1. a. $x[n] = \cos(\sqrt{3}n)$: periodic.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}} = \frac{2\sqrt{3}}{3}\pi$

b. $x[n] = \cos(\frac{3\pi}{5}n) + \sin(\frac{4\pi}{5}n)$

suppose $T_1 = \frac{2\pi}{\frac{3\pi}{5}} = \frac{10}{3}$

$T_2 = \frac{2\pi}{\frac{4\pi}{5}} = \frac{5}{2}$

$\frac{T_1}{T_2} = \frac{10}{3} \times \frac{2}{5} = \frac{4}{3}$, $x[n]$ is periodic.

thus: $T = 3 \cdot T_1 = 10$

c. $x[n] = \sin(\frac{\pi}{6}n^2)$: periodic.

suppose $T = N$. then we have $x[n] = x[n+N]$

thus: $\sin(\frac{\pi}{6}n^2) = \sin(\frac{\pi}{6}(n+N)^2)$

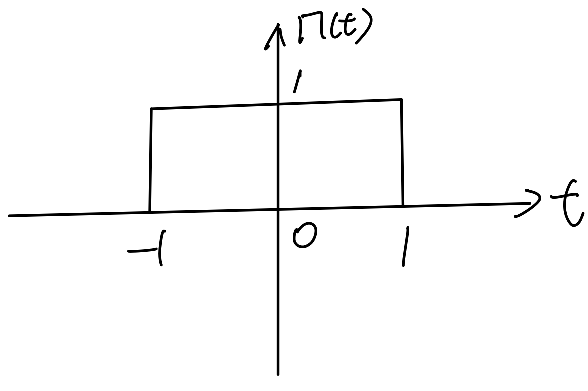
$\sin(\frac{\pi}{6}(n+N)^2) = \sin(\frac{\pi}{6}(n^2+2nN+N^2))$

$= \sin(\frac{\pi}{6}n^2 + \frac{nN}{3} + \frac{\pi}{6}N^2)$

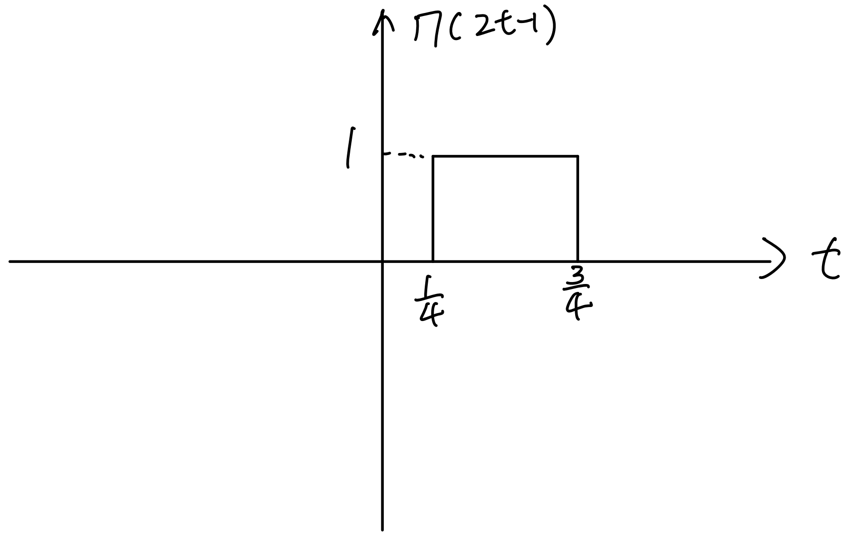
take $n=0$ we have: $\sin 0 = \sin(\frac{\pi}{6}N^2)$

thus $\frac{\pi}{6}N^2 = 2\pi \Rightarrow N = 2\sqrt{3}$.

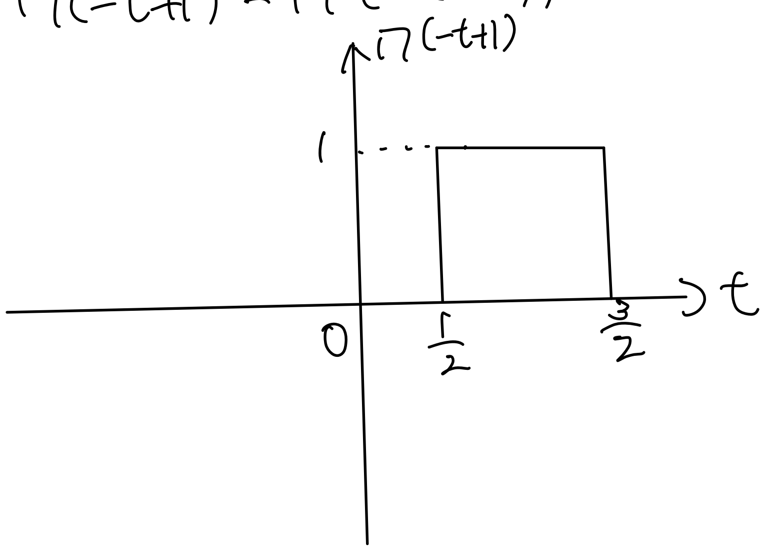
2. a.



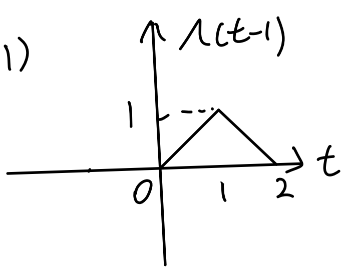
b. $\Pi(2t-1) = \Pi(2(t-\frac{1}{2}))$



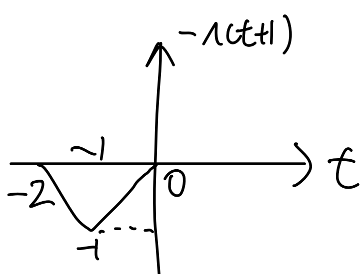
c. $\Pi(-t+1) = \Pi(-(t-1))$



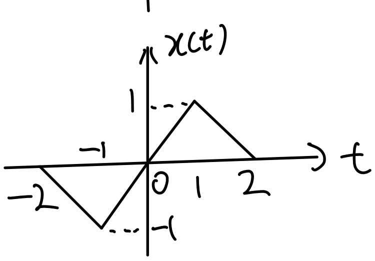
3. $\Lambda(t-1)$



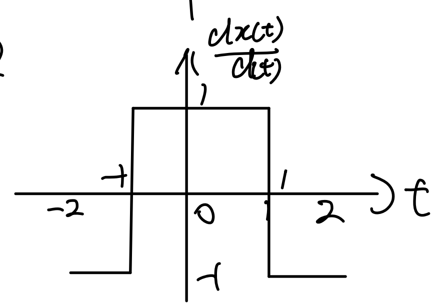
$-\Lambda(t+1)$



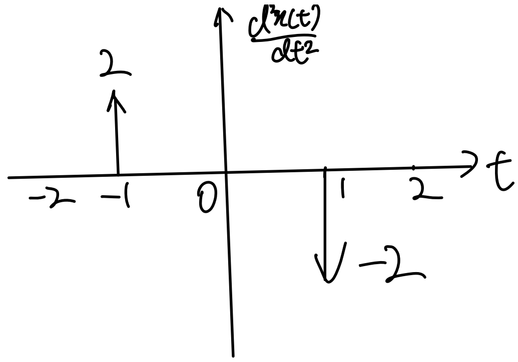
a. $x(t)$:



b. $\frac{dx(t)}{dt}$



C. $\frac{d^2x(t)}{dt^2}$



4.

$(a)\ 3u[n+3] - nu[n] + (n-3)u[n-3]$

$= 3u[n+3] - r[n] + r[n-3].$

$(b)\ 3u[n-3+3] - r[n-3] + r[n-3-3]$

$= 3u[n] - r[n-3] + r[n-6]$

$= 3u[n] - (n-3)u[n-3] + (n-6)u[n-6]$

$(c)\ 3u[-n+3] - r[-n] + r[-n-3]$

$= 3u[3-n] + nu[-n] + (-n-3)u[-n-3]$

5. a. $\cos(\pi t)[\delta(t) + \delta(t-1)].$

$= \cos(\pi t)\delta(t) + \cos(\pi t)\delta(t-1)$

$= \cos(\pi \times 0)\delta(t) + \cos(\pi \times 1)\delta(t-1)$

$= \delta(t) - \delta(t-1)$

b. $\int_{-\infty}^{\infty} \cos(\pi t)[\delta(t) + \delta(t-1)]dt$

$= \int_{-\infty}^{\infty} \cos(\pi t)\delta(t)dt + \int_{-\infty}^{\infty} \cos(\pi t)\delta(t-1)dt$

$= \cos 0 + \cos \pi = 1 - 1 = 0.$

c. suppose $t+1=u$, $\Rightarrow t=u-1$ $dt=d(u-1)=du$.

thus: $\int_{-\infty}^{\infty} f(t+1)\delta(t-2)dt = \int_{-\infty}^{\infty} f(u)\delta(u-3)du = f(3)$.

d. $\int_{0}^{\infty} f(t)[\delta(t-1)+\delta(t+1)]dt$

$= \int_{0}^{\infty} f(t)\delta(t-1)dt + \int_{0}^{\infty} f(t)\delta(t-(-1))dt$.

$= f(1) + 0 = f(1)$

6. a. not linear but is time invariant.

b. (i) $H\{x(t)\} = x(-(t-T)) = y(t)$

$y(t-t_0) = x(-(t-t_0-T)) = x(T+t_0-t)$ ①

let $v(t) = x(t-t_0)$

$H\{v(t)\} = v(-t+T)$

$= x(-t+T-t_0)$ ②

① ≠ ② thus not time invariant

(ii) let $v(t) = a_1x_1(t) + a_2x_2(t)$

$H\{v(t)\} = v(T-t) = a_1x_1(T-t) + a_2x_2(T-t)$ ①

$a_1H\{x_1(t)\} + a_2H\{x_2(t)\} = a_1x_1(T-t) + a_2x_2(T-t)$ ②

① = ② thus linear.

c. (i) $y[n-n_0] = x[n-n_0]+C$. ①

let $v[n] = x[n-n_0]$.

$H\{v[n]\} = v[n]+C = x[n-n_0]+C$ ②

① = ② thus time invarient.

(ii) let $v[n] = a_1x_1[n]+a_2x_2[n]$.

$\{v[n]\} = v[n]+C = a_1x_1[n]+a_2x_2[n]+C$ ①

$a_1 H\{x_1[n]\}+a_2H\{x_2[n]\} = a_1(x_1[n]+C)+a_2(x_2[n]+C)$ ②

if $C \neq 0$, ① ≠ ② thus non-linear.

if $C=0$ it's linear.

d. (i) $y[n-n_0] = e^{j\Omega_0(n-n_0)}x[n-n_0]$ ①

let $v[n] = x(n-n_0)$

$H\{v[n]\} = e^{j\Omega_0 n}v[n] = e^{j\Omega_0 n}x[n-n_0]$ ②

① ≠ ② thus not time invarient.

(ii) let $v[n] = a_1x_1[n]+a_2x_2[n]$

$H\{v[n]\} = e^{j\Omega_0 n}v[n] = e^{j\Omega_0 n}(a_1x_1[n]+a_2x_2[n])$ ①

$a_1H\{x_1[n]\}+a_2H\{x_2[n]\} = a_1e^{j\Omega_0 n}x_1[n]+a_2e^{j\Omega_0 n}x_2[n]$

$= e^{j\Omega_0 n}(a_1x_1[n]+a_2x_2[n])$

① = ② thus linear.

7. a. let $v(t) = a_1x_1(t) + a_2x_2(t)$

$$H\{v(t)\} = y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} v(t')\,dt'$$

$$= y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau}(a_1x_1(t')+a_2x_2(t'))\,dt' \quad ①$$

$$a_1H\{x_1(t)\} + a_2H\{x_2(t)\} = a_1\left(y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau}x_1(t')\,dt'\right)$$

$$+ a_2\left(y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau}x_2(t')\,dt'\right)$$

$$= (a_1+a_2)\,y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau}(a_1x_1(t')+a_2x_2(t'))\,dt \quad ②$$

only when $y_0=0$ we can get ① = ② for $\forall a_1\in\mathbb{C}$, $\forall a_2\in\mathbb{C}$;

therefore only when $y_0=0$ H is linear.

b. let $v(t) = x(t-t_0)$ $t_0\in\mathbb{C}\setminus\{0\}$.

$$y(t-t_0) = y_0^{-(t-t_0)/\tau} + \frac{1}{\tau}\int_0^{t-t_0} e^{-(t-t_0-t')/\tau} x(t')\,dt' \quad ①$$

$$H\{v(t)\} = y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} v(t')\,dt'$$

$$= y_0e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} x(t'-t_0)\,dt' \quad ②$$

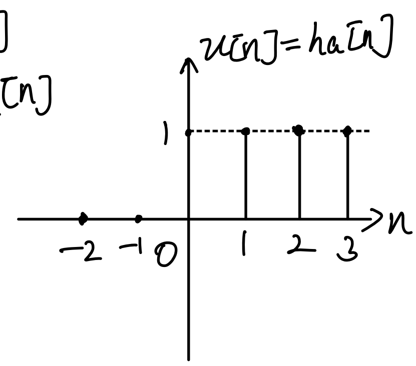
① ≠ ② therefore not time-invarient.

8. a.(i): $H_a\{x[n]\} = \sum_{k=\infty}^{n} x[k] = y[n]$

take $\delta[n]$ as input:

$$h_a[n] = \sum_{k=-\infty}^{n} \delta[n] = u[n].$$

(ii): $h_a[n]$



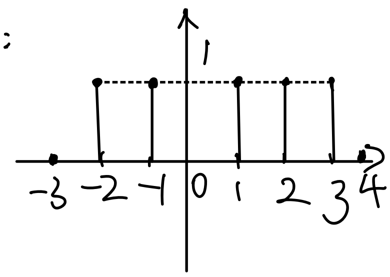
(iii): causal.

b. (i) $H_b\{x[n]\} = \sum_{k=n-2}^{n+2} x[k] = \sum_{k=-\infty}^{n+2} x[k] - \sum_{k=-\infty}^{n-3} x[k]$.

take $\delta[n]$ as input:

$$h_b[n] = \sum_{k=-\infty}^{n+2} \delta[k] - \sum_{k=-\infty}^{n-3} \delta[k] = u[n+2] - u[n-3]$$

(ii):



(iii) As can be seen that $h_b[n]$, the impulse response of it is not 0 for all $n<0$. so it is not causal.

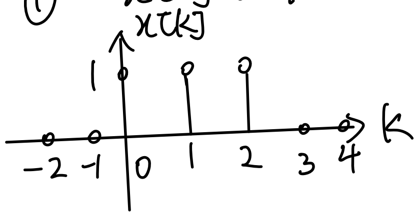
9. $$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty} x[n] * h[n] = \sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x[k]h[n-k]\right)$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k]\right)\left(\sum_{n=-\infty}^{\infty} h[n-k]\right), \quad (\text{let } n-k=m)$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k]\right)\cdot\left(\sum_{m=-\infty}^{\infty} h[m]\right) \quad (\text{replacing } k \text{ and } m \text{ by } n)$$

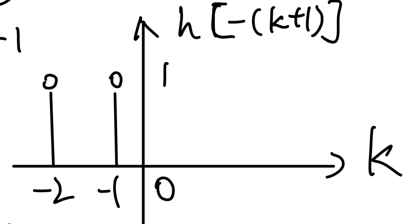
$$= \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$$

10. ① $x[k]$ vs. k

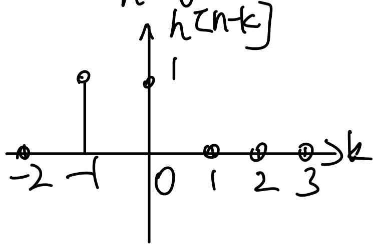


② $h[-(k-n)]$ vs. k.

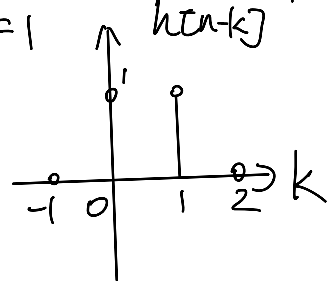
$n=-1$



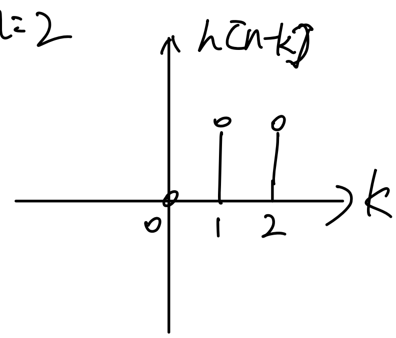
$n=0$

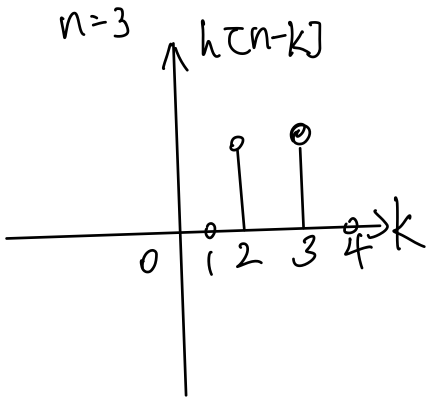


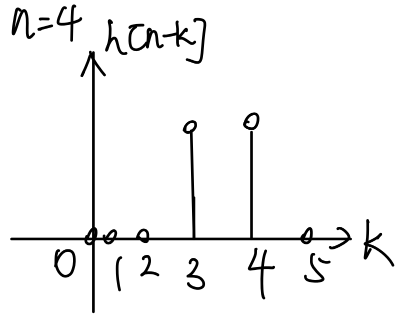
$n=1$



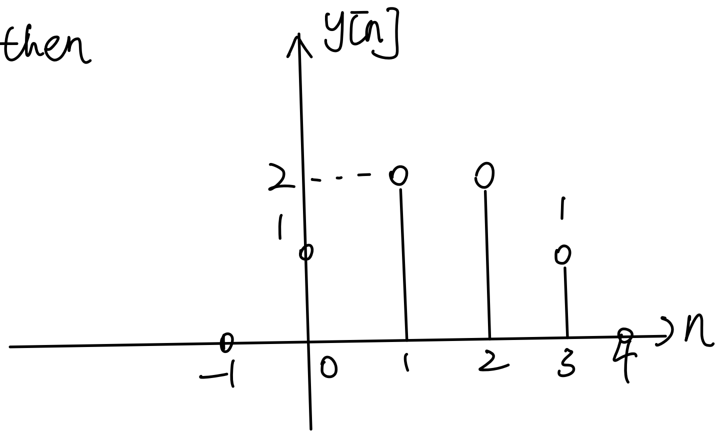
$n=2$







then



11.

$$h[n] = h_1[n] * h_2[n]$$

$$= \sum_{k=-\infty}^{\infty} h_1[k]\, h_2[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k]\, b^{n-k} u[n-k] \text{, since } u[n-k] = \begin{cases} 0 & k > n \\ 1 & k \leq n \end{cases}$$

$$\text{(therefore)} = b^n \sum_{k=-\infty}^{n} \left(\frac{a}{b}\right)^k u[k] \text{, since } u[k] = \begin{cases} 0 & k < 0 \\ 1 & k \geq 0 \end{cases}$$

$$\text{(therefore)} = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k = b^n \cdot \frac{1 - \left(\frac{a}{b}\right)^{n+1}}{1 - \left(\frac{a}{b}\right)}$$

$$= b^n \cdot \frac{\frac{b^{n+1} - a^{n+1}}{b^{n+1}}}{\frac{b-a}{b}} = \frac{b^{n+1} - a^{n+1}}{b} \cdot \frac{b}{b-a}$$

$$= \frac{b^{n+1} - a^{n+1}}{b-a}.$$

therefore we assume:

$$h[n] = \frac{b^{n+1}-a^{n+1}}{b-a}\cdot u[n]$$

$$= \frac{b}{b-a}\cdot b^n u[n] - \frac{a}{b-a}\, a^n \cdot u[n]$$

$$= \frac{b}{b-a}\, h_2[n] - \frac{a}{b-a}\, h_1[n]$$

b. $\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=-\infty}^{\infty} \frac{b^{n+1}-a^{n+1}}{b-a}\, u[n] = \frac{1}{b-a}\left(b\sum_{n=0}^{\infty} b^n - a\sum_{n=0}^{\infty} a^n\right)$

$$= \frac{1}{b-a}\left(\frac{b}{1-b} - \frac{a}{1-a}\right) = \frac{1}{b-a}\left(\frac{b-a}{(1-a)(1-b)}\right) = \frac{1}{(1-a)(1-b)} \quad ①$$

$\sum_{n=\infty}^{\infty} h_1[n] = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$, $\sum_{n=-\infty}^{\infty} h_2[n] = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$

$$\sum_{n=-\infty}^{\infty} h_1[n] \cdot \sum_{n=-\infty}^{\infty} h_2[n] = \frac{1}{1-a}\cdot\frac{1}{1-b} = \frac{1}{(1-a)(1-b)} \quad ②$$

① = ②. therefore the equation is true.

12. $x[n] = u[n] - u[n-5]$

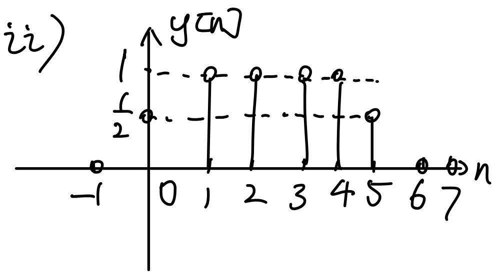
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

a. (i) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$$= \frac{1}{2}\sum_{k=0}^{4} \left(1 \times (\delta[n-k] + \delta[n-k-1])\right)$$

$$= \frac{1}{2}\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \frac{1}{2}\delta[n-5]$$

(ii)



(iii) $\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=0}^{n=4} y[n] = 5.$ thus

$\sum_{n=-\infty}^{\infty} x[n] = \sum_{n=0}^{n=4} x[n] = 5$ $\quad \sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$

verified.

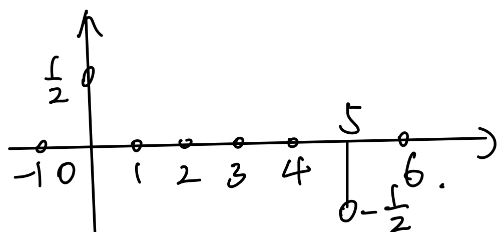
$\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=0}^{1} h[n] = 1$

b(i) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=0}^{4} h[n-k]$

$= \frac{1}{2}\left(\delta[n] - \delta[n-1] + \delta[n-1] - \delta[n-2] + \cdots + \delta[n-4] - \delta[n-5]\right)$

$= \frac{1}{2}\left(\delta[n] - \delta[n-5]\right)$

(ii)



(iii) $\sum_{-\infty}^{\infty} y[n] = 0$ $\quad$ therefore $\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$

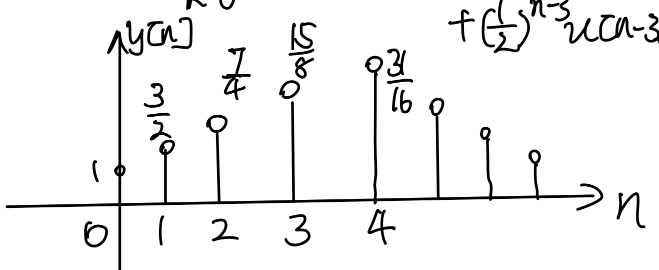
$\sum_{-\infty}^{\infty} x[n] = 5$ $\quad$ verified.

$\sum_{-\infty}^{\infty} h[n] = 0$

C. $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$= \sum_{k=0}^{4} h[n-k] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2]$

$+ \left(\frac{1}{2}\right)^{n-3} u[n-3] + \left(\frac{1}{2}\right)^{n-4} u[n-4]$



$\Longrightarrow$

it can be seen that $\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$