’Brian (1998):

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Where 59.38, 113.7, 96.96 and is precipitable water (kg m-2) determined by the method of Prata (1996). is derived following Unsworth and Monteith (1975):

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with cloudiness () derived from the potential incoming shortwave radiation (, W m-2) using the common approach described by Juszak and Pellicciotti (2013):

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can only be calculated during the day, defined as periods when is greater than a threshold (150 W m-2 in T and C), with nighttime values linearly interpolated between the daytime values. However, significant shading at CG during the morning resulted in erroneously high cloudiness values. To correct for this CG daytime was defined as being between 11:00 and 16:00, based on analysis of the diurnal cycle. Timesteps with precipitation are always given a cloudiness of 1. Meanwhile is computed from the Stefan-Boltzmann law:

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With the surface emissivity (0.97 for both snow and ice) and , the surface temperature in Kelvins.

## Turbulent fluxes

is calculated from:

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Where is air density (kg m-3), is the specific heat capacity of air at constant pressure (J kg-1 K-1), is the temperature of the ice or snow surface (°C), the air temperature (°C) at the measurement height and is the aerodynamic resistance to heat flux (s m-1). The aerodynamic resistance is calculated using the simplified solution of the Monin-Obukohv similarity theory (Mascart et al, 1995; Noilhan and Mafhouf, 1996). Full details are given in the supplementary information of Fatichi *et al.* (2012) and Fatichi (2010). The roughness lengths (m) of heat () and water vapour () used in the calculation of the aerodynamic resistance are equal in T and C (), and , with the roughness length of momentum () equal to 0.001 m for snow and ice surfaces.

is estimated from:

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Where is the latent heat of sublimation defined as = , with the latent heat of vapourisation (1000(2501.3-2.361) J kg-1) and the latent heat of melting (333700 J kg-1). The term is the surface specific humidity at saturation, is the specific humidity of air at the measurement height and the aerodynamic resistance to the vapour flux, which equals

## Incoming heat with precipitation

To estimate the precipitation amount and temperature must be known. The rain/snow temperature is assumed to be the greater/smaller value of and 0°C. Therefore is calculated as:

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Where = 4186 J kg-1 K-1 is the specific heat of water, = 2093 J kg-1 K-1 is the specific heat capacity of ice, = 1000 kg m-2 is the density of water and and are liquid and solid precipitation (m s-1).

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## Snowpack water content

The water content of the snowpack is approximated using a bucket model, in which outflow of water from the snowpack occurs when the maximum holding capacity of the snowpack is exceeded. The maximum holding capacity of the snowpack is based on the snow water equivalent, holding capacity coefficient and snow density, following the method of Belair *et al.* (2003). Snowmelt plus liquid precipitation, minus the water released from the snowpack gives the current snowpack water content (). If the surface temperature of the snow is greater than -0.01°C then the snowpack water content is presumed to be liquid, whereas otherwise it is presumed frozen. The process of melting (resulting in a negative flux) and freezing (resulting in a positive flux) of the water content of the snowpack is associated with the heat flux :

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Where (s) is the timestep and is the fraction of the snowpack water content involved in either melting or freezing. This fraction is defined using , with the water equivalent snowpack.

## Ice water content

The water content of ice is approximated with a linear reservoir model. The liquid water outflow is proportional to the ice pack water content (), which is initiated when the water content exceeds a threshold capacity, prescribed as 1% of the ice pack water equivalent. The icepack water content is the sum of ice melt and liquid precipitation, minus the water released from the ice pack. The water released is the sum of the ice pack excess water content plus the outflow from the linear reservoir, given as where is the reservoir constant which is proportional to the ice pack water equivalent. Unlike within snowpacks, is not accounted for within the ice pack, since water is presumed to percolate quickly and avoid refreezing.

## Precipitation partition

Input precipitation is required to be partitioned into solid () and liquid () precipitation, because of the differing impacts of snow and rain on the energy and mass balance. Originally T and C employed the method of Wigmosta *et al.* (1994) where the precipitation is partitioned as a function of solely , with all precipitation deemed snow under the condition , all precipitation deemed rain under the condition , and solid and liquid precipitation partitioned between both states under intermediate conditions. However, for this study the precipitation partition method described by Ding et al. (2014) was implemented in T and C. This scheme determines the precipitation partition based on the wet-bulb temperature, station elevation and relative humidity. Ding et al. (2014) found that the wet-bulb temperature was found to be a better predictor than of the precipitation type; that the temperature threshold between snow and rain is increased at higher elevations; and that the probability of sleet is reduced in conditions of low relative humidity.

## Snow and ice mass balance

The calculation of the evolution of the snow and ice mass balance is rather similar, so they will be treated together here, however the calculations are performed for snow if there is snow precipitation during a timestep or the modelled snow water equivalent at the surface is greater than zero. Net input of energy to the snow or ice pack will increase its temperature, and after the temperature has been raised to the melting point, additional energy inputs will result in melt. The change in the average temperature of the ice or snowpack () is controlled using:

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Where is the time step (h) and (mm w.e.) is the water equivalent mass of the ice or snowpack before melting. Energy inputs into an isothermal ice/snow pack result in melt ( in mm w.e.) calculated from:

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The water equivalent mass of the snow/ice pack after melting () is updated conserving the mass balance following:

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Here , which is the evaporation/sublimation from ice/snow in mm w.e. The snow density is assumed to be constant with depth and calculations are performed in a single snowpack layer. The snow density evolves over time using the method proposed by Verseghy (1991) and improved by Belair et al. (2003). In this parameterisation the snow density increases exponentially over time due to gravitational settling and is updated when fresh snow is added to the snowpack. Two parameters are required in this scheme, and , which represent the maximum snow density (kg m-3) under melting and freezing conditions, respectively. The depth of the ice pack can be increased through the formation of ice from the snowpack, which is prescribed to occur if the snow density increases to greater than 500 kg m-3 (a density associated with the firn to ice transition) and at a rate of 0.037 mm h-1 (Cuffey and Paterson, 2010). The density of ice is assumed constant with depth and given a value of 916.2 kg m-3.