1. a. $x[n]=\cos(\sqrt{3}n)$: periodic.

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{3}}=\frac{2\sqrt{3}}{3}\pi$$

b. $x[n]=\cos(\frac{3\pi}{5}n)+\sin(\frac{4\pi}{5}n)$

suppose $T_1=\frac{2\pi}{\frac{3\pi}{5}}=\frac{10}{3}$

$T_2=\frac{2\pi}{\frac{4\pi}{5}}=\frac{5}{2}$

$\frac{T_1}{T_2}=\frac{10}{3}\times\frac{2}{5}=\frac{4}{3}$, $x[n]$ is periodic.

thus: $T=3\cdot T_1=10$

c. $x[n]=\sin(\frac{\pi}{6}n^2)$: periodic.

suppose $T=N$. then we have $x[n]=x[n+N]$

thus: $\sin(\frac{\pi}{6}n^2)=\sin(\frac{\pi}{6}(n+N)^2)$

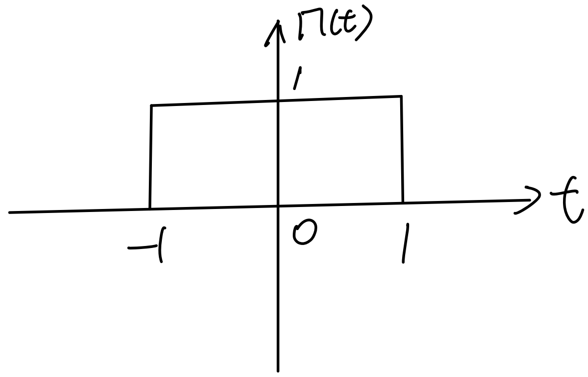
$\sin(\frac{\pi}{6}(n+N)^2)=\sin(\frac{\pi}{6}(n^2+2nN+N^2))$

$=\sin(\frac{\pi}{6}n^2+\frac{nN}{3}+\frac{\pi}{6}N^2)$

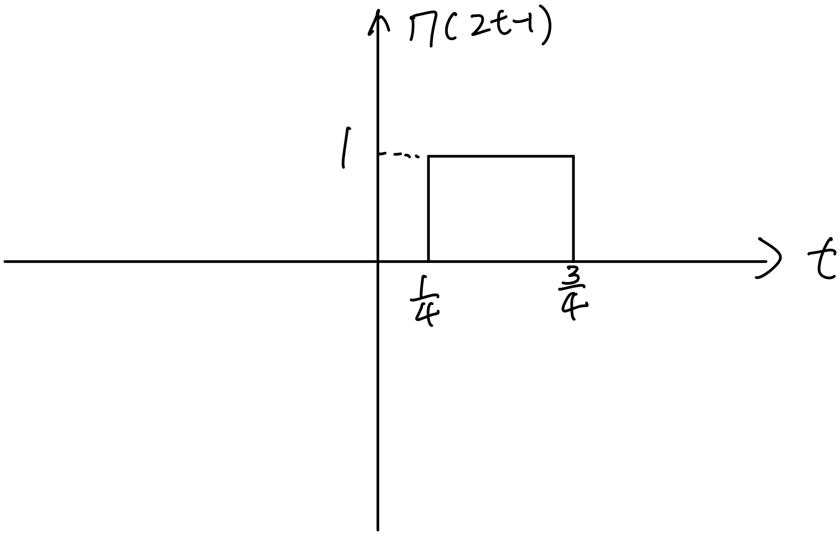
take $n=0$ we have: $\sin 0=\sin(\frac{\pi}{6}N^2)$

thus $\frac{\pi}{6}N^2=2\pi \Rightarrow N=2\sqrt{3}$.

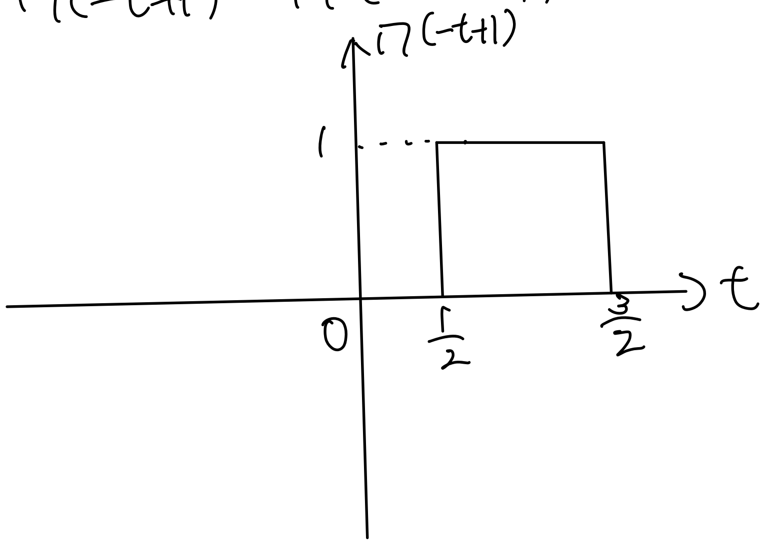
2. a.



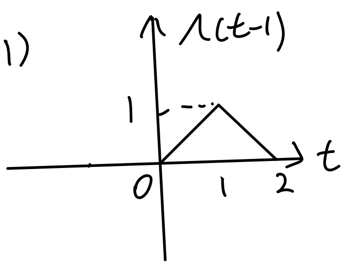
b. $\Pi(2t-1) = \Pi(2(t-\frac{1}{2}))$



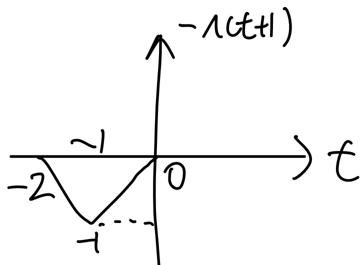
c. $\Pi(-t+1) = \Pi(-(t-1))$



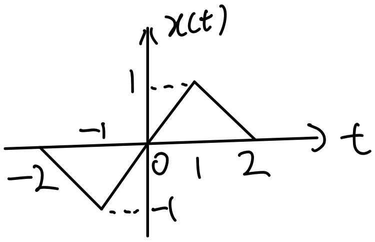
3. $\Lambda(t-1)$



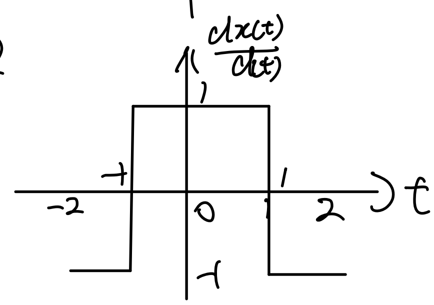
$-\Lambda(t+1)$



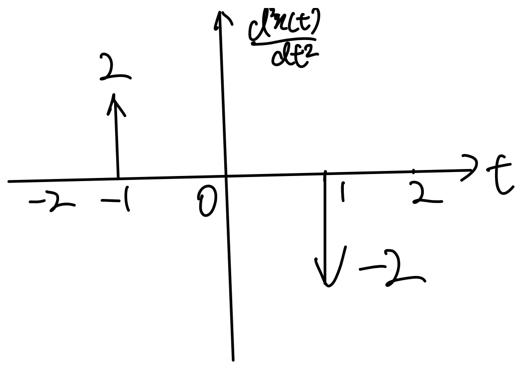
a. $x(t)$:



b. $\frac{dx(t)}{dt}$



C. $\frac{d^2x(t)}{dt^2}$



4.

(a) $3u[n+3] - nu[n] + (n-3)u[n-3]$
$= 3u[n+3] - r[n] + r[n-3]$.

(b) $3u[n-3+3] - r[n-3] + r[n-3-3]$
$= 3u[n] - r[n-3] + r[n-6]$
$= 3u[n] - (n-3)u[n-3] + (n-6)u[n-6]$

(c) $3u[-n+3] - r[-n] + r[-n-3]$
$= 3u[3-n] + nu[-n] + (-n-3)u[-n-3]$

5. a. $\cos(\pi t)[\delta(t) + \delta(t-1)]$.
$= \cos(\pi t)\delta(t) + \cos(\pi t)\delta(t-1)$
$= \cos(\pi \times 0)\delta(t) + \cos(\pi \times 1)\delta(t-1)$
$= \delta(t) - \delta(t-1)$

b. $\int_{-\infty}^{\infty} \cos(\pi t)[\delta(t) + \delta(t-1)]dt$
$= \int_{-\infty}^{\infty} \cos(\pi t)\delta(t)dt + \int_{-\infty}^{\infty} \cos(\pi t)\delta(t-1)dt$
$= \cos 0 + \cos \pi = 1 - 1 = 0$.

c. suppose $t+1=u$, $\Rightarrow t=u-1$ $dt=d(u-1)=du$.

thus: $\int_{-\infty}^{\infty} f(t+1)\delta(t-2)dt=\int_{-\infty}^{\infty} f(u)\delta(u-3)du=f(3)$.

d. $\int_{0}^{\infty} f(t)\,[\delta(t-1)+\delta(t+1)]dt$

$=\int_{0}^{\infty} f(t)\delta(t-1)\,dt+\int_{0}^{\infty} f(t)\,\delta(t-(-1))\,dt.$

$=\ f(1)+0\ =f(1)$

6. a. not linear but is time invariant.

b. (i) $H\{x(t)\}=x(-(t-T))=y(t)$

$y(t-t_0)=x(-(t-t_0-T))=x(T+t_0-t)$ ①

let $v(t)=x(t-t_0)$

$H\{v(t)\}=v(-t+T)$

$=x(-t+T-t_0)$ ②

①≠② thus not time invariant

(ii) let $v(t)=a_1x_1(t)+a_2x_2(t)$

$H\{v(t)\}=v(T-t)=a_1x_1(T-t)+a_2x_2(T-t)$ ①

$a_1H\{x_1(t)\}+a_2H\{x_2(t)\}=a_1x_1(T-t)+a_2x_2(T-t)$ ②

①=② thus linear.

c. (i) $y[n-n_0] = x[n-n_0]+C$. ①

let $v[n] = x[n-n_0]$.

$H\{v[n]\} = v[n]+C = x[n-n_0]+C$ ②

① = ② thus time invarient.

(ii) let $v[n] = a_1x_1[n] + a_2x_2[n]$.

$\{v[n]\} = v[n]+C = a_1x_1[n]+a_2x_2[n]+C$ ①

$a_1H\{x_1[n]\} + a_2H\{x_2[n]\} = a_1(x_1[n]+C)+a_2(x_2[n]+C)$ ②

if $C \neq 0$, ① ≠ ② thus non-linear.

if $C=0$ it's linear.

d. (i) $y[n-n_0] = e^{j\Omega_0(n-n_0)}x[n-n_0]$ ①

let $v[n] = x(n-n_0)$

$H\{v[n]\} = e^{j\Omega_0 n}v[n] = e^{j\Omega_0 n}x[n-n_0]$ ②

① ≠ ② thus not time invarient.

(ii) let $v[n] = a_1x_1[n] + a_2x_2[n]$

$H\{v[n]\} = e^{j\Omega_0 n}v[n] = e^{j\Omega_0 n}(a_1x_1[n]+a_2x_2[n])$ ①

$a_1H\{x_1[n]\} + a_2H\{x_2[n]\} = a_1e^{j\Omega_0 n}x_1[n] + a_2e^{j\Omega_0 n}x_2[n]$

$= e^{j\Omega_0 n}(a_1x_1[n]+a_2x_2[n])$

① = ② thus linear.

7. a. let $v(t) = a_1x_1(t) + a_2x_2(t)$

$$H\{v(t)\} = y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} v(t')\,dt'$$

$$= y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} (a_1x_1(t') + a_2x_2(t'))\,dt' \quad ①$$

$$a_1H\{x_1(t)\} + a_2H\{x_2(t)\} = a_1\left(y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} x_1(t')\,dt'\right)$$

$$+ a_2\left(y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} x_2(t')\,dt'\right)$$

$$= (a_1+a_2)\,y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} (a_1x_1(t') + a_2x_2(t'))\,dt \quad ②$$

only when $y_0 = 0$ we can get ① = ② for $\forall a_1 \in \mathbb{C}$, $\forall a_2 \in \mathbb{C}$;

therefore only when $y_0 = 0$ H is linear.

b. let $v(t) = x(t-t_0)$ $t_0 \in \mathbb{C}\setminus\{0\}$.

$$y(t-t_0) = y_0^{-(t-t_0)/\tau} + \frac{1}{\tau}\int_0^{t-t_0} e^{-(t-t_0-t')/\tau} x(t')\,dt' \quad ①$$

$$H\{v(t)\} = y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} v(t')\,dt'$$

$$= y_0 e^{-t/\tau} + \frac{1}{\tau}\int_0^t e^{-(t-t')/\tau} x(t'-t_0)\,dt' \quad ②$$

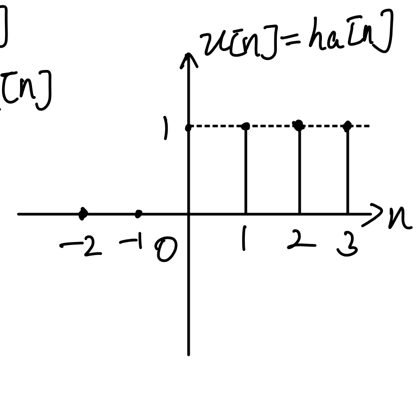
① ≠ ② therefore not time-invarient.

8. a.(i): $H_a\{x[n]\} = \sum_{k=\infty}^{n} x[k] = y[n]$

take $\delta[n]$ as input:

$$h_a[n] = \sum_{k=-\infty}^{n} \delta[n] = u[n].$$

(ii): $h_a[n]$



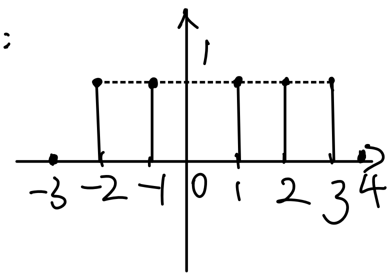
(iii): causal.

b. (i) $H_b\{x[n]\} = \sum_{k=n-2}^{n+2} x[k] = \sum_{k=-\infty}^{n+2} x[k] - \sum_{k=-\infty}^{n-3} x[k]$.

take $\delta[n]$ as input:

$$h_b[n] = \sum_{k=-\infty}^{n+2} \delta[k] - \sum_{k=-\infty}^{n-3} \delta[k] = u[n+2] - u[n-3]$$

(ii):



(iii) as can be seen that $h_b[n]$, the impulse response of it is not 0 for all $n<0$. so it is not causal.

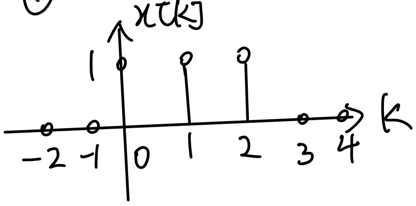
9. $$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty} x[n] * h[n] = \sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x[k]h[n-k]\right)$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k]\right)\left(\sum_{n=-\infty}^{\infty} h[n-k]\right), \text{ (let } n-k=m)$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k]\right)\cdot\left(\sum_{m=-\infty}^{\infty} h[m]\right) \quad \text{(replacing } k \text{ and } m \text{ by } n)$$

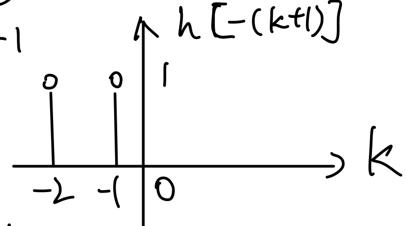
$$= \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$$

10. ① $x[k]$ vs. k

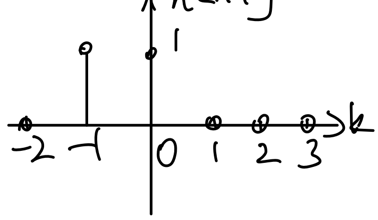


② $h[-(k-n)]$ vs. k.

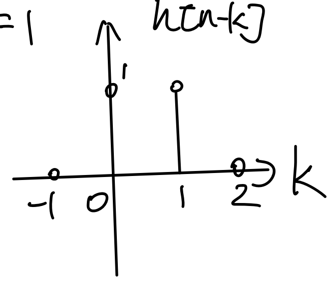
$n=-1$



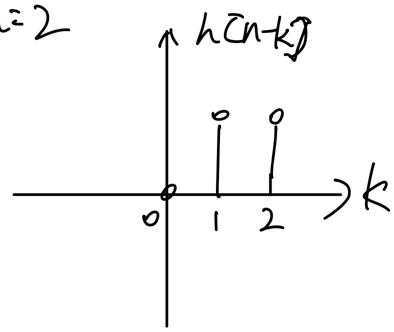
$n=0$

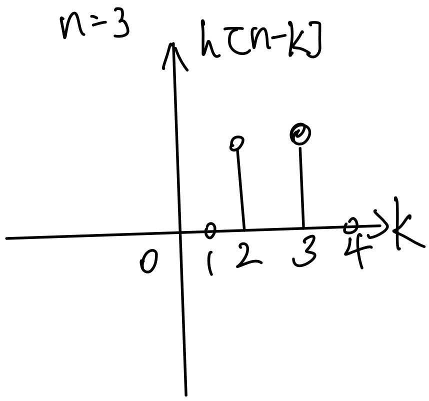


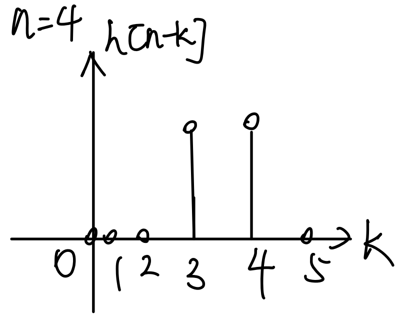
$n=1$



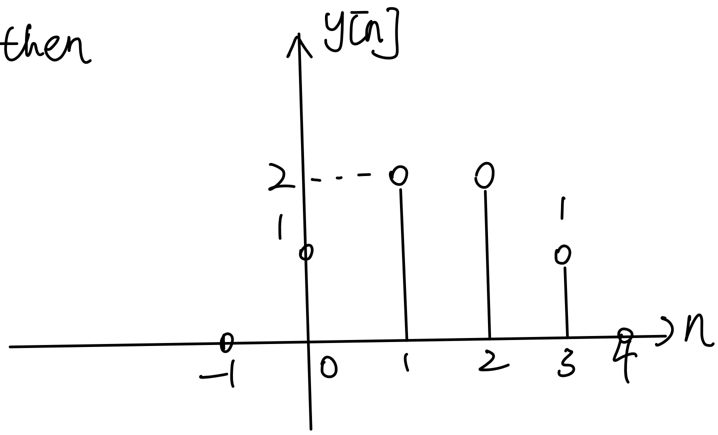
$n=2$







then



11. $$h[n] = h_1[n] * h_2[n]$$

$$= \sum_{k=-\infty}^{\infty} h_1[k]\, h_2[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k]\, b^{n-k} u[n-k] \text{ , since } u[n-k] = \begin{cases} 0 & k>n \\ 1 & k\le n \end{cases}$$

$$(\text{therefore}) = b^n \sum_{k=-\infty}^{n} \left(\frac{a}{b}\right)^k u[k] \text{ , since } u[k] = \begin{cases} 0 & k<0 \\ 1 & k\ge 0 \end{cases}$$

$$(\text{therefore}) = b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k = b^n \cdot \frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\left(\frac{a}{b}\right)}$$

$$= b^n \cdot \frac{\frac{b^{n+1}-a^{n+1}}{b^{n+1}}}{\frac{b-a}{b}} = \frac{b^{n+1}-a^{n+1}}{b} \cdot \frac{b}{b-a}$$

$$= \frac{b^{n+1}-a^{n+1}}{b-a}.$$

therefore we assume:

$$h[n] = \frac{b^{n+1} - a^{n+1}}{b-a} \cdot u[n]$$

$$= \frac{b}{b-a} \cdot b^n u[n] - \frac{a}{b-a} a^n \cdot u[n]$$

$$= \frac{b}{b-a} h_2[n] - \frac{a}{b-a} h_1[n]$$

b. $\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=-\infty}^{\infty} \frac{b^{n+1} - a^{n+1}}{b-a} u[n] = \frac{1}{b-a}\left(b \sum_{n=0}^{\infty} b^n - a \sum_{n=0}^{\infty} a^n\right)$

$$= \frac{1}{b-a}\left(\frac{b}{1-b} - \frac{a}{1-a}\right) = \frac{1}{b-a}\left(\frac{b-a}{(1-a)(1-b)}\right) = \frac{1}{(1-a)(1-b)} \quad ①$$

$$\sum_{n=-\infty}^{\infty} h_1[n] = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad \sum_{n=-\infty}^{\infty} h_2[n] = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$$

$$\sum_{n=-\infty}^{\infty} h_1[n] \cdot \sum_{n=-\infty}^{\infty} h_2[n] = \frac{1}{1-a} \cdot \frac{1}{1-b} = \frac{1}{(1-a)(1-b)} \quad ②$$

① = ②. therefore the equation is true.

12. $x[n] = u[n] - u[n-5]$

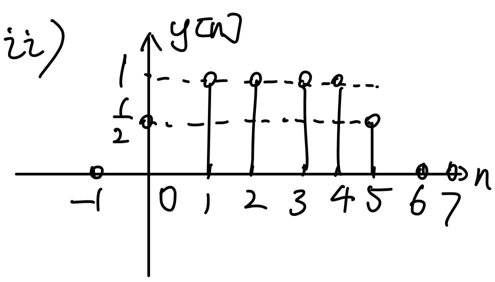
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

a. (i) $y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

$$= \frac{1}{2} \sum_{k=0}^{4} 1 \times \cdot (\delta[n-k] + \delta[n-k-1])$$

$$= \frac{1}{2}\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \frac{1}{2}\delta[n-5]$$

(ii)



(iii) $\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=0}^{n=4} y[n] = 5.$ thus

$\sum_{n=-\infty}^{\infty} x[n] = \sum_{n=0}^{n=4} x[n] = 5$ $\quad \sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$

verified.

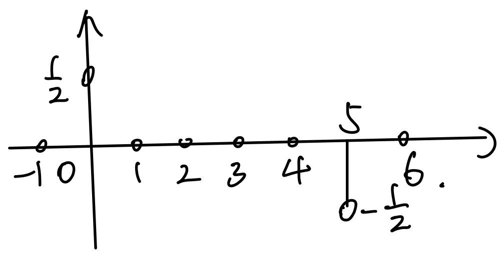
$\sum_{n=-\infty}^{\infty} h[n] = \sum_{n=0}^{1} h[n] = 1$

b (i) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{4} h[n-k]$

$= \frac{1}{2}(\delta[n] - \delta[n-1] + \delta[n-1] - \delta[n-2] + \cdots + \delta[n-4] - \delta[n-5])$

$= \frac{1}{2}(\delta[n] - \delta[n-5])$

(ii)



(iii) $\sum_{-\infty}^{\infty} y[n] = 0$ $\quad$ therefore $\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$

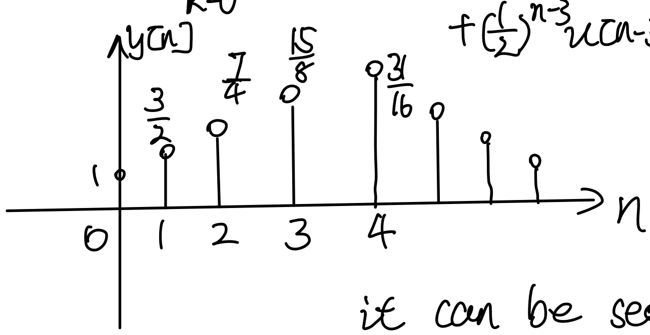
$\sum_{-\infty}^{\infty} x[n] = 5$ $\quad$ verified.

$\sum_{-\infty}^{\infty} h[n] = 0$

C. $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$= \sum_{k=0}^{4} h[n-k] = \left(\frac{1}{2}\right)^{n} u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2]$

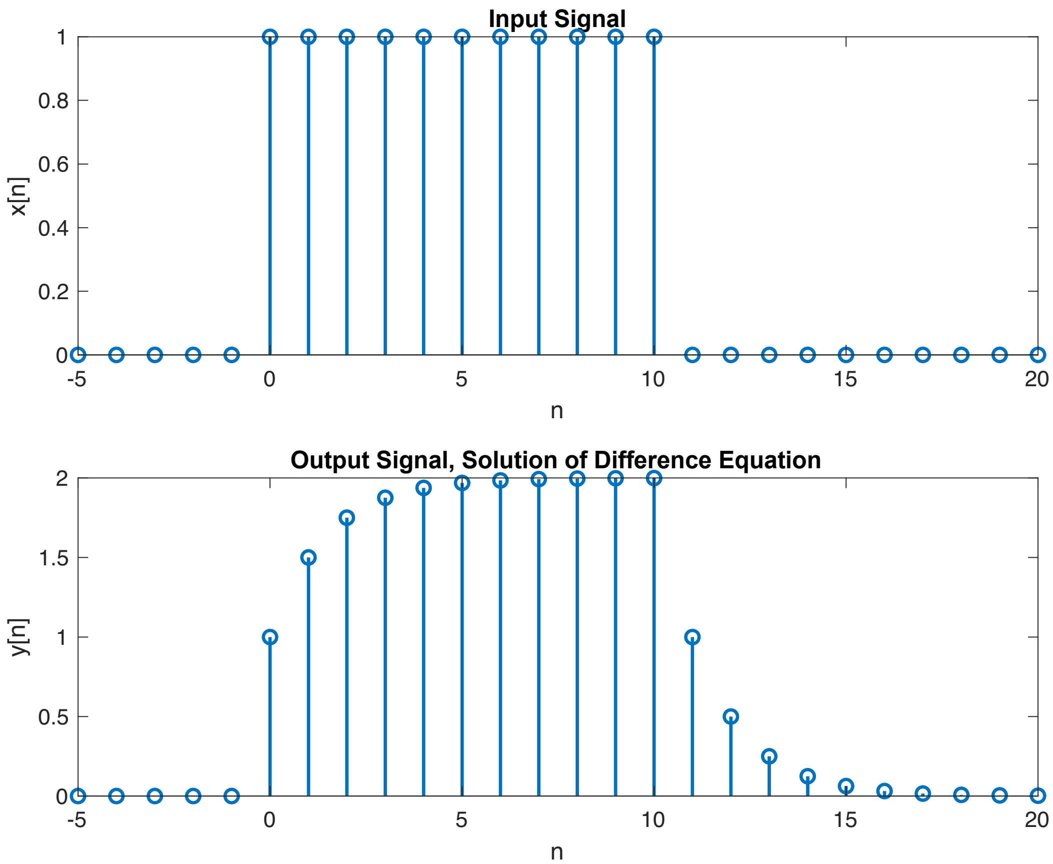
$+ \left(\frac{1}{2}\right)^{n-3} u[n-3] + \left(\frac{1}{2}\right)^{n-4} u[n-4]$



$\Rightarrow$

it can be seen that $\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$

Figures 1



Input Signal
x[n]
n
Output Signal, Solution of Difference Equation
y[n]
n

Figure2:

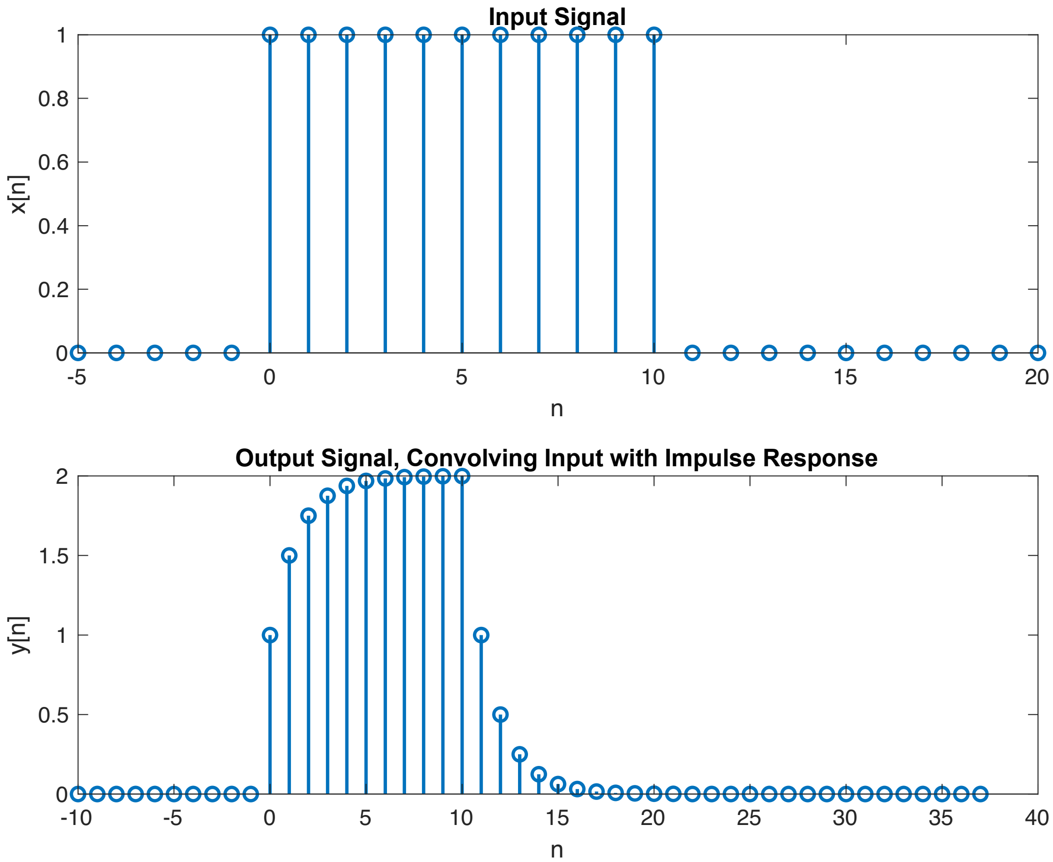
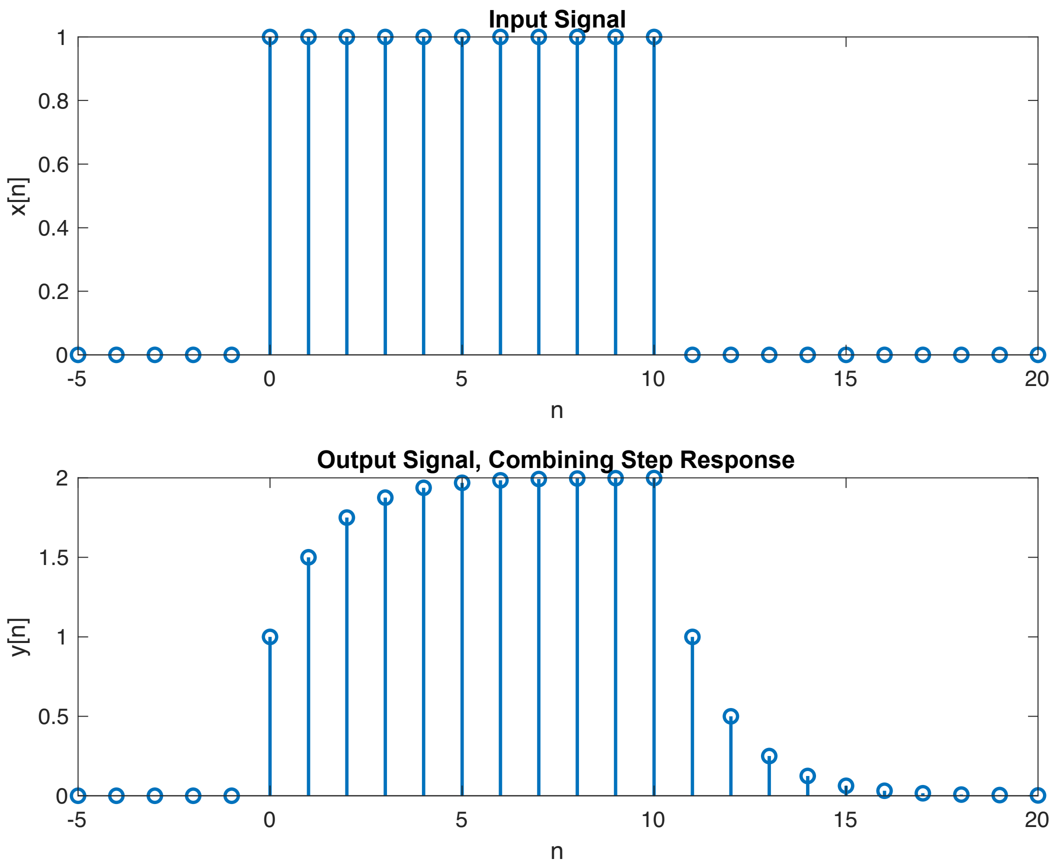

Input Signal
x[n]
n
Output Signal, Convolving Input with Impulse Response
y[n]
n

Figure 3



Codes:

Running script: HW2.m:

```
%Create input signal (rectangular pulse)
n_x1 = -5;
n_x2 = 20;
n_x = n_x1:n_x2;
n_on = 0;
n_off = 11;
x = u(n_x-n_on)-u(n_x-n_off);
%Define system:
a = 0.5; %exponential parameter
n_0 = 0; %Time at which we start solving the equation.
k_0 = find(n_x==n_0); %Index of time at which we start solving eqn.
n_yA1 = n_x1;
n_yA2 = n_x2;
n_yA = n_yA1 : n_yA2;
```

```
y_A = zeros(size(n_yA));
a_fo = [1 -a];
b_fo = 1;% The a and b parameters from the general filter command.
y_A(k_0:end) = filter(b_fo,a_fo,x(k_0:end));
figure(1);
subplot(211);
stem(n_x,x);
lw = 1.5;
l = get(gca, 'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('x[n]');
title('Input Signal');
subplot(212);
stem(n_yA,y_A);
lw = 1.5;
l = get(gca,'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('y[n]');
title('Output Signal, Solution of Difference Equation');
c = 1e-5; % cut-off value
n_h2 = ceil(log(c)/log(abs(a))); % cut-off index
n_h1 = n_x1; nh = n_h1:n_h2; % impulse response indexing
n_yB1 = n_x1+n_h1; n_yB2 = n_x2+n_h2; n_yB = n_yB1:n_yB2; % output indexing
h_fo_trunc = hfo(nh,a);
y_B = conv(x,h_fo_trunc);
figure(2);
subplot(211);
stem(n_x,x);
lw = 1.5;
l = get(gca, 'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('x[n]');
title('Input Signal');
subplot(212);
stem(n_yB,y_B);
lw = 1.5;
```

```
l = get(gca,'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('y[n]');
title('Output Signal, Convolving Input with Impulse Response');
y_C = sfo(n_x-n_on,a)-sfo(n_x-n_off,a);
figure(3);
subplot(211);
stem(n_x,x);
lw = 1.5;
l = get(gca, 'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('x[n]');
title('Input Signal');
subplot(212);
stem(n_x,y_C);
lw = 1.5;
l = get(gca,'children');
set(l,'linewidth',lw);
xlabel('n');ylabel('y[n]');
title('Output Signal, Combining Step Response');
```

u.m:

```
function y = u(x)
%unit step function
y = double(x>=0);
end
```

hfo.m:

```
function h = hfo(n,a)
h = (a.^n).*u(n);
end
```

sfo.m:

```
function s = sfo(n,a)
s = 1/(1-a)*(1-a.^(n+1)).*u(n);
end
```