

## Calculus I – Limit

### Problem 1: Limit Tricks

- (a) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ .
- (b) Evaluate  $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$ .
- (c) Evaluate  $\lim_{x \rightarrow 0^+} (e^x - 1)^{\frac{1}{\ln x}}$ .
- (d) Evaluate  $\lim_{x \rightarrow 0} (1 + |\sin x|)^{\frac{1}{x}}$ .
- (e) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{2^x + 3^x + 5^x}{3} \right)^{\frac{1}{x}}$ .

**Problem 2: Limit Tricks**

Let  $\alpha$  and  $\beta$  be two constants. Suppose

$$\lim_{x \rightarrow -\infty} \left( \sqrt{x^2 + 3x + 2} - \alpha x - \beta \right) = 0.$$

Find  $\alpha$  and  $\beta$ .

**Problem 3: Continuity and Differentiability**

Let

$$f(x) = \begin{cases} (1+x)^{\frac{1}{x}}, & x \neq 0, \ x > -1, \\ a, & x = 0. \end{cases}$$

- (a) Find the value of  $a$  such that  $f(x)$  is continuous at  $x = 0$ .
- (b) Find  $\lim_{x \rightarrow \infty} f(x)$ .
- (c) Compute  $f'(x)$ , for  $x \neq 0$ .
- (d) Is  $f(x)$  differentiable at  $x = 0$ ? If  $f(x)$  is differentiable at  $x = 0$ , then find  $f'(0)$ .

**Problem 4: Derivatives**

Consider the function  $f(x) = xe^{\frac{1}{x}}$  for  $x \neq 0$ .

- (a) Find  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0^-} f(x)$ .
- (b) Find all the vertical asymptotes of  $y = f(x)$ .
- (c) Find the slant asymptote(s) of  $y = f(x)$ .
- (d) Find  $f'(x)$ . Write down the interval(s) of increase and interval(s) of decrease of  $y = f(x)$ .
- (e) Find  $f''(x)$ . Write down the interval(s) on which  $y = f(x)$  is concave upward and the interval(s) on which  $y = f(x)$  is concave downward.