Convolutional Neural Networks III

CS7150, Spring 2025

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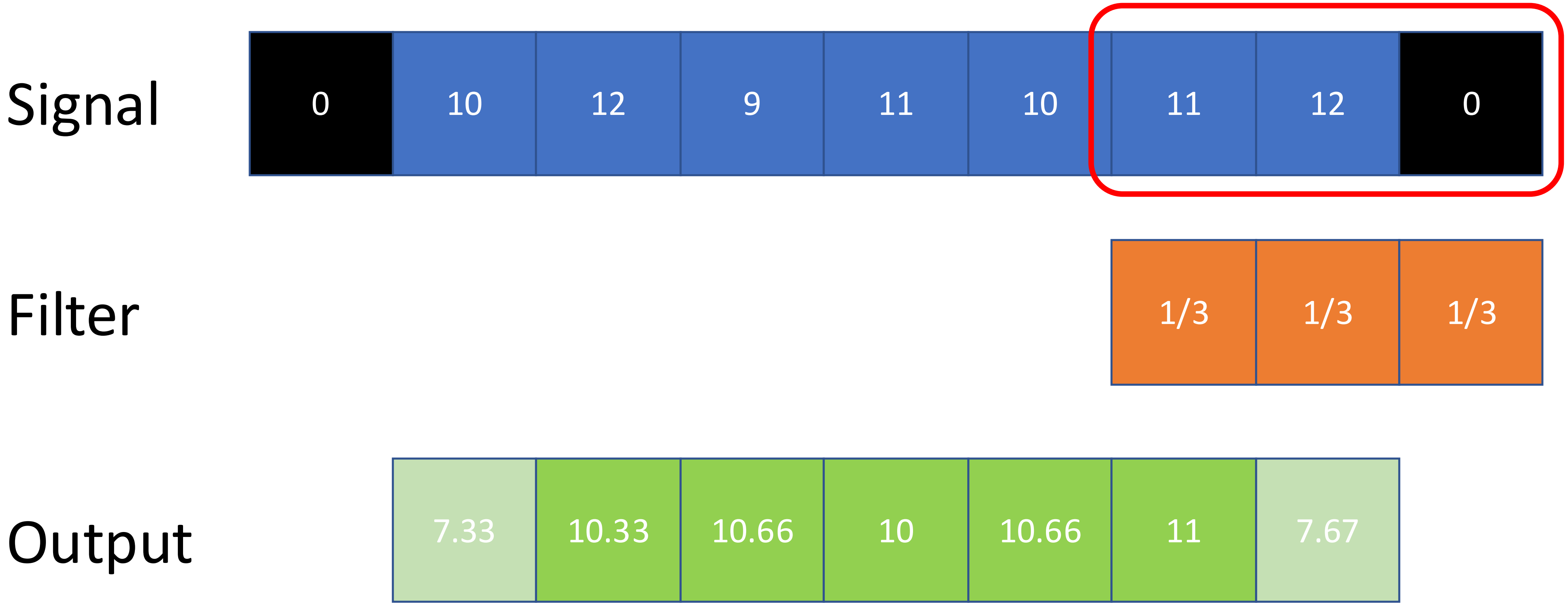
Northeastern University

Recap

Padding: 1D Case

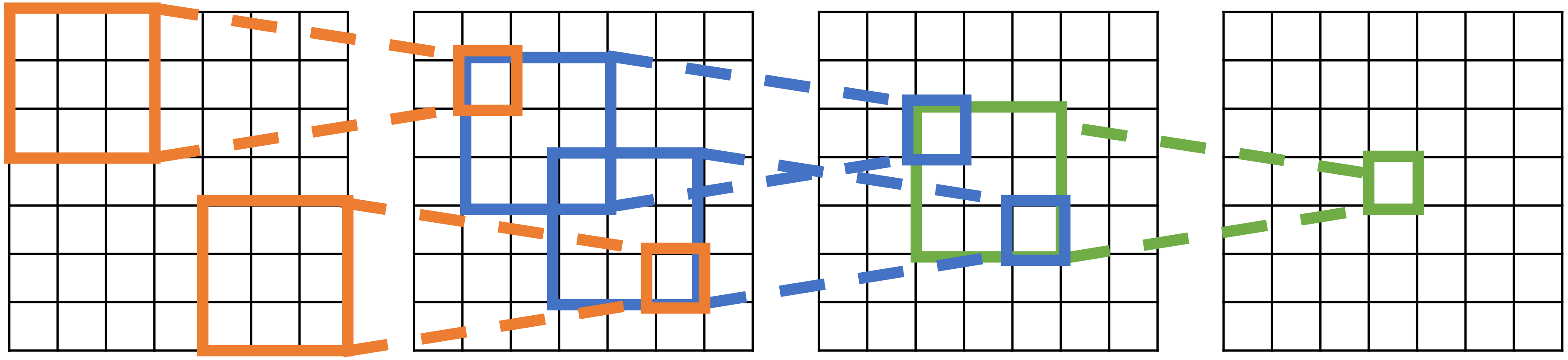


Input: 7 vs output: 7

Slide inspired by : J Johnson

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



Input

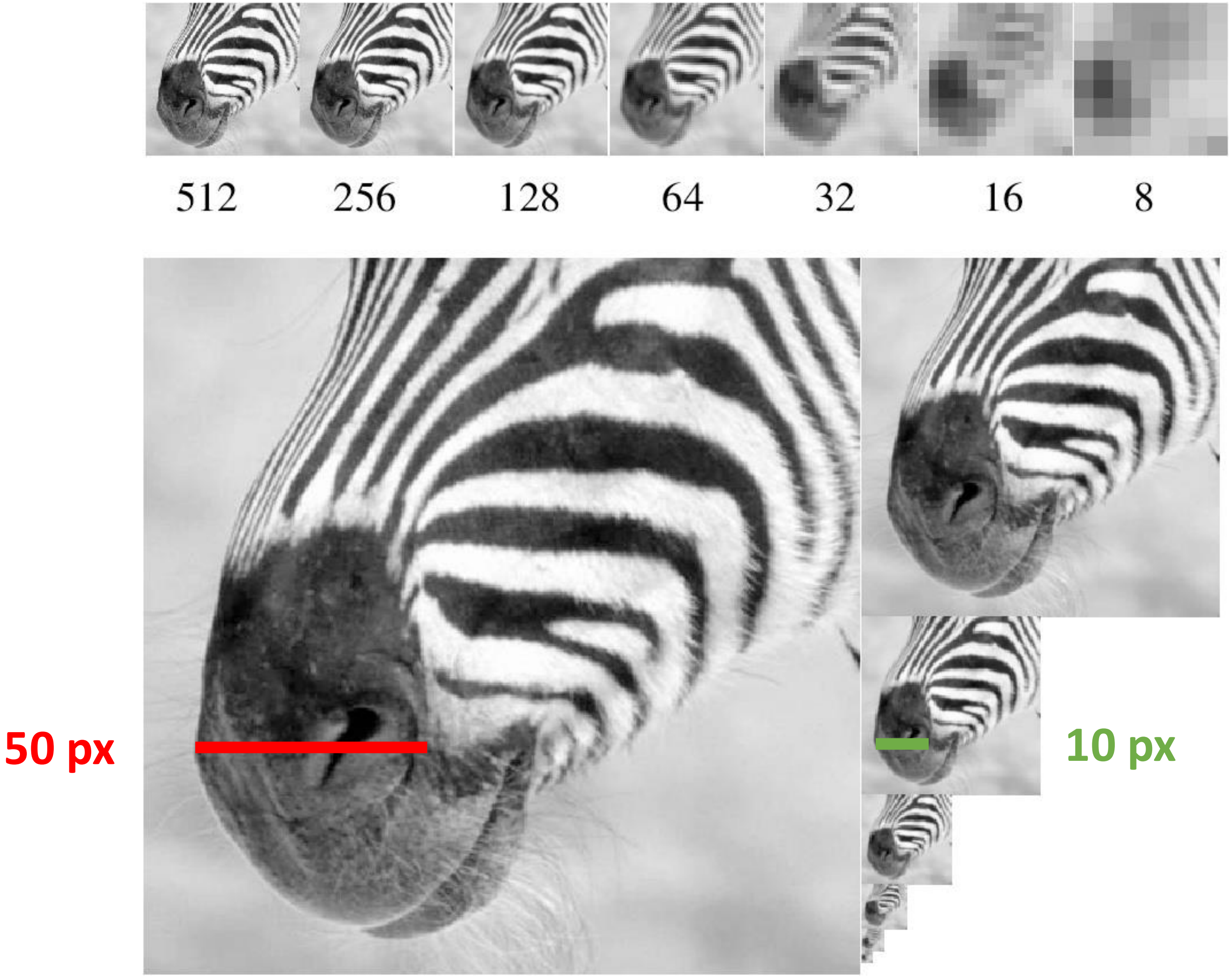
Output

Problem: For large images we need many layers for each output to "see" the whole image image

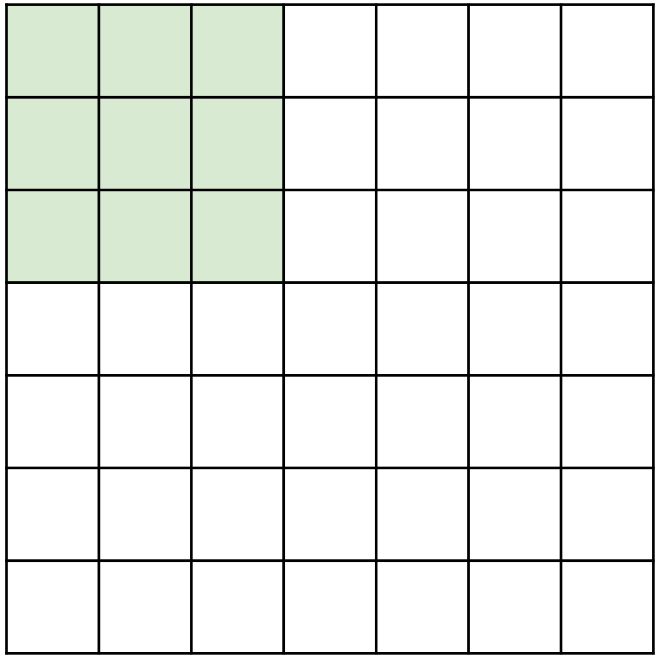
Solution: Downsample inside the network

Slide credit: D Fouhey & J Johnson

Idea: Image/Feature Pyramid

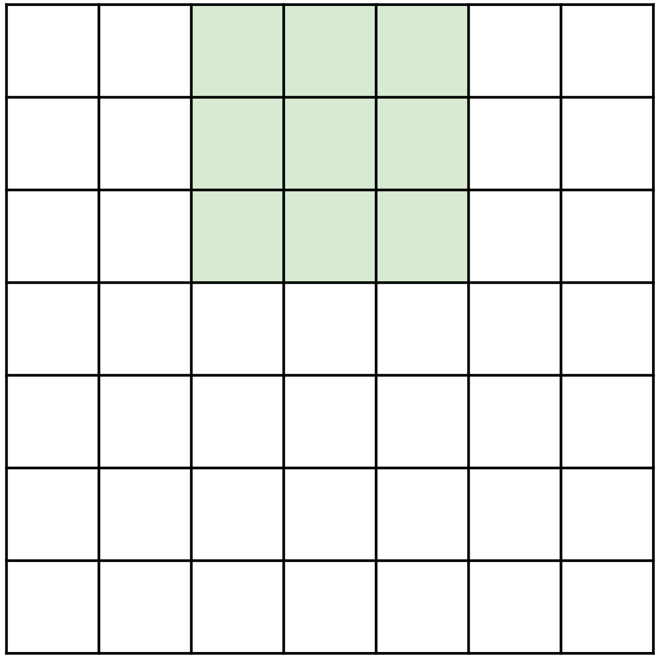


Strided Convolution



Input: 7x7
Filter: 3x3
Stride: 2

Strided Convolution



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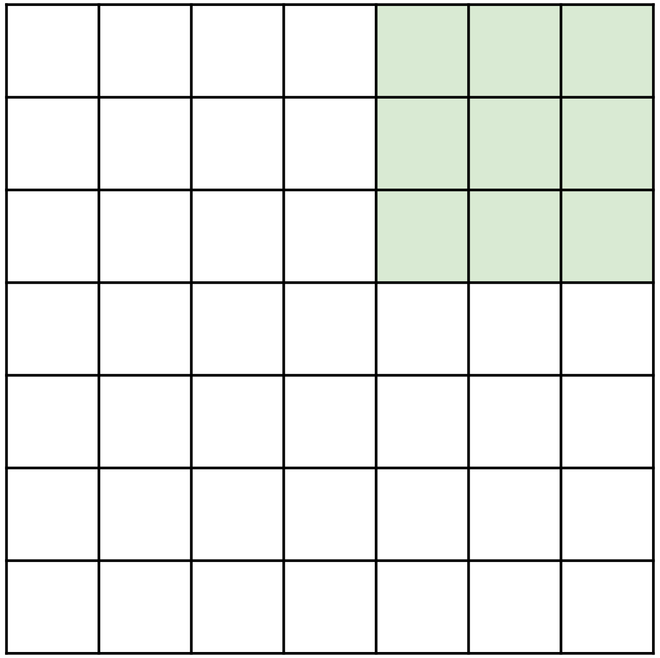


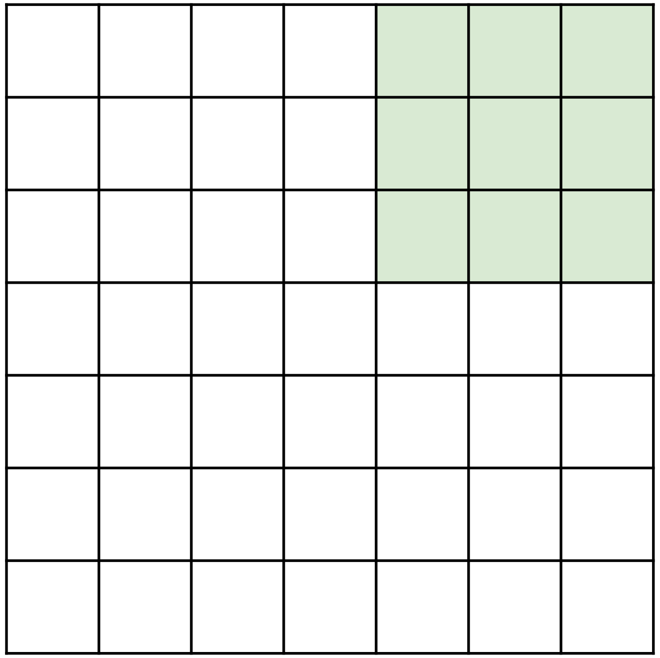
Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

Strided Convolution



Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

In general:

Input: W

Filter: K

Padding: P

Stride: S

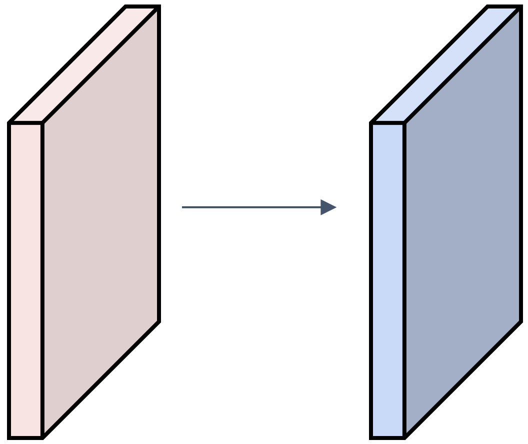
Output: Ceil((W – K + 1 + 2P) / S)

Slide credit: D Fouhey & J Johnson

Today’s Class

- More about convolution layer
- Pooling layer
- Batch normalization layer

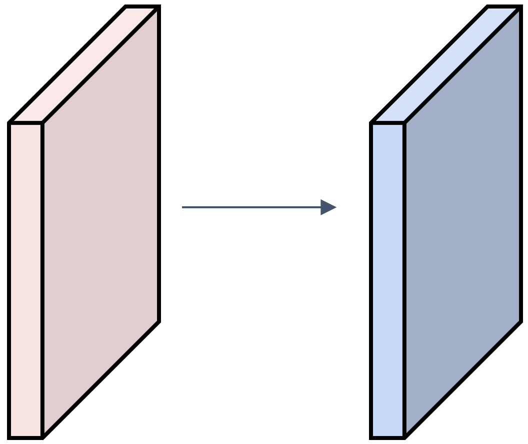
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: ?

Convolution Example



Input volume: 3 x **32** x **32**

10 5x5 filters with stride **1**, pad **2**

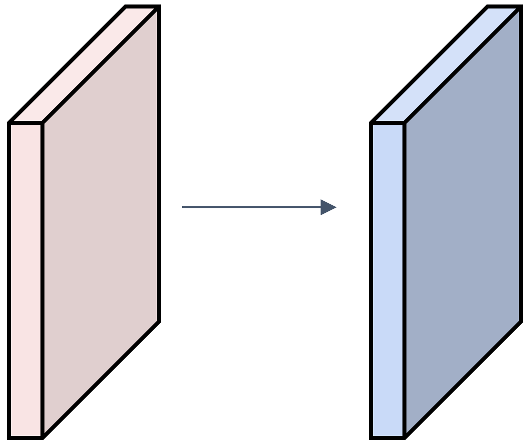
Output volume size:

Ceil((**32**+2***2**-**5+1**)/**1**) = 32 spatially, so

10 x 32 x 32

Slide credit: D Fouhey & J Johnson

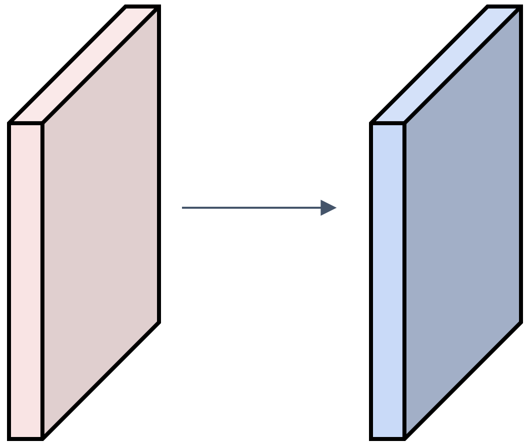
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: ?

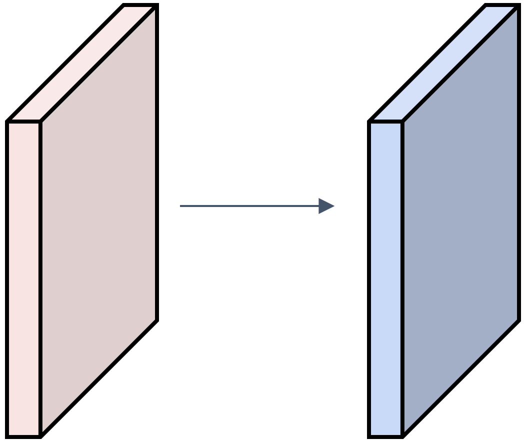
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: **760**
Parameters per filter: **3*5*5** + 1 (for bias) = **76**
10 filters, so total is **10** * **76** = **760**

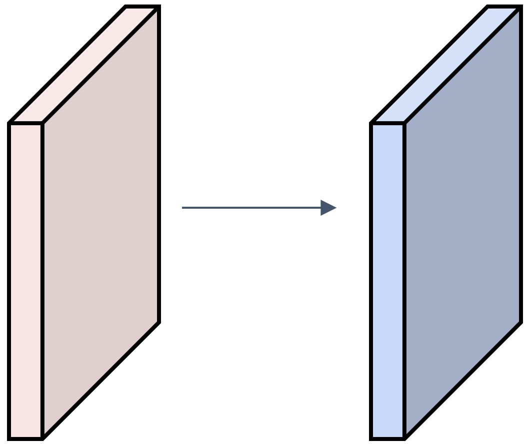
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: 760
Number of multiply-add operations: ?

Convolution Example



Input volume: **3** x 32 x 32
10 **5x5** filters with stride 1, pad 2

Output volume size: **10 x 32 x 32**
Number of learnable parameters: 760
Number of multiply-add operations: **768,000**
10*32*32 = 10,240 outputs; each output is the inner product of two **3**x**5x5** tensors (75 elems); total = 75*10240 = **768K**

Slide credit: D Fouhey & J Johnson

Convolution Summary

Input: C_{in} x H x W
Hyperparameters:
- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W
giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:
- H' = Ceil((H – K + 2P + 1) / S)
- W' = Ceil((W – K + 2P + 1) / S)

Slide credit: D Fouhey & J Johnson

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:
- **Kernel size**: K_H x K_W
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Output size: C_{out} x H' x W' where:
- H' = Ceil((H – K + 2P + 1) / S)
- W' = Ceil((W – K + 2P + 1) / S)

Common settings:

$K_H = K_W$ (Small square filters)

P = (K – 1) / 2 ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

K = 5, P = 2, S = 1 (5x5 conv)

K = 1, P = 0, S = 1 (1x1 conv)

K = 3, P = 1, S = 2 (Downsample by 2)

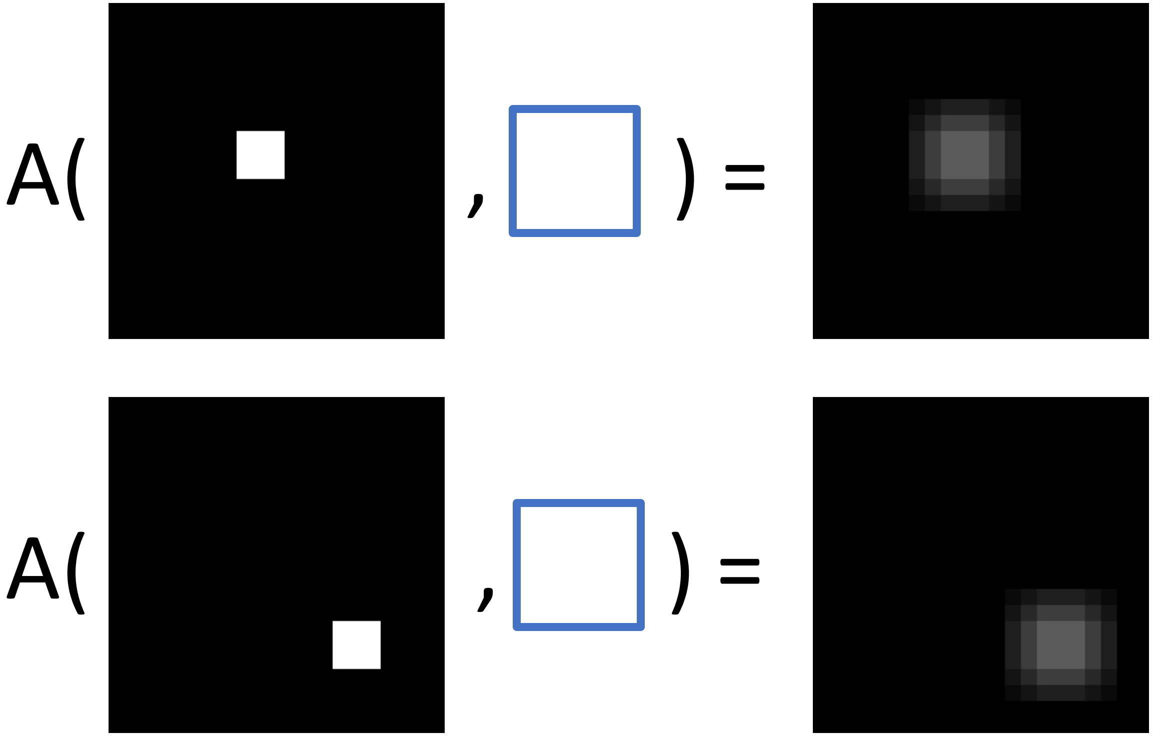
Slide credit: D Fouhey & J Johnson

Properties: Shift-Invariant

Assume: I image, f filter

Shift-invariant: shift(apply(I,f)) = apply(shift(I,f))

Intuitively: only depends on filter neighborhood



Slide credit: J Johnson

Forward pass of a convolution layer

```
x_padded = pad_input(x)
H_out, W_out = compute_output_dimension()
N, C, H, W = x.shape
F, C, HH, WW = w.shape
# shape of (F, C’), where C’=C * HH * WW
w_reshape = reshape_to_F_Cp(w).T
for i in range(W_out):
  for j in range(H_out):
    startx, starty = get_top_left_position()
    # shape of (N, C, HH, WW)
    x_data = get_patch(x_padded, startx, starty, HH, WW)
    # shape of (N, C’), where C’=C* HH * WW
    x_data_reshape = reshape_to_N_Cp(x_data)
    # shape of each location is (N, F)
    out[:, :, j, i] = dot_product(x_data_reshape, w_reshape) + b
```

Gradients of weights in convolution

2	3	2	4
5	6	6	8
3	2	1	0
1	2	3	4

input

1	2	3
4	5	6
7	8	9

conv kernel

Gradients of weights in convolution

2	3	2	4
5	6	6	8
3	2	1	0
1	2	3	4

input

1	2	3
4	5	6
7	8	9

conv kernel

Gradients for this particular weight

Gradients of weights in convolution

2	3	2	4
5	6	6	8
3	2	1	0
1	2	3	4

input

1	2	3
4	5	6
7	8	9

conv kernel

Gradients for this particular weight

Sum the gradients from different patches (determined by the padding and stride).

The same idea applies to the bias term.

Gradients of input in convolution

2	3	2	4
5	6	6	8
3	2	1	0
1	2	3	4

input

1	2	3
4	5	6
7	8	9

conv kernel

Gradients for this particular input

(why we ever need to compute the gradient for the input?)

Gradients of input in convolution

2	3	2	4
5	6	6	8
3	2	1	0
1	2	3	4

input

1	2	3
4	5	6
7	8	9

conv kernel

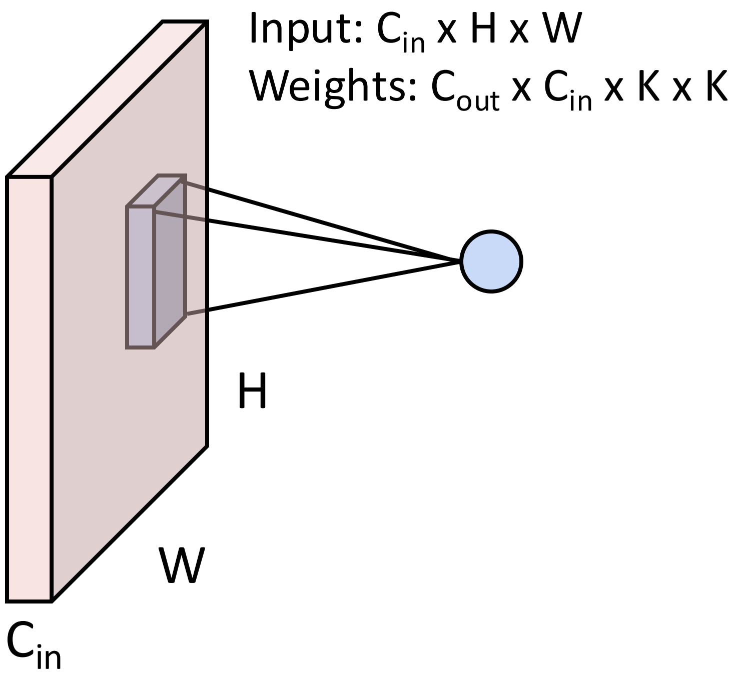
10	11	12
13	14	15
16	17	18

Gradients for this particular input

Sum the gradients from different convolution kernels.

Other types of convolution

So far: 2D Convolution

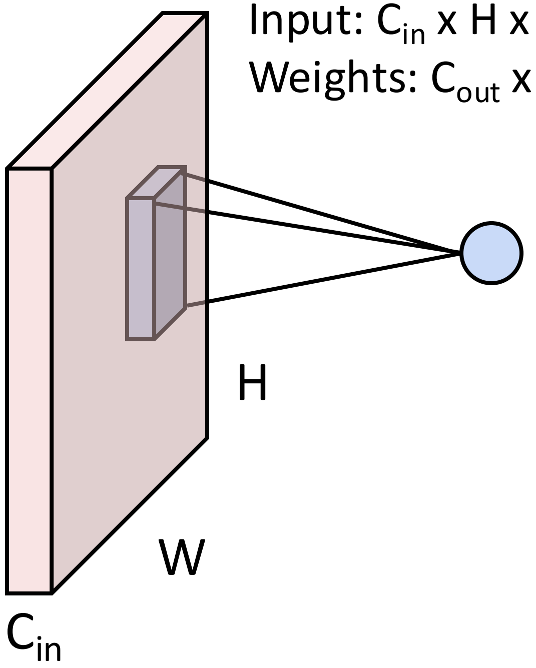


Slide credit: D Fouhey & J Johnson

Other types of convolution

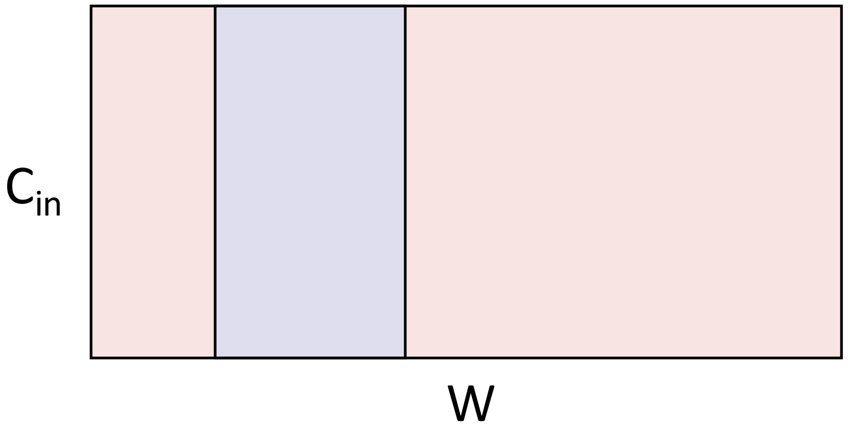
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

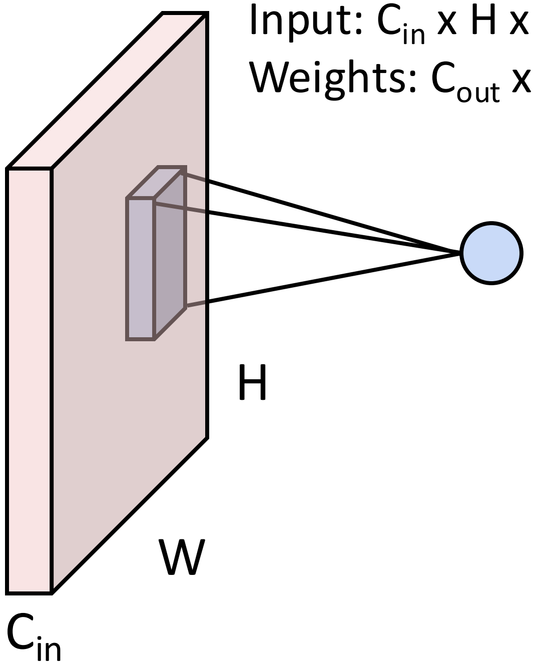
Input: C_{in} x W
Weights: C_{out} x C_{in} x K



Other types of convolution

So far: 2D Convolution

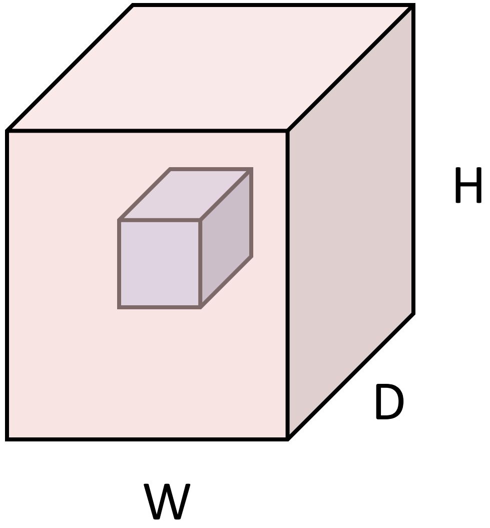
Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



3D Convolution

Input: C_{in} x H x W x D
Weights: C_{out} x C_{in} x K x K x K

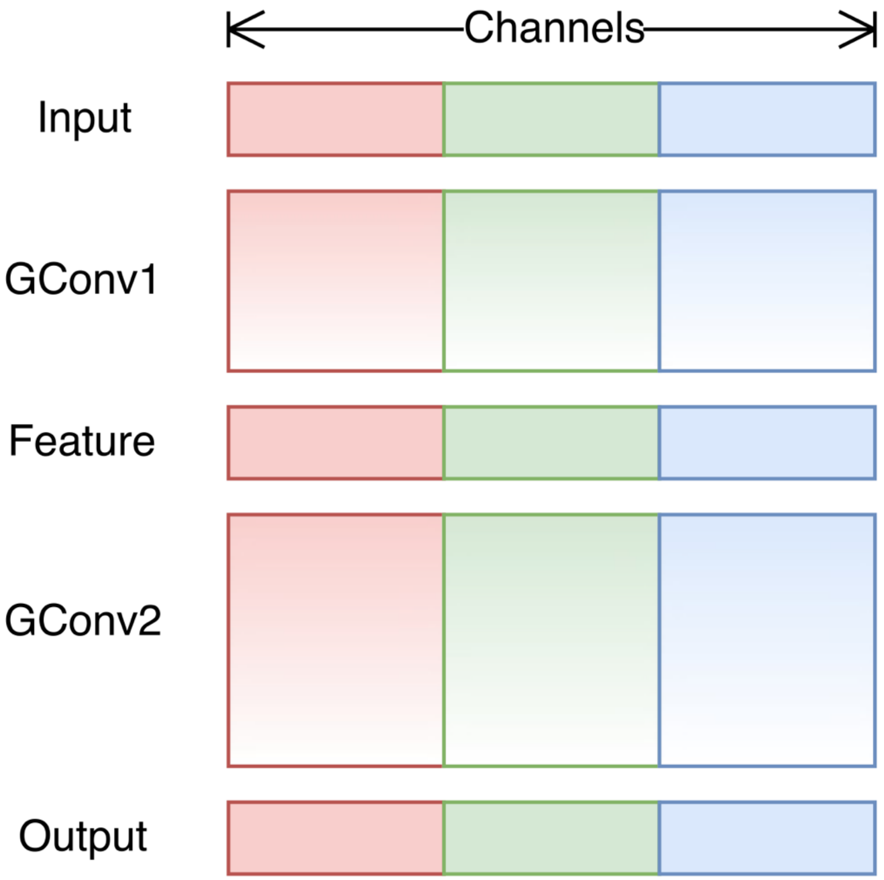
C_{in}-dim vector at each point in the volume



Slide credit: D Fouhey & J Johnson

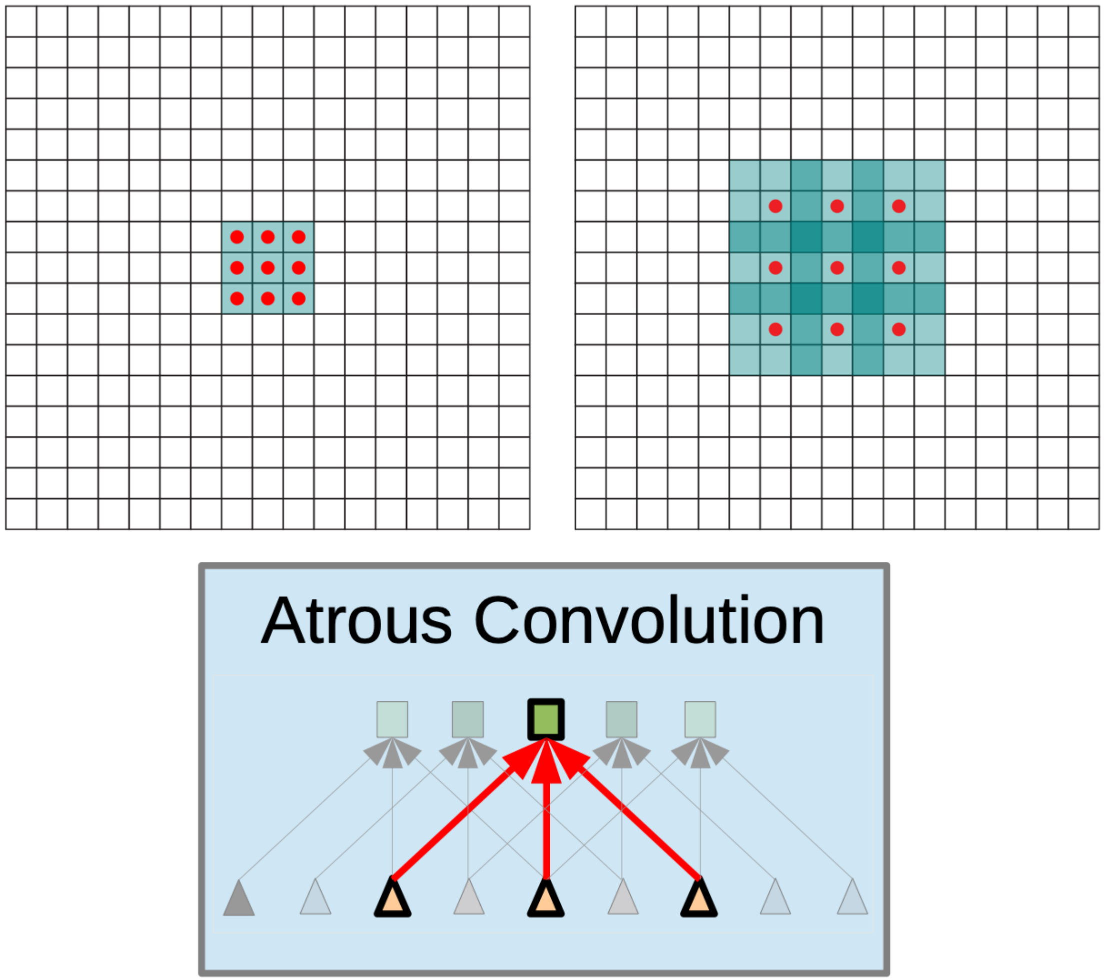
Other types of convolution

Group-based convolution



[Zhang et al., CVPR 2017]

Dilated convolution



[Yu and Koltun. ICLR 2016]
[Chen et al., ICLR 2015]

PyTorch Convolution Layer

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

PyTorch Convolution Layers

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

Conv1d

CLASS `torch.nn.Conv1d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

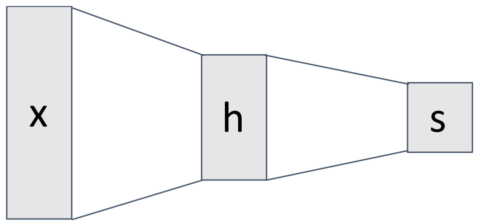
Conv3d

CLASS `torch.nn.Conv3d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')` [SOURCE]

Slide credit: D Fouhey & J Johnson

Components of a Convolutional Network

Fully-Connected Layers

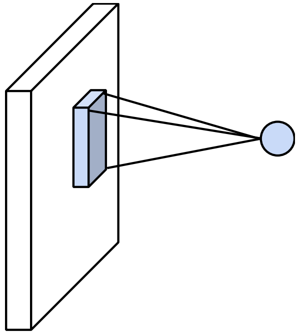


$$y = Wx + b$$

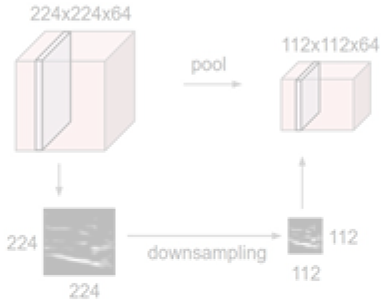
Activation Function

$$y = \max(0, x)$$

Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Slide credit: D Fouhey & J Johnson

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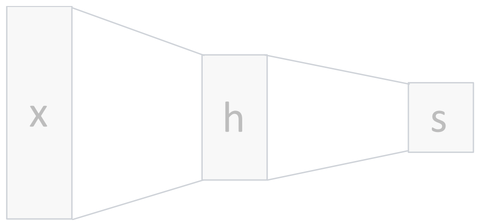

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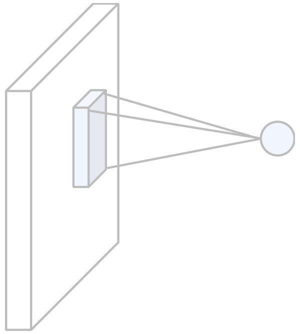
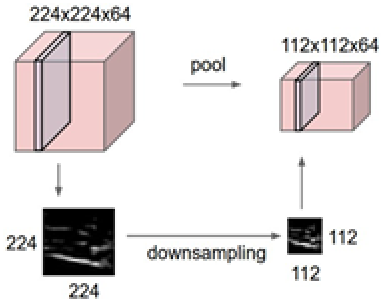


Pooling Layers



Normalization

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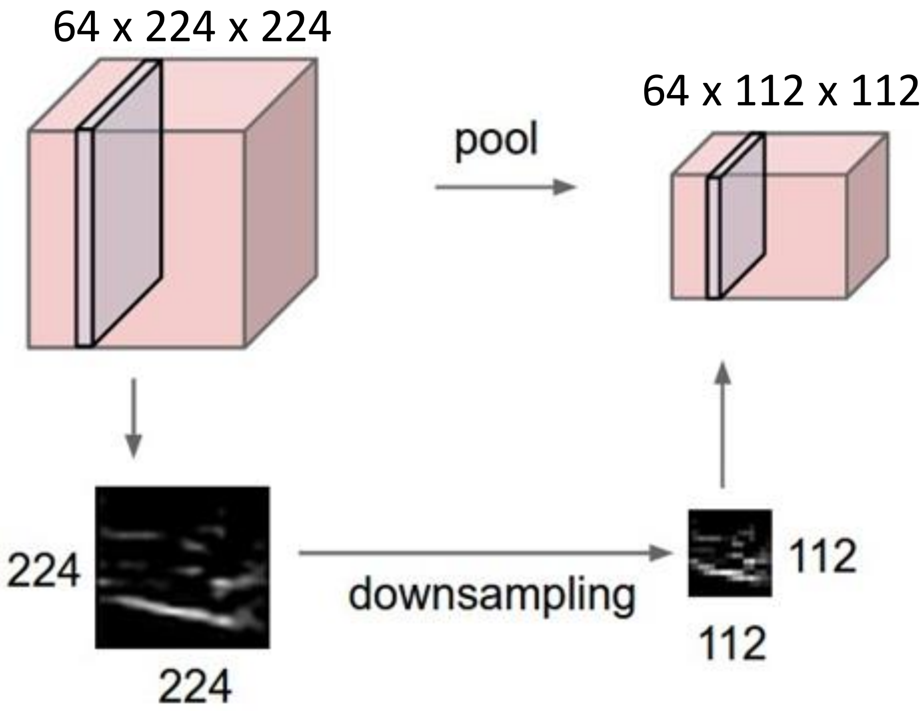
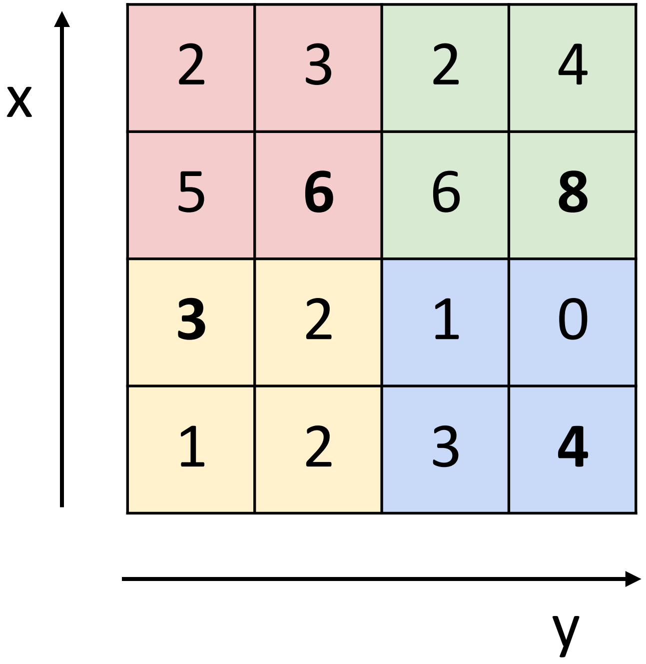
Pooling Layers: Downampling

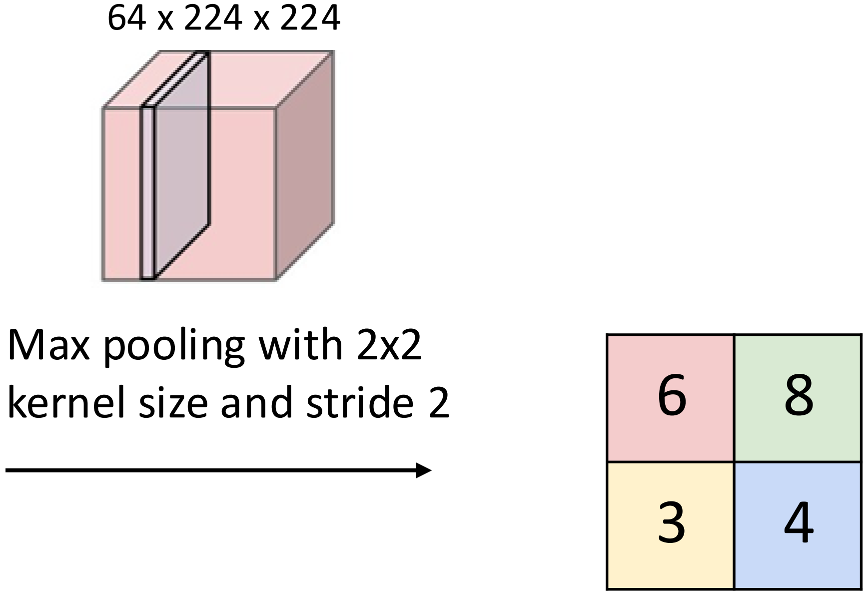


Hyperparameters:
Kernel Size
Stride
Pooling function

Max Pooling

Single depth slice



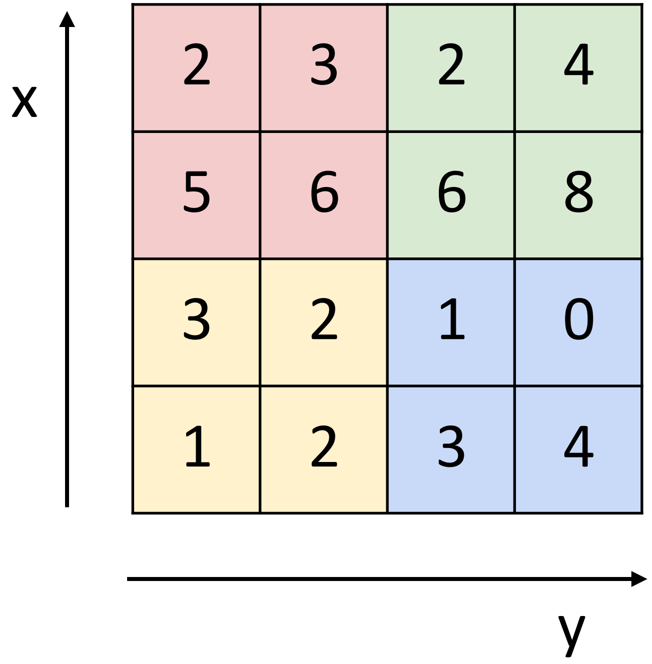


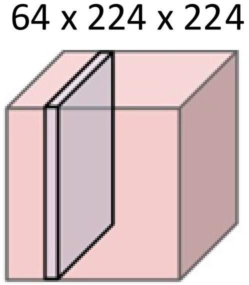
Introduces **invariance** to small spatial shifts

No learnable parameters!

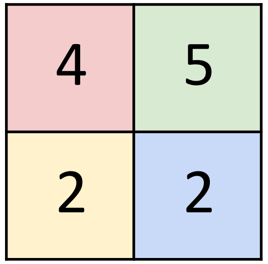
Average Pooling

Single depth slice





Avg pooling with 2x2 kernel size and stride 2



Introduces **invariance** to small spatial shifts

No learnable parameters!

Slide credit: D Fouhey & J Johnson

Pooling Summary

Input: C x H x W

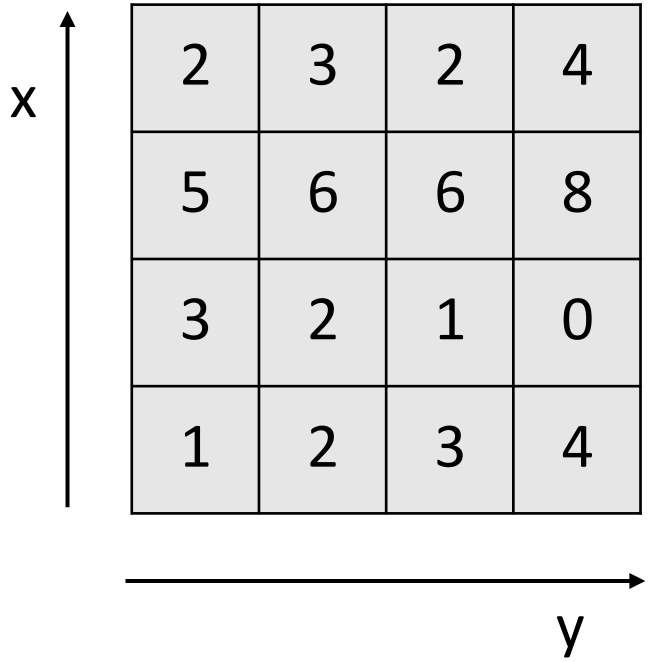
Hyperparameters:
- Kernel size: K
- Stride: S
- Pooling function (max, avg)

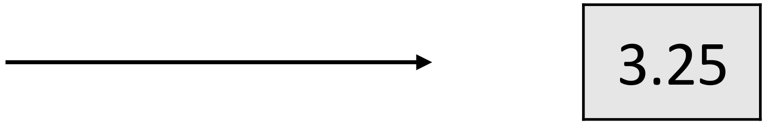
Output: C x H' x W' where
- H' = Ceil((H – K + 2P + 1) / S)
- W' = Ceil((W – K + 2P + 1) / S)

Learnable parameters: None!

Common settings:
max, K = 2, S = 2
max, K = 3, S = 2 (AlexNet)

Global Average Pooling





Average per channel (1 channel here).

Gradients?

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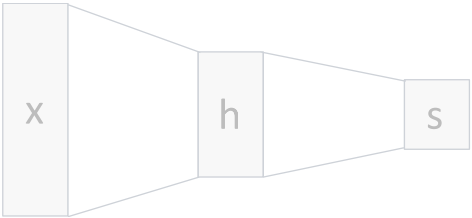

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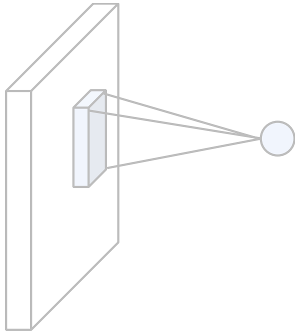
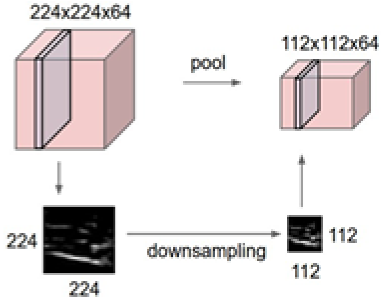
Convolution Layers



Pooling Layers



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Slide credit: D Fouhey & J Johnson

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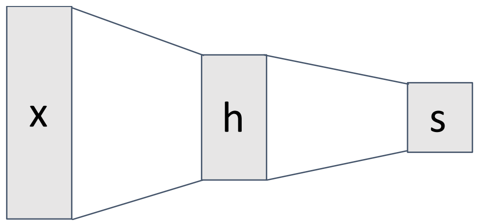

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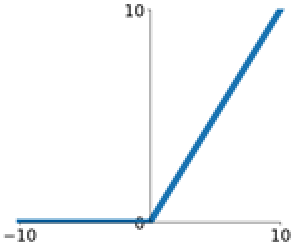
Activation Function



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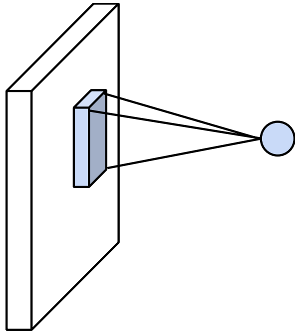
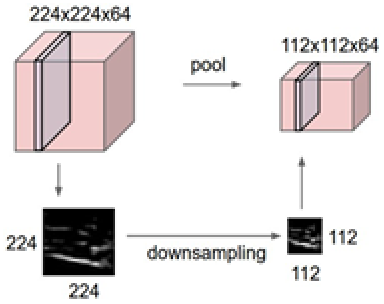
Convolution Layers



Pooling Layers



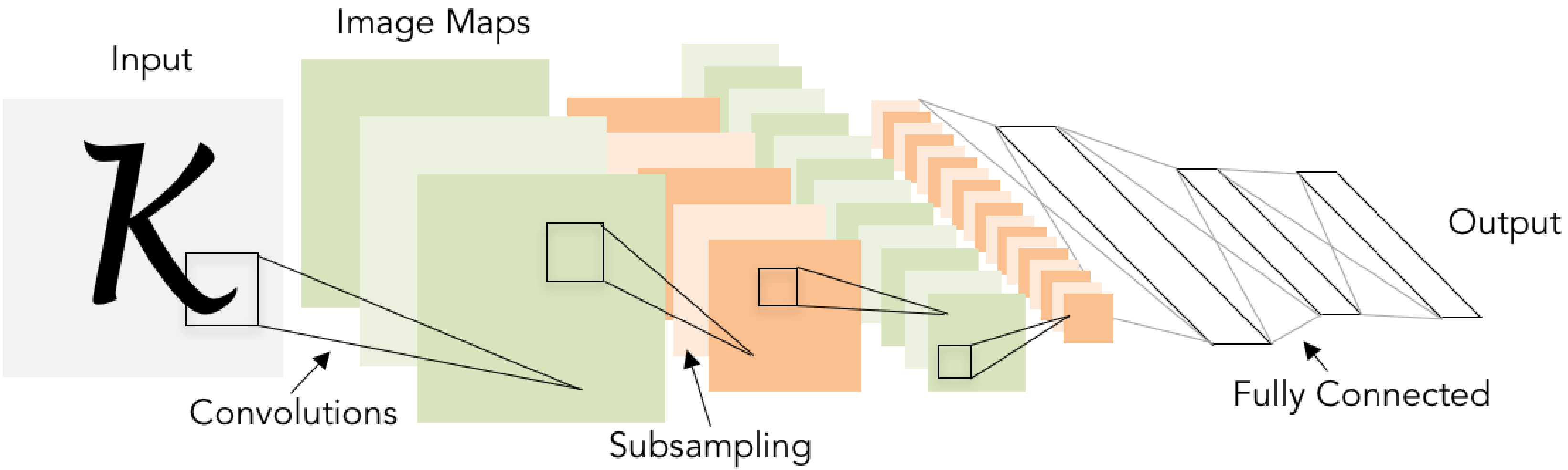
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Convolutional Networks

Classic architecture:
[Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

Example: LeNet-5

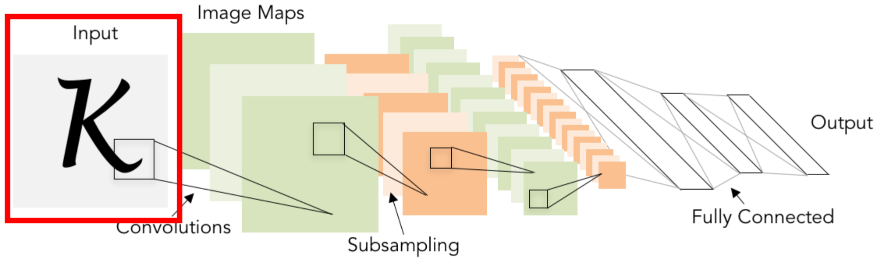


Lecun et al, "Gradient-based learning applied to document recognition", 1998

Slide credit: D Fouhey & J Johnson

Example: LeNet-5

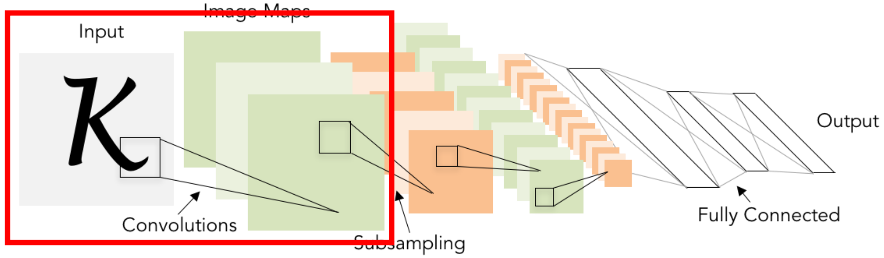
Layer	Output Size	Weight Size
Input	1 x 28 x 28	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Slide credit: D Fouhey & J Johnson

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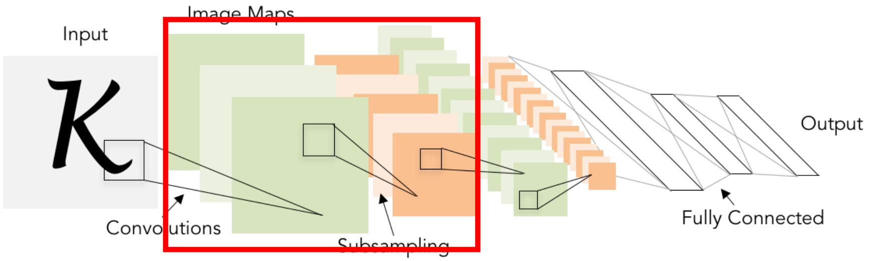
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

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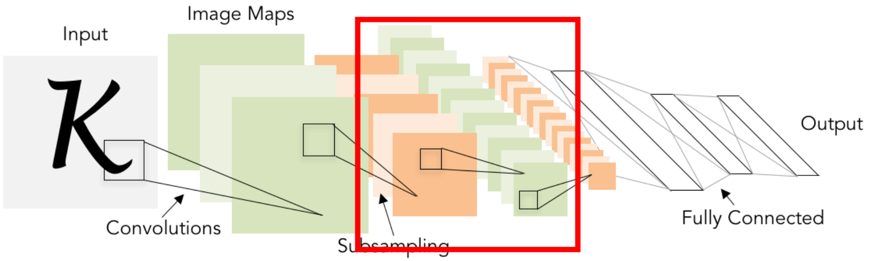
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	



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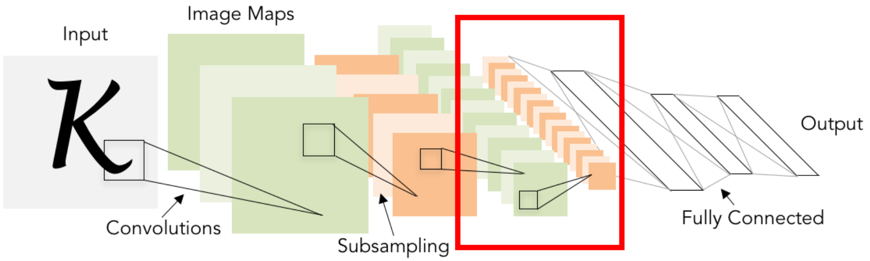
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	



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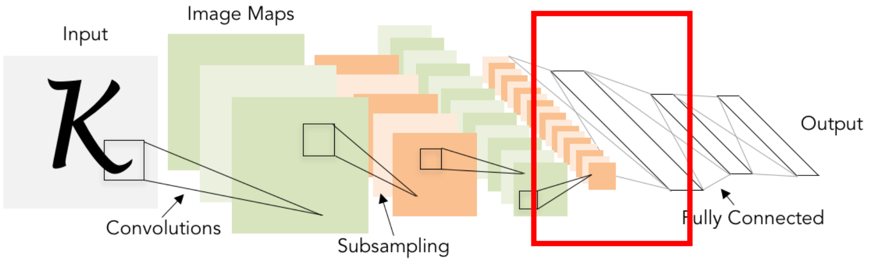
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	



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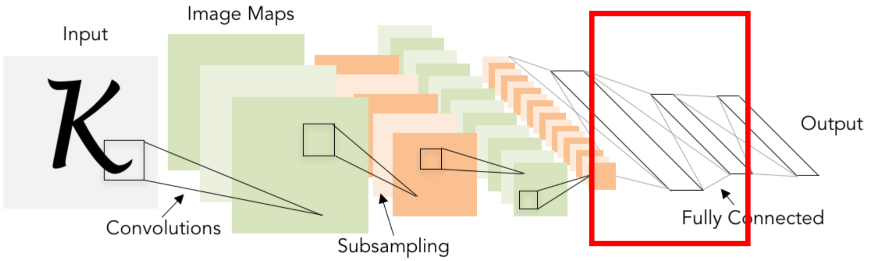
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	



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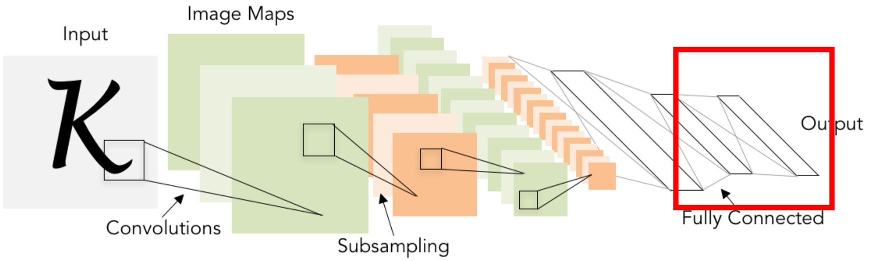
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10



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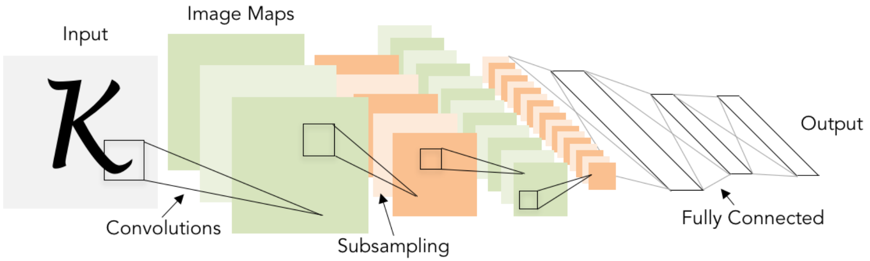


As we go through the network:

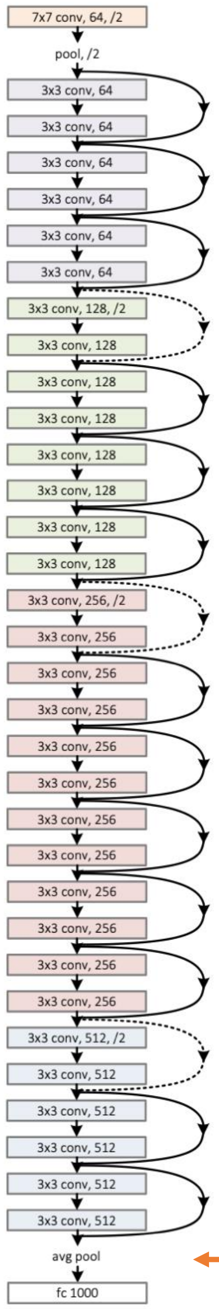
Spatial size **decreases**
(using pooling or strided conv)

Number of channels **increases**
(total “volume” is preserved!)

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Slide credit: D Fouhey & J Johnson

ResNet



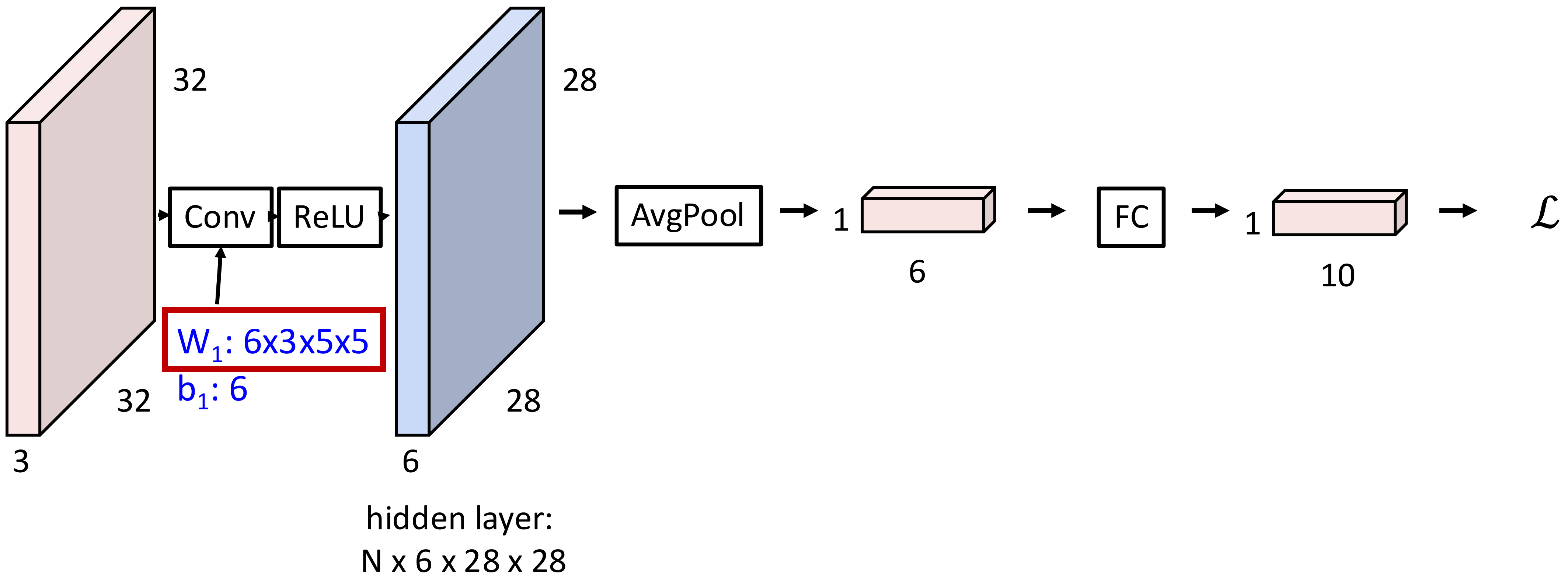
Why is it a better idea than flattening the feature map before the classifier (a fully-connect layer)?

Number of parameters.

Spatial dimension of the input.

global average pooling

Backpropagation for CNNs

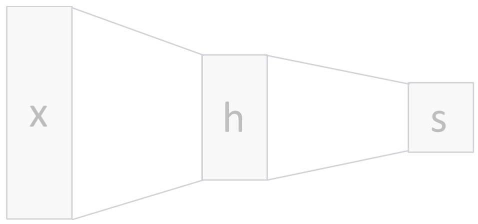


What does its computational graph look like?

Problem: Deep Networks very hard to train!

Components of a Convolutional Network

Fully-Connected Layers

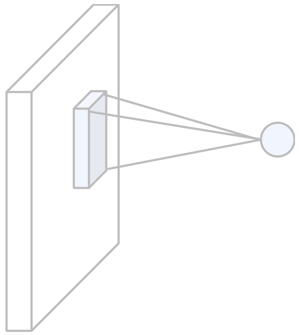


$$y = Wx + b$$

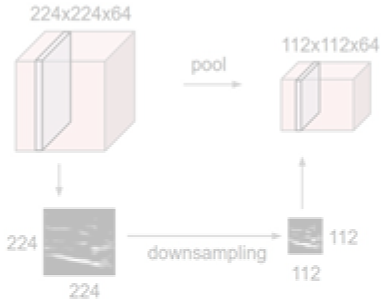
Activation Function

$$y = \max(0, x)$$

Convolution Layers



Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Slide credit: D Fouhey & J Johnson

Batch Normalization

Idea: “Normalize” the outputs of each layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization (hypothesis)

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

Idea: "Normalize" the outputs of each layer so they have zero mean and unit variance

Why? Helps reduce "internal covariate shift", improves optimization (hypothesis)

We can normalize a batch of activations like this:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

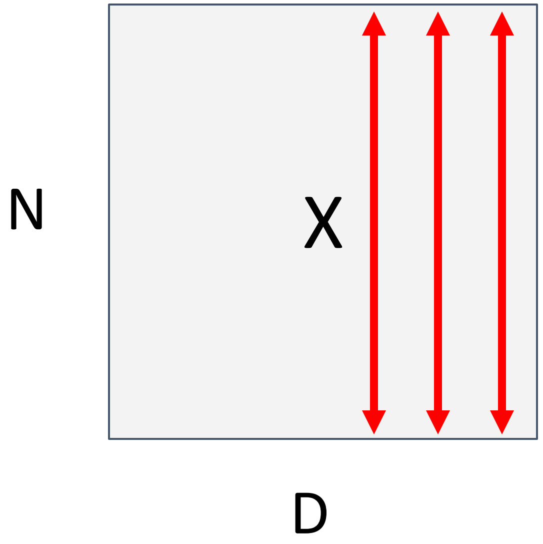
This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Slide credit: D Fouhey & J Johnson

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$



$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} \left(x_{i,j} - \mu_j\right)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

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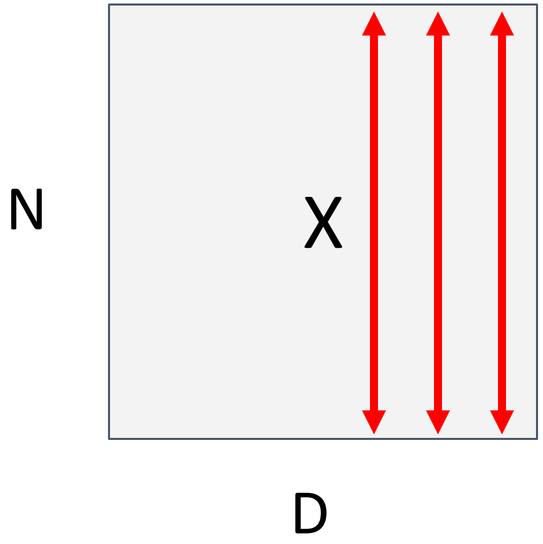

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Normalized x, Shape is N x D

Problem: What if zero-mean, unit variance is too restrictive?

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

Batch Normalization

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^D$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

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Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Slide credit: D Fouhey & J Johnson

Batch Normalization

Problem: Estimates depend on minibatch; can't do this at test-time!

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^{D}$$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

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Slide credit: D Fouhey & J Johnson

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$\gamma, \beta \in \mathbb{R}^D$

Learning $\gamma = \sigma, \beta = \mu$ will recover the identity function (in expectation)

$\mu_j =$ (Running) average of values seen during training

Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Slide credit: D Fouhey & J Johnson

Batch Normalization: Test-Time

Input: $x \in \mathbb{R}^{N \times D}$

Learnable scale and shift parameters:

$$\gamma, \beta \in \mathbb{R}^D$$

During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

$\mu_j =$ (Running) average of values seen during training — Per-channel mean, shape is D

$\sigma_j^2 =$ (Running) average of values seen during training — Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Slide credit: D Fouhey & J Johnson

Batch Normalization for ConvNets

Batch Normalization for **fully-connected** networks

$$x : N \times D$$

Normalize

$$\mu, \sigma : 1 \times D$$
$$\gamma, \beta : 1 \times D$$
$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

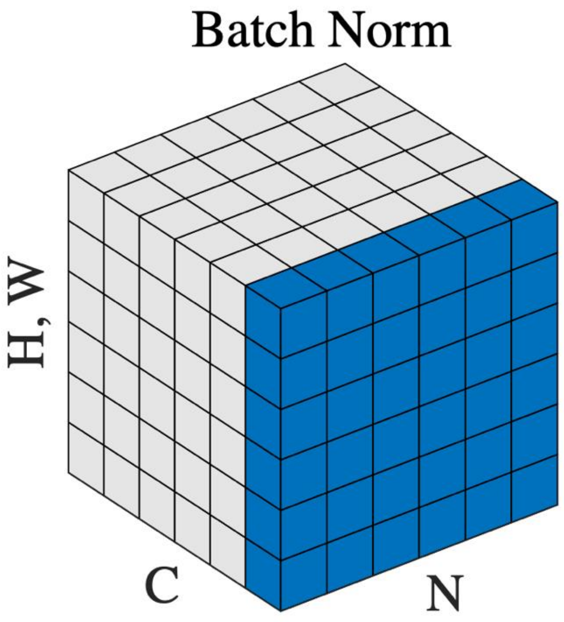
Batch Normalization for **convolutional** networks (Spatial Batchnorm, BatchNorm2D)

$$x : N \times C \times H \times W$$

Normalize

$$\mu, \sigma : 1 \times C \times 1 \times 1$$
$$\gamma, \beta : 1 \times C \times 1 \times 1$$
$$y = \frac{(x - \mu)}{\sigma}\gamma + \beta$$

Batch Normalization for ConvNets



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$$x : N \times C \times H \times W$$

Normalize

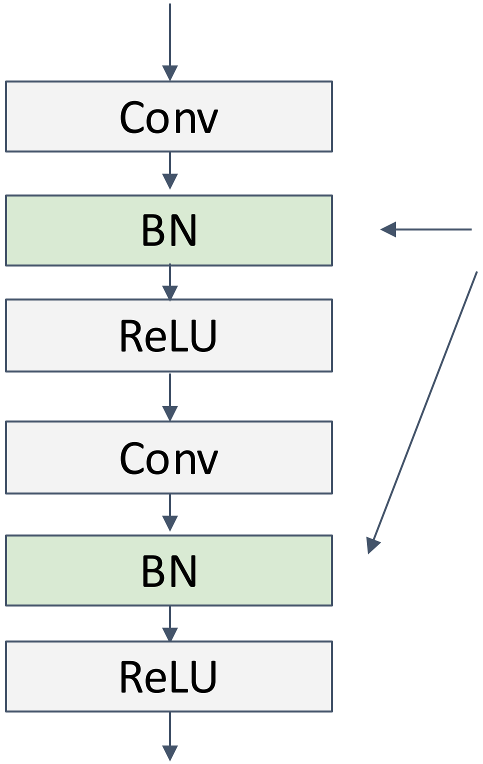
$$\mu, \sigma : 1 \times C \times 1 \times 1$$

$$\gamma, \beta : 1 \times C \times 1 \times 1$$

$$y = \frac{(x - \mu)}{\sigma} \gamma + \beta$$

Slide adapted from: D Fouhey & J Johnson

Batch Normalization

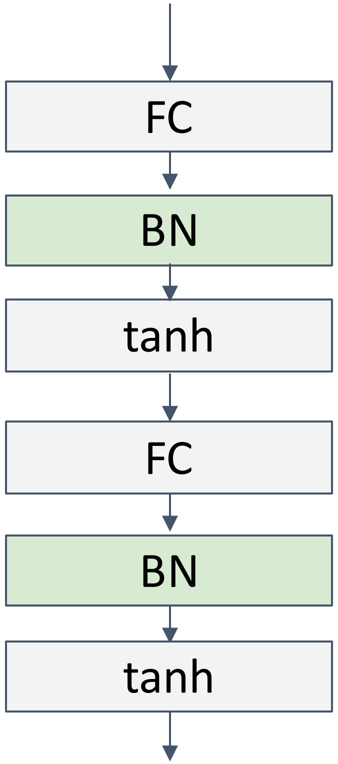


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

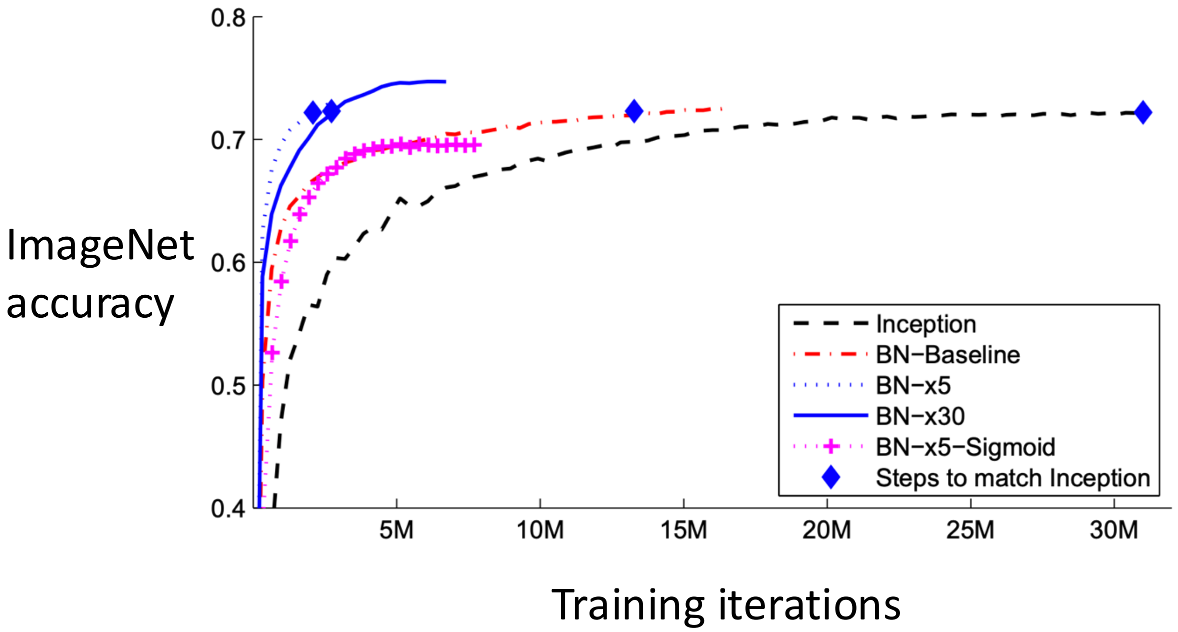
$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Slide credit: D Fouhey & J Johnson

Batch Normalization



- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Free at test-time: can be fused with conv!



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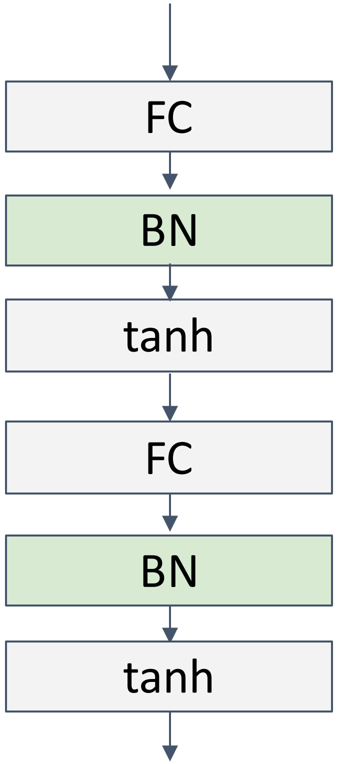
Batch Normalization

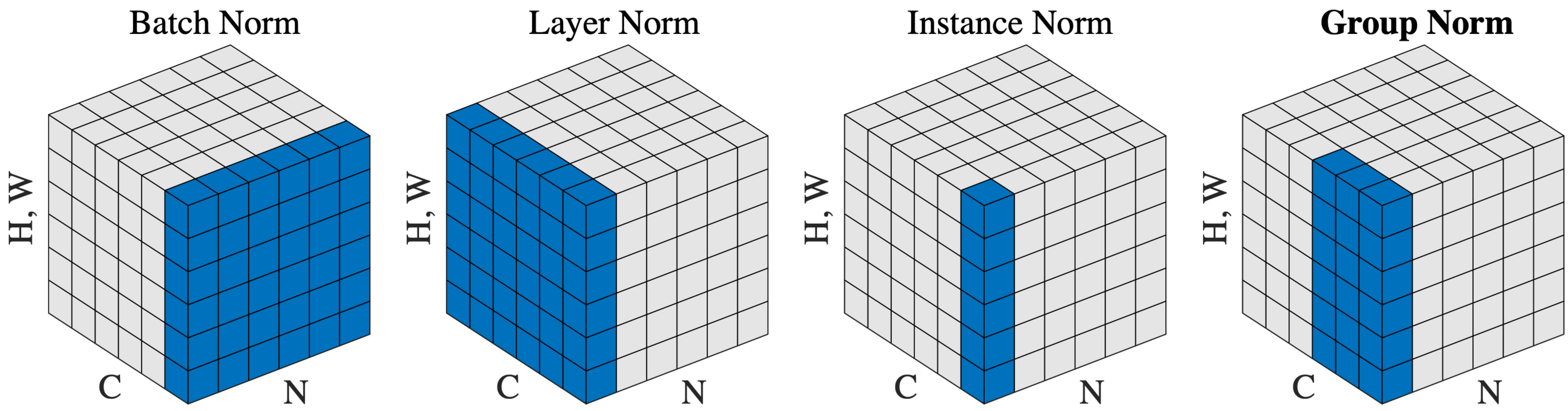


Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015

- Makes deep networks **much** easier to train!
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Free at test-time: can be fused with conv!
- Not well-understood theoretically (yet)
- Behaves differently during training and testing: this is a very common source of bugs!

Slide credit: D Fouhey & J Johnson

Different Normalization Layers



[Wu and He. Group Normalization. ECCV 2018. Best paper honorable mention.]

Devils in the details (mainly for PyTorch)

1. To reliably estimate BN statistics (running mean and average), you need at least 8 samples on each GPU
2. If you don’t have enough samples on each GPU
    1. Synchronized BN layers
    2. Group normalization layers
3. Instance normalization layers are useful for some applications: such as style transfer, dense correspondence.

Next Class

More about
Convolutional Neural Networks