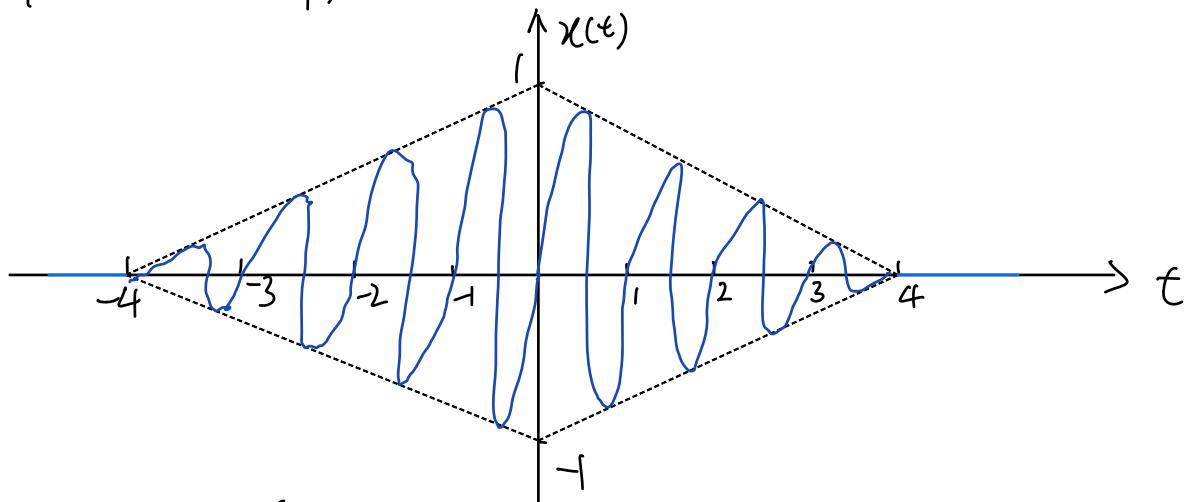


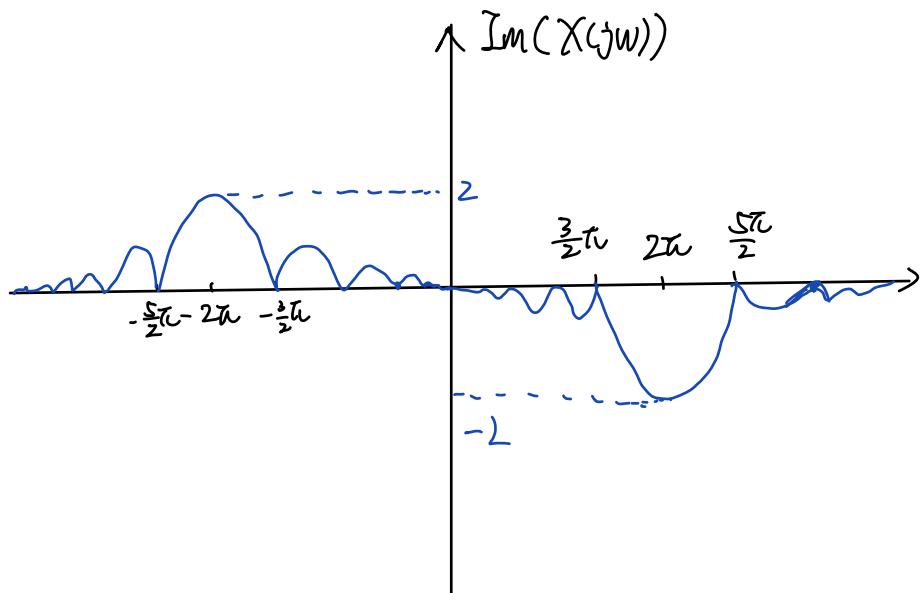
$$I.(a) \chi(t) = \Lambda\left(\frac{t}{4}\right) \sin(2\pi t)$$



$$\chi_s(jw) = \mathcal{F} \left\{ \Lambda\left(\frac{t}{4}\right) \right\} = 4 \operatorname{sinc}\left(\frac{2w}{\pi}\right)$$

$$\chi(jw) = \mathcal{F} \left\{ \Lambda\left(\frac{t}{4}\right) \sin(2\pi t) \right\} = \frac{j}{2} \left( 4 \operatorname{sinc}\left(\frac{2(w+2\pi)}{\pi}\right) - 4 \operatorname{sinc}\left(\frac{2(w-2\pi)}{\pi}\right) \right)$$

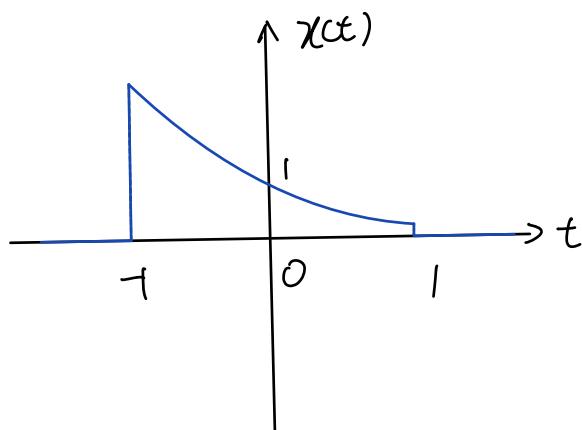
$$= j \left[ 2 \operatorname{sinc}^2\left(\frac{2(w+2\pi)}{\pi}\right) - 2 \operatorname{sinc}^2\left(\frac{2(w-2\pi)}{\pi}\right) \right]$$



$\chi(t)$ : real & odd

$\chi(jw)$ : imaginary & odd.

(b)



$$\begin{aligned}x(t) &= e^{-t}(u(t+1) - u(t-1)) \\&= e \cdot e^{-(t+1)} u(t+1) - e^{-(t-1)} u(t-1) \cdot e^{-1}\end{aligned}$$

$$\begin{aligned}X(j\omega) &= e \cdot e^{j\omega} \left( \frac{1}{1+j\omega} \right) - \frac{1}{e} \cdot e^{j\omega} \left( \frac{1}{1+j\omega} \right) \\&= (e^{j\omega+1} - e^{-j\omega-1}) \left( \frac{1}{1+j\omega} \right) = \frac{e^{j\omega+1} - e^{-j\omega-1}}{1+j\omega}\end{aligned}$$

$x(t)$  real ,  $X(j\omega)$  has conjugate symmetry.

$$\begin{aligned}2.(a.) \quad X_1(j\omega) &= |X_1(j\omega)| e^{j\pi} \\&= \Pi\left(\frac{\omega}{2\pi}\right) e^{j\pi}\end{aligned}$$

$$x(t) = e^{j\pi} \text{sinc}(t) = -\text{sinc}(t).$$

$x(t)$  real & even

$X(j\omega)$  real & even

$$\begin{aligned}
 (b) \quad X_2(jw) &= |X_2(jw)| e^{j\angle X_2(jw)} \\
 &= \pi\left(\frac{w+\frac{\pi}{2}}{\pi}\right) \cdot e^{j\frac{\pi}{2}} + \pi\left(\frac{w-\frac{\pi}{2}}{\pi}\right) e^{-j\frac{\pi}{2}} \\
 x(t) &= \frac{1}{2} \operatorname{sinc}\left(\frac{t}{2}\right) \cdot e^{j\frac{\pi}{2}t} \cdot e^{j\frac{\pi}{2}} + \frac{1}{2} \operatorname{sinc}\left(\frac{t}{2}\right) e^{-j\frac{\pi}{2}t} \cdot e^{-j\frac{\pi}{2}} \\
 &= \frac{j}{2} \operatorname{sinc}\left(\frac{t}{2}\right) \cdot (e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}) \\
 &= 2j \cdot \frac{1}{2j} \cdot \frac{j}{2} \operatorname{sinc}\left(\frac{t}{2}\right) \cdot (e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}) \\
 &= \operatorname{sinc}\left(\frac{t}{2}\right) \left( \frac{e^{-j\frac{\pi}{2}t} - e^{j\frac{\pi}{2}t}}{2j} \right) \\
 &= \operatorname{sinc}\left(\frac{t}{2}\right) \operatorname{sinc}\left(\frac{\pi}{2}t\right)
 \end{aligned}$$

$x(t)$  : odd, real

$X_2(jw)$  : odd, imaginary.

$$3. \quad a. \quad X(j\omega) = F\{x(t)\}$$

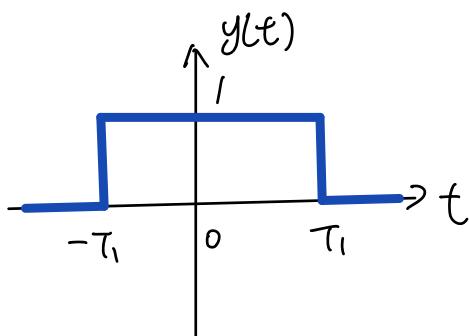
$$= F\{\delta(t+\tau_1) - \delta(t-\tau_1)\}$$

$$= e^{-j\omega(\tau_1)} \cdot 1 - e^{j\omega\tau_1} \cdot 1$$

$$= \frac{e^{j\omega\tau_1} - e^{-j\omega\tau_1}}{2j} \cdot 2j$$

$$= j \cdot 2 \sin(\omega\tau_1)$$

$$b.$$



$$c. \quad Y(j\omega) = \frac{1}{j\omega} X(j\omega) + \pi X(j\omega) \delta(\omega)$$

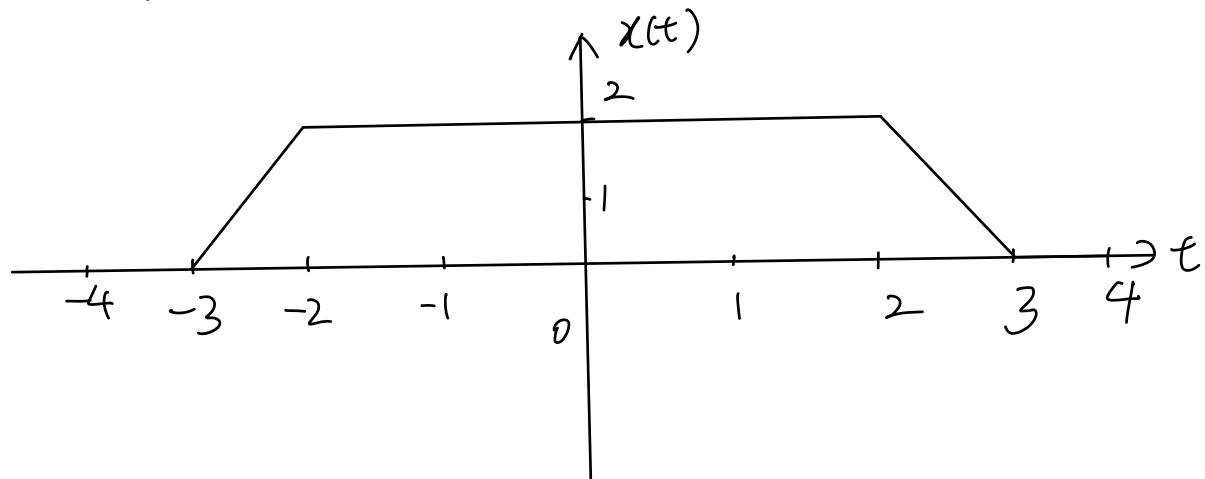
$$= \frac{1}{j\omega} \cdot 2j \sin(\tau_1\omega) + \pi \cdot 2j \cdot \sin(\tau_1 \cdot 0) \delta(\omega)$$

$$= \frac{2 \sin(\tau_1\omega)}{\omega} = \frac{2\tau_1 \sin(\omega\tau_1)}{\omega\tau_1}$$

$$= \frac{2\tau_1 \sin\left(\frac{\omega\tau_1\pi}{\pi}\right)}{\pi\omega\tau_1} = 2\tau_1 \sin\left(\frac{\omega\tau_1}{\pi}\right)$$

4. a.  $x_1(t) = \Pi\left(\frac{t}{2}\right)$ ,  $x_2(t) = \Pi\left(\frac{t}{4}\right)$

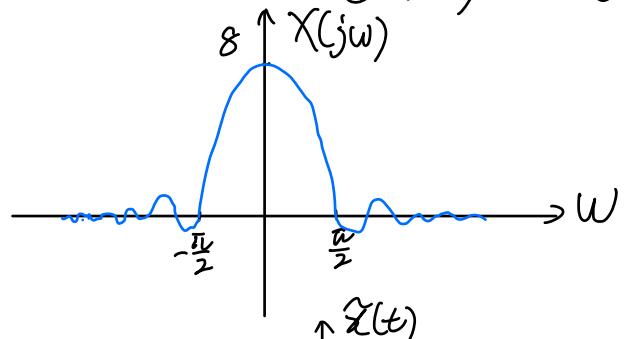
$$x(t) = x_1(t) * x_2(t)$$



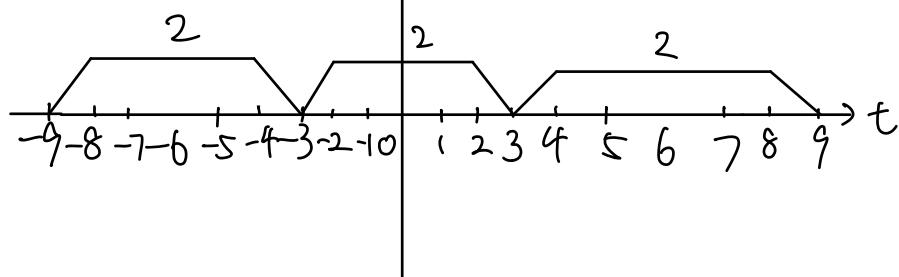
b.  $X(j\omega) = X_1(j\omega) X_2(j\omega)$

$$= 2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right) \cdot 2T_2 \operatorname{sinc}\left(\frac{\omega T_2}{\pi}\right)$$

$$= 4T_1 T_2 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right) \operatorname{sinc}\left(\frac{\omega T_2}{\pi}\right)$$



c.

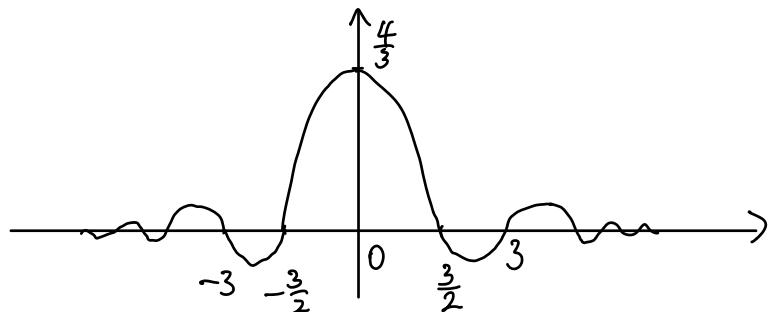


d.  $T_0 = 6$ . ' by sampling at  $\omega = \omega_0 k$ . ( $\omega_0 = \frac{2\pi}{T_0}$ )

$$X(j\omega)|_{\omega=\omega_0 k} = 8 \operatorname{sinc}\left(\frac{\omega_0 k}{\pi}\right) \operatorname{sinc}\left(\frac{2\omega_0 k}{\pi}\right) = 6 a_k$$

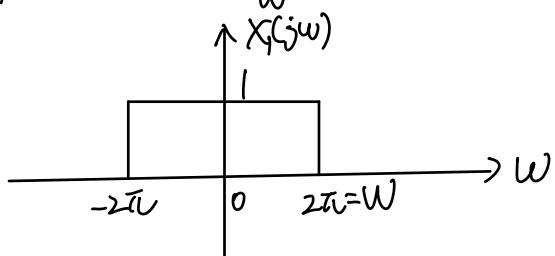
$$a_k = \frac{4}{3} \operatorname{sinc}\left(\frac{2\pi k}{6\pi}\right) \operatorname{sinc}\left(\frac{4\pi k}{6\pi}\right)$$

$$= \frac{4}{3} \operatorname{sinc}\left(\frac{k}{3}\right) \cdot \operatorname{sinc}\left(\frac{2k}{3}\right)$$

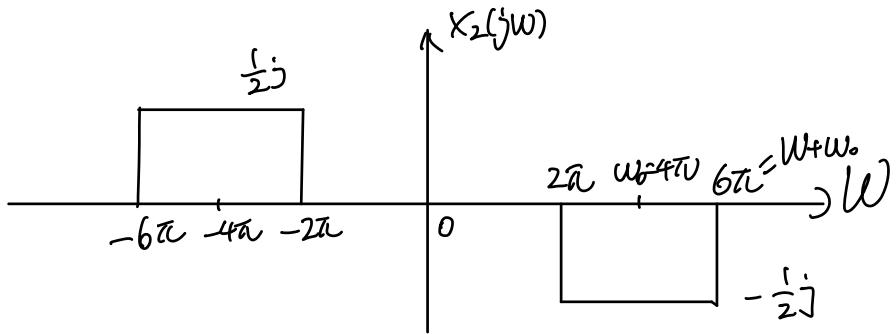


$$\text{S. a. } x(t) = \frac{\omega}{\pi} \operatorname{sinc}\left(\frac{\omega t}{\pi}\right) = 2 \operatorname{sinc}(2t)$$

$$X(j\omega) = \Pi\left(\frac{\omega}{2W}\right) = \Pi\left(\frac{\omega}{4\pi}\right)$$



$$\begin{aligned} X_2(j\omega) &= \mathcal{F}\left\{ x(t) \sin(\omega_0 t) \right\} = \frac{j}{2} [X(j(\omega + \omega_0)) - X(j(\omega - \omega_0))] \\ &= \frac{j}{2} [X(j(\omega + 4\pi)) - X(j(\omega - 4\pi))] \\ &= \frac{j}{2} [\Pi\left(\frac{\omega + 4\pi}{4\pi}\right) - \Pi\left(\frac{\omega - 4\pi}{4\pi}\right)] \end{aligned}$$



$$b. E x_1 = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X_2(jw)|^2 dw$$

$$= \frac{w}{\pi}$$

$$E x_2 = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X_2(jw)|^2 dw$$

$$= \frac{w}{2\pi}$$

$$c. \langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{\infty} x_1(t) x_2^*(t) dt$$

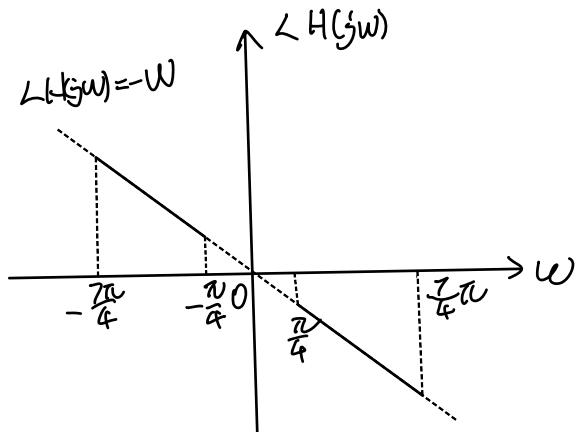
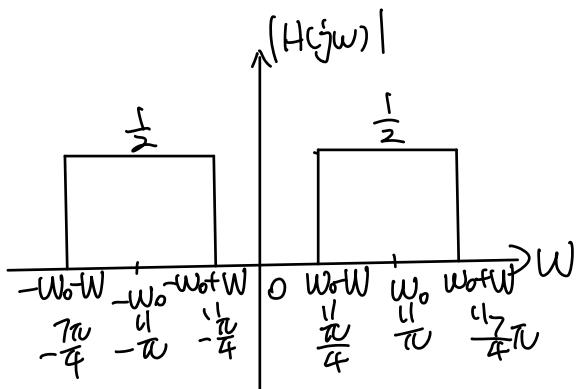
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(jw) X_2^*(jw) dw$$

$$= 0$$

$$6. a. H(jw) = \frac{1}{2} e^{-jw\tau_0} \left[ \pi \left( \frac{w+w_0}{2W} \right) + \pi \left( \frac{w-w_0}{2W} \right) \right]$$

$$= \frac{1}{2} \pi \left( \frac{w+w_0}{2W} \right) e^{-jw\tau_0} + \frac{1}{2} \pi \left( \frac{w-w_0}{2W} \right) e^{-jw\tau_0}$$

let  $W = \frac{3}{4}\pi$ ,  $w_0 = \pi$ ,  $\tau_0 = 1$ .



b. band pass filter

$$c. y(t) = x(t) * h(t) = 1 * h(t) + 2\sin(t) * h(t) + \cos(2\pi t) * h(t)$$

$$y(jw) = 2\pi\delta(w) \cdot H(jw) + 2\pi j [\delta(w+1) - \delta(w-1)] \cdot H(jw)$$

$$+ \pi [\delta(w-2\pi) + \delta(w+2\pi)] \cdot H(jw)$$

$$= 0 + 2\pi j [\delta(w+1) - \delta(w-1)] \cdot H(jw) + 0$$

$$= 2\pi j [\delta(w+1) - \delta(w-1)] \cdot H(jw)$$

$$= 2\pi j [\delta(w+1) - \delta(w-1)] \cdot \left( \frac{1}{2} e^{-jw\tau_0} \left[ \pi \left( \frac{w+w_0}{2W} \right) + \pi \left( \frac{w-w_0}{2W} \right) \right] \right)$$

$$(w_0 = \pi, W = \frac{3}{4}\pi, \tau_0 = 1)$$

$$= \pi j e^{-jw} \cdot \left\{ \delta(w+1) \left[ \pi \left( \frac{w+\pi}{2 \cdot \frac{3}{4}\pi} \right) + \pi \left( \frac{w-\pi}{2 \cdot \frac{3}{4}\pi} \right) \right] - \delta(w-1) \left[ \pi \left( \frac{w+\pi}{2 \cdot \frac{3}{4}\pi} \right) + \pi \left( \frac{w-\pi}{2 \cdot \frac{3}{4}\pi} \right) \right] \right\}$$

$$= e^{-jw} \cdot \pi j (\delta(w+1) - \delta(w-1)) = e^{-jw} \cdot \pi j (\delta(w+1) - \delta(w-1))$$

$$\text{thus : } y(t) = \sin(t-1)$$

$$7. \text{ a. } y(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t-t_0)$$

$$h(t) = \frac{1}{2} g(t) + \frac{1}{2} g(t-t_0)$$

$$\text{b. } H(j\omega) = \frac{1}{2} (F\{g(t)\} + F\{g(t-t_0)\})$$

$$= \frac{1}{2} (1 + e^{-j\omega t_0})$$

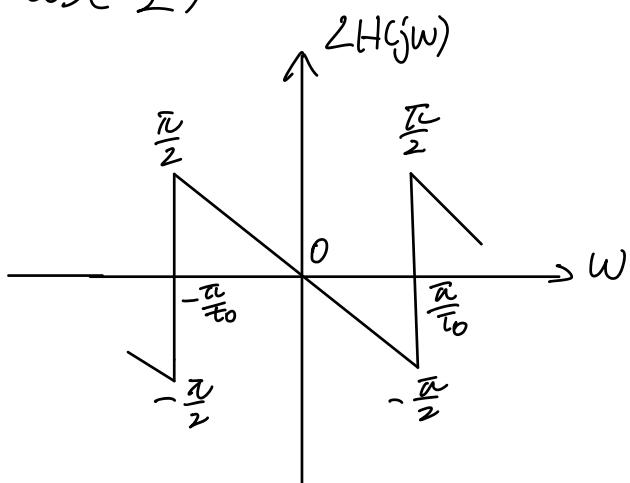
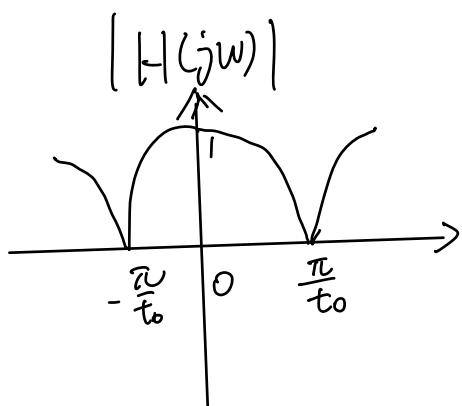
$$= \frac{1}{2} + \frac{1}{2} e^{-j\omega t_0}$$

$$= e^{\frac{-j\omega t_0}{2}} \left( \frac{e^{\frac{j\omega t_0}{2}} + e^{-\frac{j\omega t_0}{2}}}{2} \right)$$

$$= e^{\frac{-j\omega t_0}{2}} \cdot \cos\left(\frac{\omega t_0}{2}\right)$$

$$|H(j\omega)| = \left| e^{\frac{-j\omega t_0}{2}} \cdot \cos\left(\frac{\omega t_0}{2}\right) \right| = \left| \cos\left(\frac{\omega t_0}{2}\right) \right|$$

$$\angle H(j\omega) = \begin{cases} -\frac{t_0}{2}\omega & \cos\left(\frac{\omega t_0}{2}\right) \geq 0 \\ -\frac{t_0}{2}\omega + \pi & \cos\left(\frac{\omega t_0}{2}\right) < 0. \end{cases}$$



$$C. \quad x(t) = \sin(\omega_0 t)$$

$$y(t) = \sin(\omega_0 t) * h(t)$$

$$= \sin(\omega_0 t) * \frac{1}{2} \delta(t) + \sin(\omega_0 t) * \frac{1}{2} \delta(t-t_0)$$

$$Y(t) = j\pi [s(\omega + \omega_0) - s(\omega - \omega_0)] \cdot \frac{1}{2} + \frac{1}{2} j\pi [s(\omega + \omega_0) - s(\omega - \omega_0)] \cdot e^{j\omega_0 t}$$

$$= \frac{1}{2} j\pi [s(\omega + \omega_0) - s(\omega - \omega_0)] (1 + e^{-j\omega_0 t})$$

$$y(t) = \frac{1}{2} \sin(\omega_0 t) + \frac{1}{2} \sin(\omega_0 t - \omega_0 t_0)$$

$$= \frac{1}{2} (\sin(\omega_0 t) + \sin(\omega_0 t - \omega_0 t_0))$$

$$= \frac{1}{2} (\sin(\omega_0 t) + \sin(\omega_0 t) \cos(\omega_0 t_0) - \cos(\omega_0 t) \sin(\omega_0 t_0))$$

$$= \frac{1}{2} (\sin(\omega_0 t) [1 + \cos(\omega_0 t_0)] - \cos(\omega_0 t) \sin(\omega_0 t_0))$$

this requires  $\cos(\omega_0 t_0) = -1$  and  $\sin(\omega_0 t_0) = 0$ .

therefore  $\omega_0 t_0 = 2k\pi + \pi$   
 $\omega_0 = \frac{2k\pi + \pi}{t_0}, k \in \mathbb{Z}$ .

$$8. a. \quad h(t) = \frac{t}{\tau} x(t).$$

thus

$$H(j\omega) = \frac{1}{\tau} \left( j \frac{dX(j\omega)}{d\omega} \right) = \frac{j}{\tau} \cdot \frac{d}{d\omega} \left( \frac{1}{1+j\omega\tau} \right)$$

$$= \frac{j}{\tau} \cdot \frac{-1}{(1+j\omega\tau)^2} \cdot j\tau$$

$$= -\frac{1}{(1+j\omega\tau)^2}$$

$$b. \quad y(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) \neq \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t), \quad X(j\omega) = \frac{1}{1+j\omega\tau} = F\{x(t)\} \\ = F\left\{\frac{1}{\tau} e^{-\frac{t}{\tau}} u(t)\right\}$$

$$Y(j\omega) = X(j\omega) \cdot X(j\omega)$$

$$= \frac{1}{(1+j\omega\tau)^2} = H(j\omega)$$

therefore  $y(t) = h(t) = \frac{t}{\tau^2} e^{-\frac{t}{\tau}} u(t).$

q.  $x(t) = x_e(t) + x_o(t)$  from "right-sided":  $x(t) = x(t)u(t)$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)] = \frac{1}{2} [x(t)u(t) + x(-t)u(-t)], \\ \Rightarrow x_e(t)u(t) = \frac{1}{2} x(t)u(t) \Rightarrow x_e(t) = \frac{1}{2} x(t)$$

$x(t)$  real thus:

$$\begin{aligned} F\{x_e(t)\} &= \frac{1}{2} [X(j\omega) + X(-j\omega)] \\ &= \frac{1}{2} [X(j\omega) + X^*(j\omega)] \\ &= \operatorname{Re}[X(j\omega)] \end{aligned}$$

thus

$$F^{-1}\{\operatorname{Re}[X(j\omega)]\} = x_e(t)$$

for  $t \leq 0, \quad x(t) = 0.$

thus:  $x(t) = \begin{cases} 2F^{-1}\{\operatorname{Re}[X(j\omega)]\}, & t > 0 \\ 0 & t \leq 0. \end{cases}$

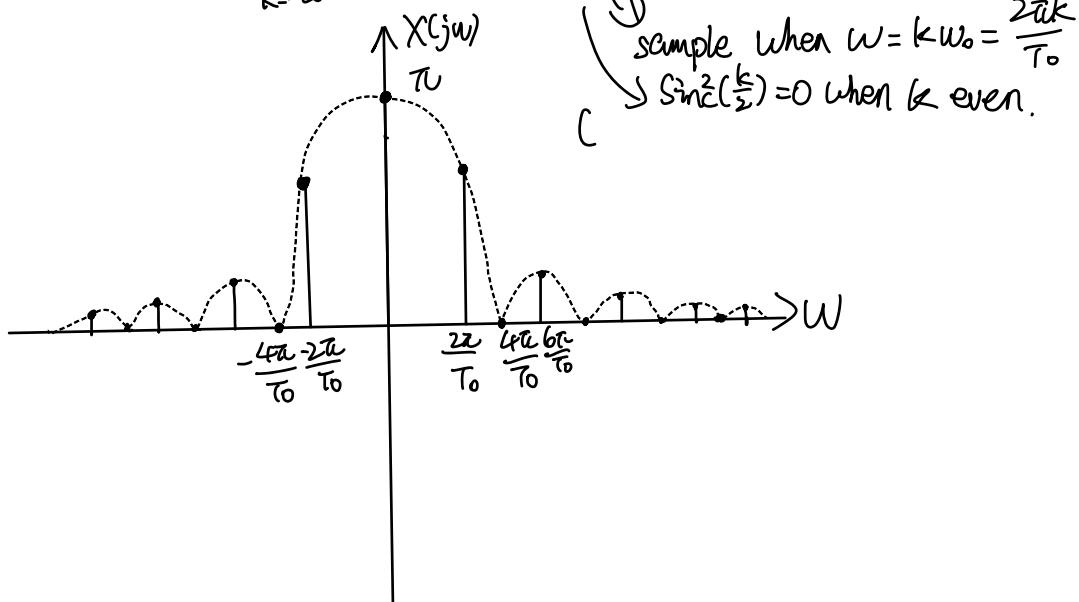
$$10. \quad a_k = \frac{4\pi^2}{T_0} \operatorname{sinc}^2\left(\frac{k 2\pi T_0}{4\pi T_0}\right) \cdot \frac{1}{2T_1}$$

$$= \frac{1}{2} \operatorname{sinc}^2\left(\frac{k}{2}\right)$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$X(jw) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(w - kw_0)$$

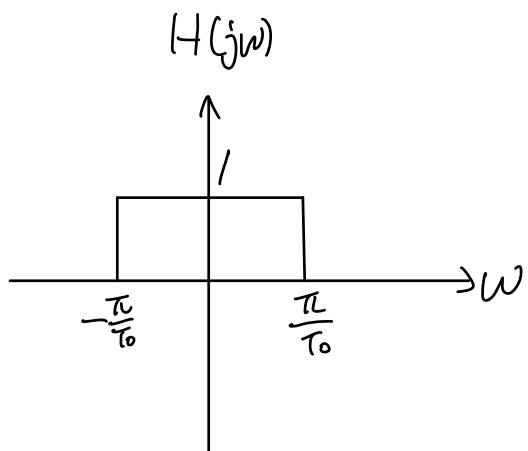
$$= \pi \sum_{k=-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{k}{2}\right) \delta(w - kw_0)$$



$$b. \quad h(t) = \frac{1}{T_0} \operatorname{sinc}\left(\frac{t}{T_0}\right)$$

$\downarrow F$

$$H(jw) = \Pi\left(\frac{T_0 w}{2\pi}\right)$$

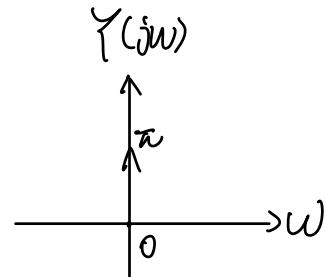


C.  $h(t)$  aperiodic

$$y(t) = x(t) * h(t)$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = \pi S(\omega)$$

$$y(t) = \frac{1}{2}, \text{ periodic}$$

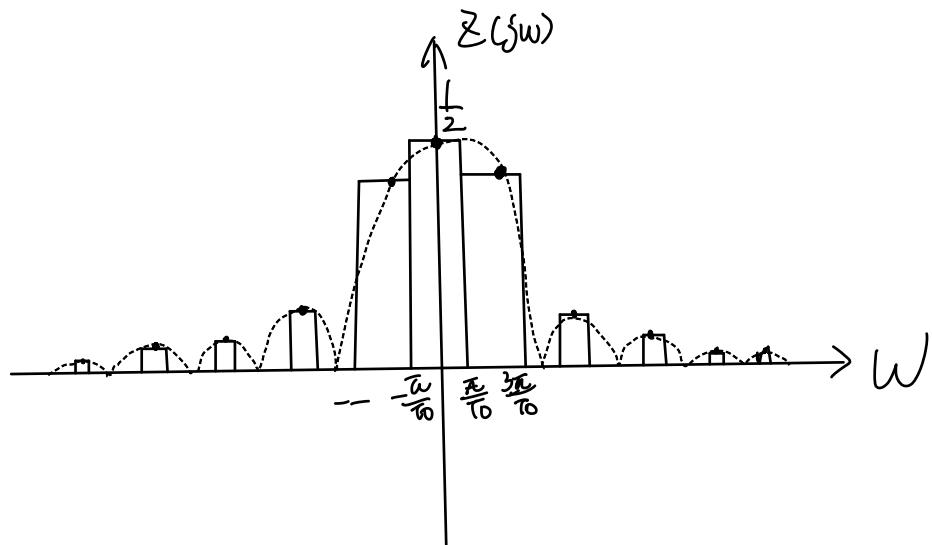


d.  $z(t) = x(t)Xh(t)$

$$Z(j\omega) = \frac{1}{2\pi} [X(j\omega) * H(j\omega)]$$

$$= \frac{1}{2} \operatorname{Im} \left( \frac{W}{2\pi j T_0} \right) * \sum_{k=-\infty}^{\infty} \sin^2 \left( \frac{k}{2} \right) \delta(\omega - kw_0)$$

$$= \frac{1}{2} \sum_{k=-\infty}^{\infty} n \left( \frac{\omega - k \frac{2\pi}{T_0}}{2\pi/T_0} \right) \sin^2 \left( \frac{k}{2} \right) \quad \omega = k \frac{2\pi}{T_0}$$



according to the plot above,

the area/integration of pulse-trains of the Fourier transform  
is not equal, thus  $Z(t)$  is not periodic.

```

function y = lambda(t)
% unit triangular pulse
y=1-abs(t);
y(t>=1)=0;
y(t<=-1)=0;
end

deltat=1/300;
tmax=1.5;
t=-tmax:deltat:tmax;
x=lambda(2*t);
[X,omega]=CTFT_approx(x,t);
phi = 0;

figure(1);
subplot(211);
plot(t,x);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Time \it{t}\rm (s)');
ylabel('|\it{x}\rm(\it{t}\rm)|');
grid on;
title('Message Signal and Spectrum');

subplot(212);
plot(omega/(2*pi),abs(X));
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\it{X}\rm(\it{j}\rm\omega)|');
grid on;

fc = 50;
omegac = 2*pi*fc;
y=x.*cos(omegac*t);
[Y,omega2]=CTFT_approx(y,t);
figure(2);
subplot(211);
plot(t,y);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Time \it{t}\rm (s)');
ylabel('|\it{y}\rm(\it{t}\rm)|');
grid on;
title('Modulated Signal and Spectrum');

subplot(212);
plot(omega/(2*pi),abs(Y));
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\it{Y}\rm(\it{j}\rm\omega)|');
grid on;

v=1/2*x*cos(phi)+1/2*x.*cos(2*omegac*t+phi);
[V,omega3]=CTFT_approx(v,t);
figure(3);
subplot(211);

```

```

plot(t,v);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Time \it{t}\rm (s)');
ylabel('|\it{v}\rm(\it{t}\rm)|');
grid on;
title('Demodulated Signal and Spectrum');

subplot(212);
plot(omega3/(2*pi),abs(V));
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\it{V}\rm(\it{j}\rm\omega)|');
grid on;

fn=5;omegan=2*pi*fn;
hlpf=hsolpfcd(t,omegan);
[H,omega4]=CTFT_approx(hlpf,t);
magnitude=abs(H);
phase=angle(H);
figure(4);
subplot(311)
plot(t,hlpf);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Time \it{t}\rm (s)');
ylabel('|\it{h}\rm(\it{t}\rm)|');
grid on;
title('Impulse Response of LPF');
subplot(312);
plot(omega4/(2*pi),magnitude);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|\it{H}\rm(\it{j}\rm\omega)|');
grid on;

subplot(313);
plot(omega4/(2*pi),phase);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('<\it{H}\rm(\it{j}\rm\omega)>');
grid on;

tw=-2*tmax:deltat:2*tmax;
w=deltat*conv(v,hlpf);
[W,omega9]=CTFT_approx(w,tw);

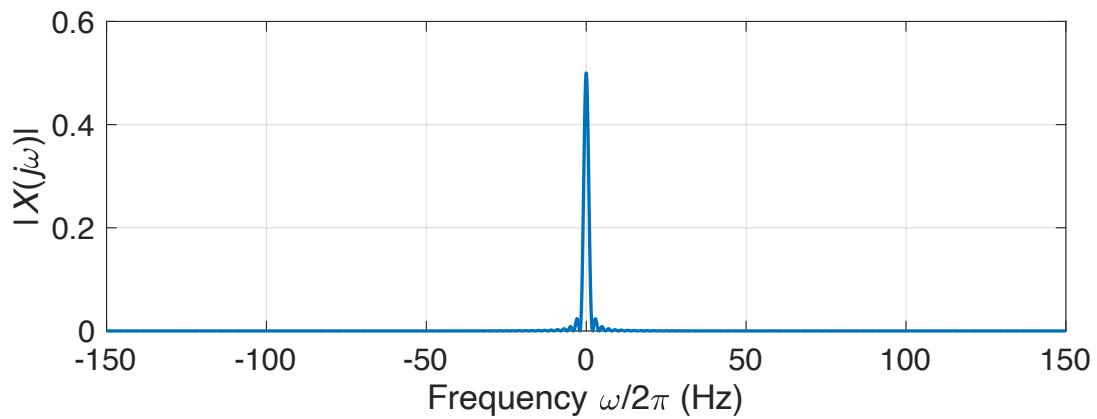
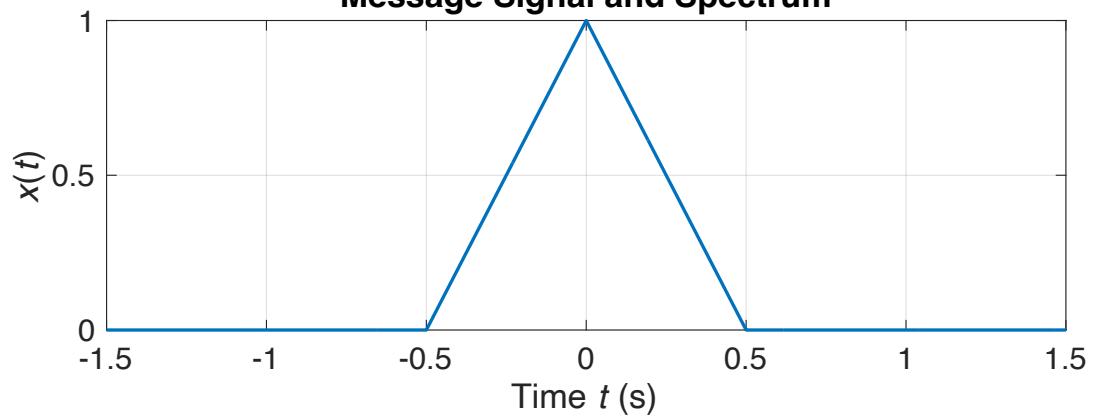
figure(5);
subplot(211);
plot(tw,w);
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Time \it{tw}\rm (s)');
ylabel('|\it{w}\rm(\it{t}\rm)|');
%ylim([0,0.5])
xlim([-1.5,1.5])

```

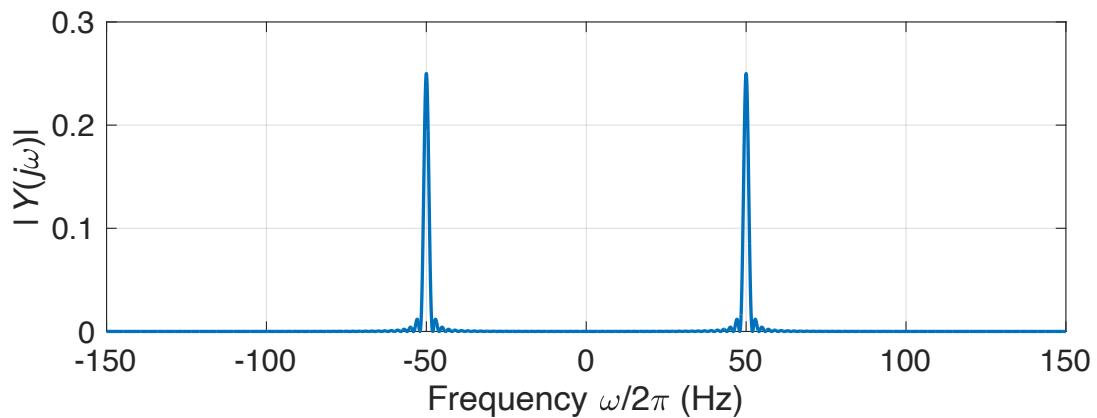
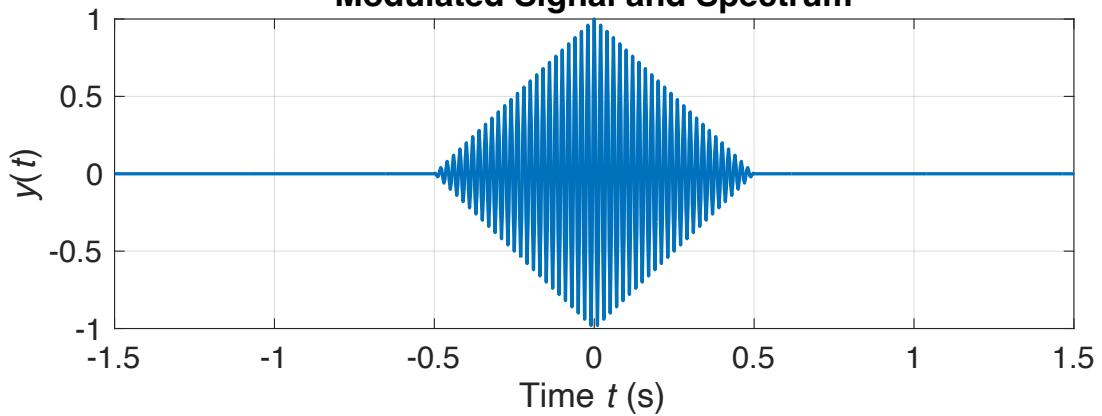
```
grid on;
title('Filtered Signal and Spectrum');

subplot(212);
plot(omega9/(2*pi),abs(W));
l=get(gca,'children');set(l,'linewidth',1.5)
set(gca,'FontName','arial');set(gca,'FontSize',14);
xlabel('Frequency \omega/2\pi (Hz)');
ylabel('|W(\omega)|');
%ylim([0,0.5])
grid on;
```

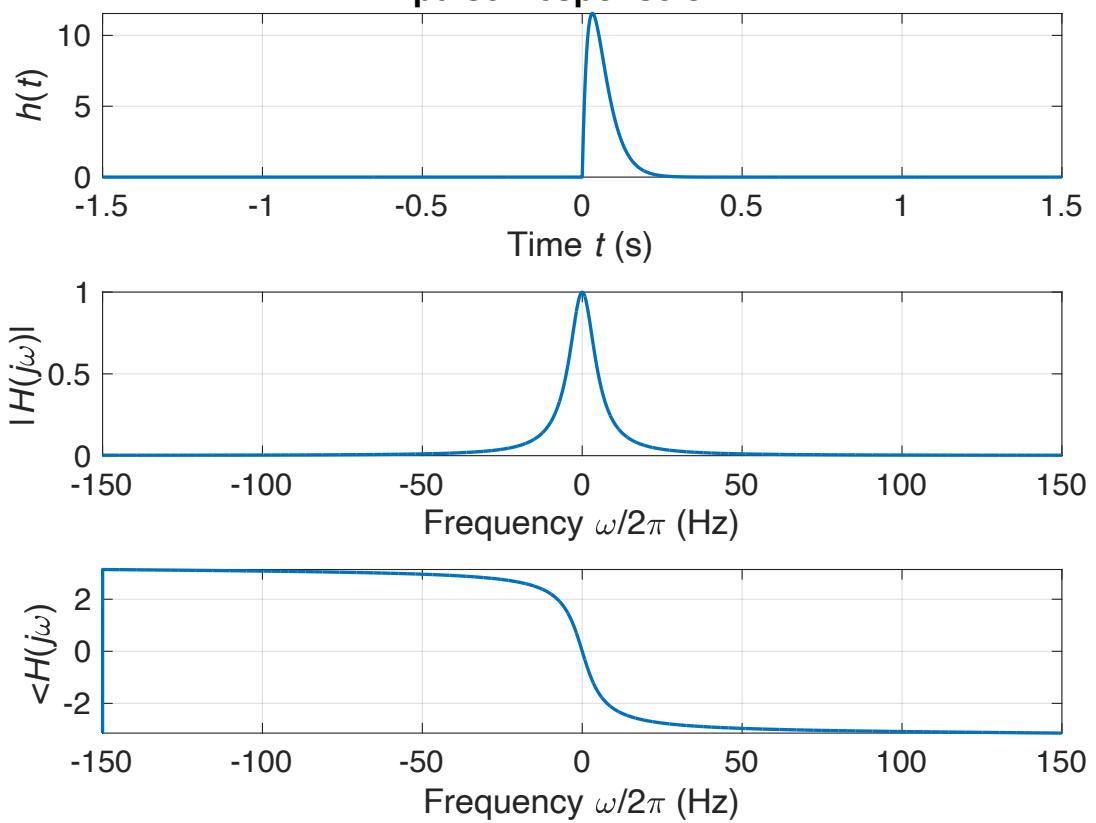
**Message Signal and Spectrum**



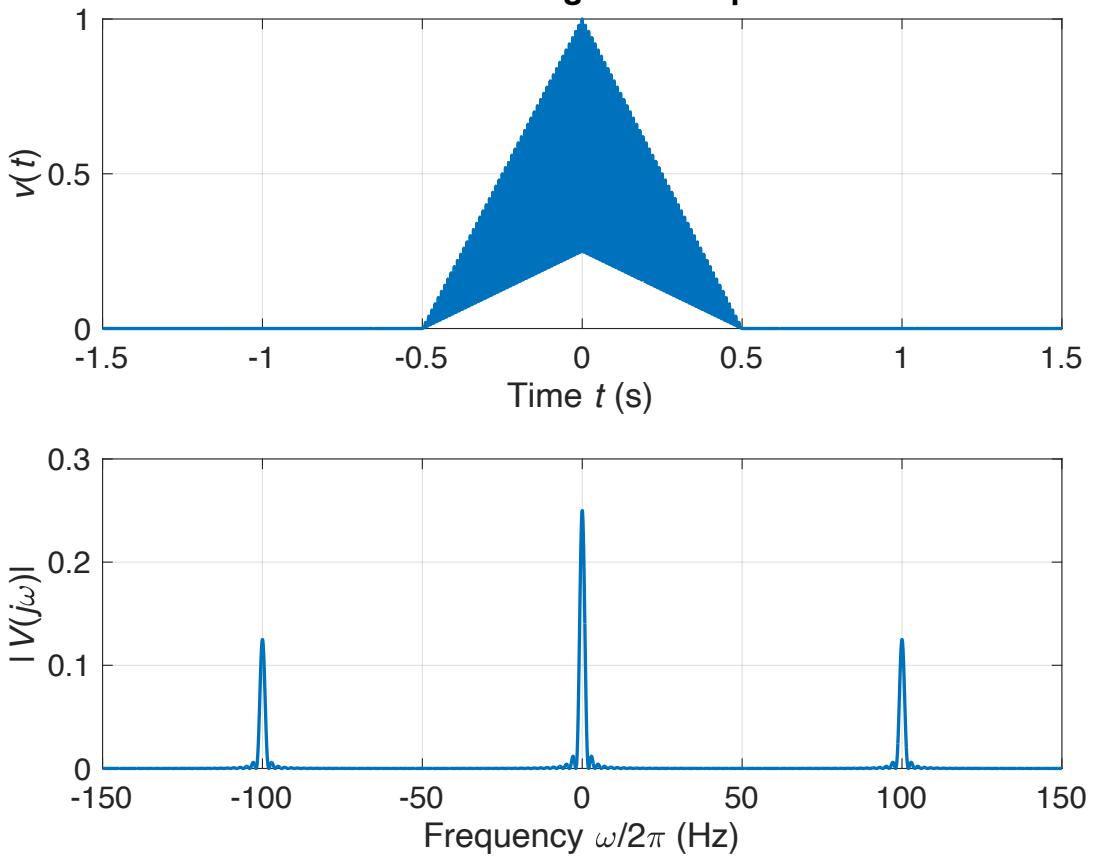
**Modulated Signal and Spectrum**

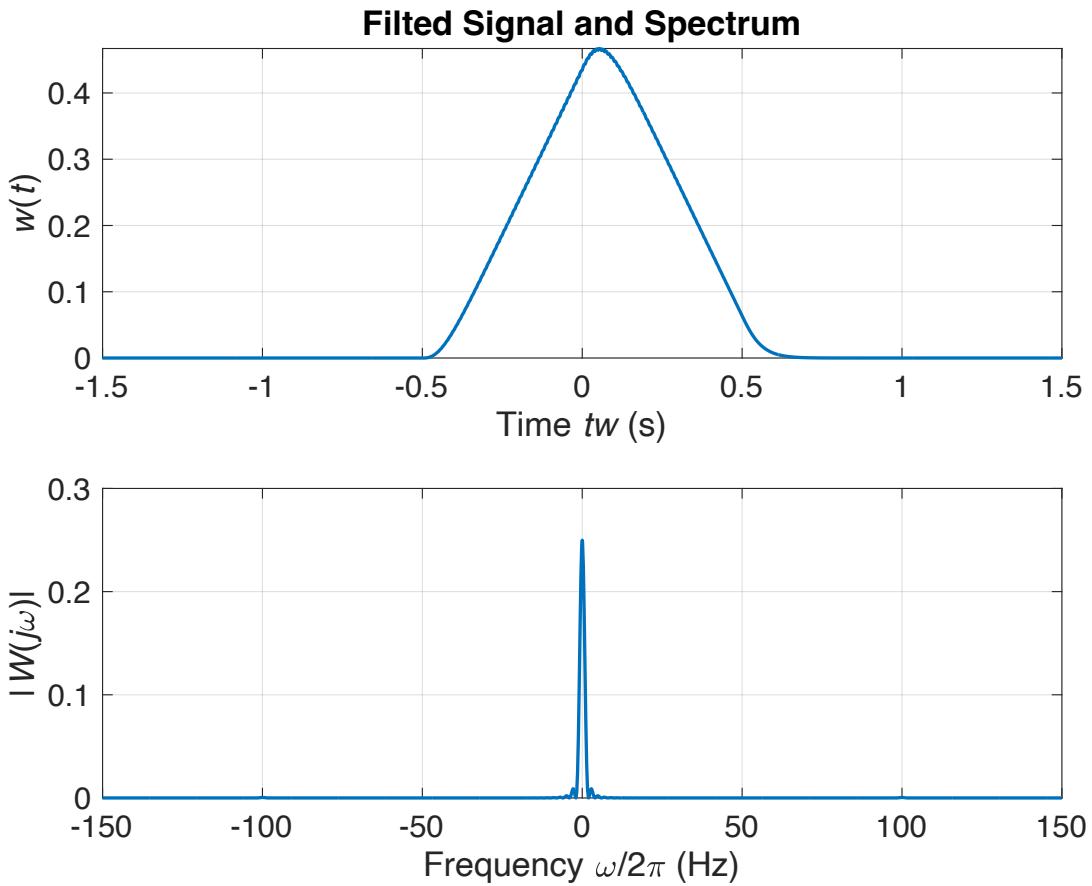


### Impulse Response of LPF



### Demodulated Signal and Spectrum





5.

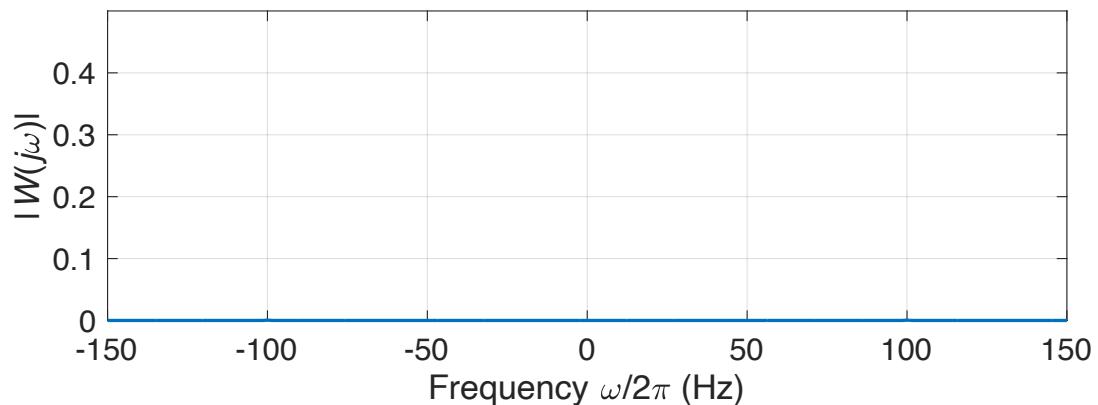
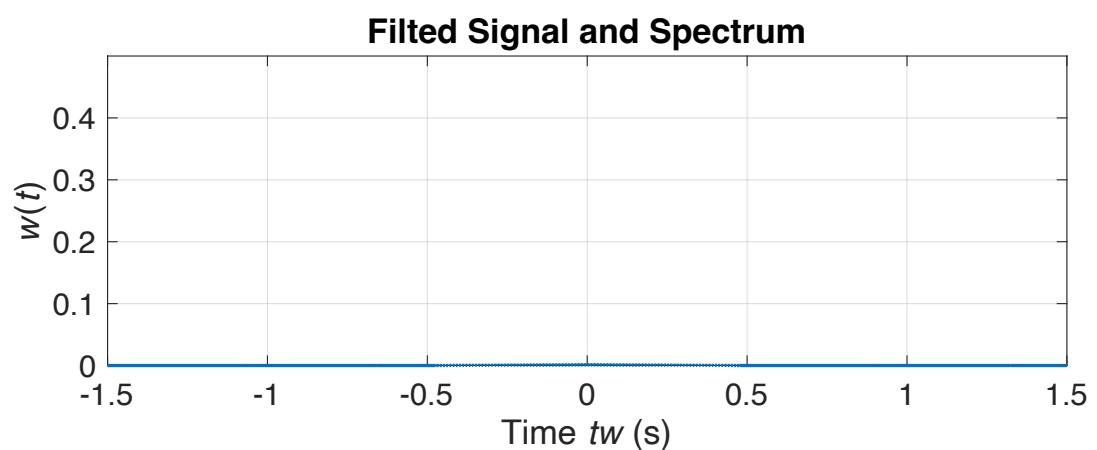
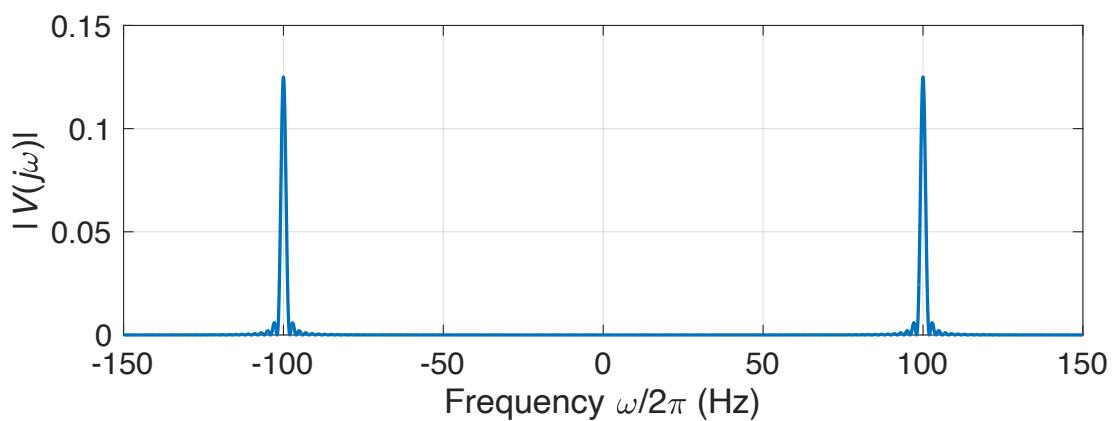
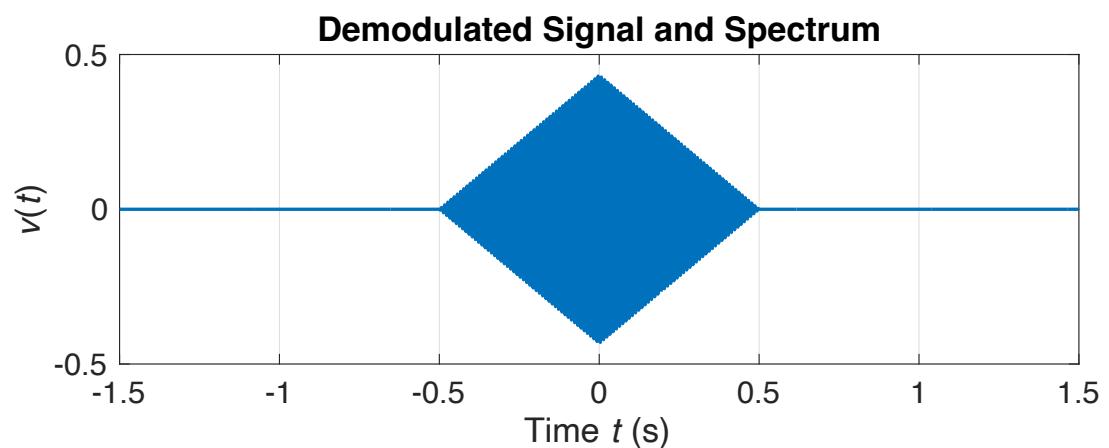
The filtered signal  $w(t)$  has 1. Small leakage of the 2 wc component; 2. Amplitude distortion;  
3. Time delay. Possible improvement: reduce the LPF cut off frequency; use a higher order filter with a higher cut-off frequency.

6.

Phase offsets of  $\phi=0, \pi/2$ , and  $\pi$  yield lowpass filter outputs  $w(t) = \frac{1}{2}x(t)$ , 0, and  $-1/2x(t)$ .  
Phase of  $\pi/2$  is worst since no signal is recovered.

7.

$\Phi=1/2\pi$ :



Phi = pi:

