Report on A 2-spine decomposition of the critical Galton-Watson tree and a probabilistic proof of Yaglom's theorem

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This paper gives a relatively short and self-contained application of recent 2-spine techniques, providing a new proof of Yaglom's theorem that the population at time n of a Galton-Watson process conditioned to survive, rescaled by $1/n$, converges in distribution to an exponential random variable.

I like the paper. It has a modest goal, but it does a good job of explaining succinctly why Yaglom's theorem should be true and translating that into mathematics. It is useful both for a new point of view on Yaglom's theorem, and as an introduction to two-spine theory. Of course—and as this paper points out—there is already a nice proof of Yaglom's theorem, due to Lyons, Pemantle and Peres, so it is reasonable to ask whether we need another one. The authors claim that this one is more "generic" and they have generalized it to more complicated critical branching systems in another paper. On the other hand, this proof has a weakness relative to the L-P-P proof: it does not show that $nP(Z_n > 0)$ converges, and uses that fact. Maybe there is a way to deduce it from the analysis here, but I did not see it.

Overall I would like to see the paper published. ECP is a good journal, and I cannot say this paper is a top drawer contribution, so maybe the editors will take a different view from my own; but it is well written, interesting and useful, so for my part I would accept it, subject to some changes outlined below.

1. pp2, first paragraph: "random element" should be "random variable" (twice)

2. pp2, 3 lines from the end, "followed-up paper" should be "follow-up paper" (also at the bottom of pp5)

3. pp3, line 3: delete "on the exponential convergence"

4. pp4, just after (1.5): To deduce the result from your heuristic, you seem to be assuming that looking at $\ddot{Y}$ is equivalent, or at least similar, to looking at the conditioned process. Why is that true? Is there any heuristic reason why this biasing should be similar to conditioning to survive?

5. pp4, "It would be interesting to compare" should be "It is interesting to compare"

6. pp7, line 7, I think C_u should be $\dot{C}_u$ here, twice.

7. pp8, halfway down, "Using this analysis, one can verify that" should be "Using this analysis, we verify that"

8. pp8, proof of Thm 1.2, the first paragraph could be reduced to "Note that $\{(X_m(t))_{0\le m\le n}; G_n\} =^d (Z_m)_{0\le m\le n}$ and $\{(X_m(t))_{0\le m\le n}; \ddot{G}_n\} =^d (\ddot{Z}_m)_{0\le m\le n}$." There is no need for so much detail.

9. pp9, first display, there is no "d" required here, this is a discrete space, you can just write $\frac{\ddot{G}_n(t)}{G_n(t)}$.

10. pp9, end of proof of Thm 1.2, the QED symbol should be *after* (2.5).

11. pp9, start of section 2.2, "notations" should be "notation" (notation is a continuum, like water). Also change "give the precise meaning that (1.5) admits a size-biased add-on structure for the size-biased μ-Galton-Watson tree" to "give a precise meaning to (1.5)"

12. pp9, Proof of Prop 2.1, this is again a bit over-elaborate at the start. Could write simply "For any particle $u = u_1 \dots u_n$, write $[\emptyset, u] := \{u_1 \dots u_j : j = 0, \dots, n\}$. The particles in $\dot{T}$ can be separated according to their nearest spine ancestor; we write

$$\dot{A}_k = \{u \in \dot{T} : |[\emptyset, u] \cap \dot{V}| = k\}.$$

Then

$$X_n(\dot{T}) = \sum_{k=0}^{n} X_n(\dot{A}_k)."$$

13. pp10, top: Change "For convention" to "and", also change "transform" to "transforms"

14. pp10, paragraph after (2.7), $\ddot{V}' - \ddot{V} \cap \ddot{V}'$ should be $\ddot{V}' \setminus \ddot{V}$, and also "is separated" should be "can be separated"

15. pp10, again the discussion before (2.8) can be shortened in the same way as at the start of the proof.

16. pp10, (2.9): Not sure I see the second equality. I think the last $E[e^{-\lambda \dot{Z}_m}]$ should be $E[e^{-\lambda \dot{Z}_{n-m}}]$. What am I missing? Even if I'm right, this doesn't seem to cause major problems, but it does need to be chased through the rest of the document.

17. pp11, top, delete "The proof is complete."

18. pp11, halfway down, after "Therefore" add "since $x - \ln x$ is decreasing on $[0, 1]$"

19. pp11, bottom, change "By dividing $\|F\|_\infty$ on the both side of (3.1)" to "By dividing both sides of (3.1) by $\|F\|_\infty$"

20. pp11/12, Proof of Lemma 3.2, you can take $k = 0$ as the base case here, which is trivial. Then your proof is shorter.

21. pp12, halfway, change "verify that, e also satisfies" to "verify that e satisfies"

22. pp12, end of proof of Lemma 1.3, you cite [1, Lemma 2.6], but this is a five-year-old preprint that does not appear to have been peer-reviewed. Please provide a better reference that has been peer-reviewed.

23. pp13, first display, there should be a dot on $Z_{\lfloor un \rfloor}$.