1. a differentiator : $H\{x(t)\} = y(t) = \frac{dx}{dt}$.

by eigenfunction property: $H\{x(t)\} = \frac{dx}{dt} = \lambda x(t)$

where λ is a constant, $\lambda \in \mathbb{C}$,

$x(t)$ is the eignfunction.

the solution of this first order linear constant-coefficient differential equation : $x(t) = e^{\lambda t}$

therefore $x(t) = e^{st}$ is the only eignfunction.

(by substitute $\lambda \to s$).

2. (a.) $b_k = a_k H(jk\omega_0) = \frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) \cdot A = \frac{2T_1}{T_0} \cdot A \cdot \operatorname{sinc}\left(\frac{2T_1 k}{T_0}\right)$

thus: $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$

$$= \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$= a_0 e^{j \cdot 0 \cdot \omega_0 t} = a_0 = \frac{2T_1}{T_0} \cdot A \cdot \operatorname{sinc}\left(\frac{2T_1 \cdot 0}{T_0}\right) = \frac{2T_1}{T_0} A = A\eta.$$

when $\eta = 1$, $y(t) = y(t)_{max} = A$.

(b). $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$

$$= a_{-1} e^{j \cdot -1 \cdot \omega_0 (t-t_0)} H(-j\omega_0) + a_1 e^{j\omega_0 (t-t_0)} H(j\omega_0)$$

$$= a_{-1} e^{-j\omega_0 (t-t_0)} + a_1 e^{j\omega_0 (t-t_0)}$$

$$= \frac{2T_1}{T_0} \cdot A \cdot \operatorname{sinc}\left(-\frac{2T_1}{T_0}\right) \cdot e^{-j\omega_0 (t-t_0)} + \frac{2T_1}{T_0} A \cdot \operatorname{sinc}\left(\frac{2T_1}{T_0}\right) e^{j\omega_0 (t-t_0)}$$

$$= \eta \cdot A \cdot \operatorname{sinc}\left(\frac{2T_1}{T_0}\right)\left(e^{-j\omega_0 (t-t_0)} + e^{j\omega_0 (t-t_0)}\right)$$

$$= \eta \cdot A \cdot \mathrm{sinc}\left(\frac{2T_1}{T_0}\right) \cdot 2\cos\left(\frac{2\pi}{T_0}(t-t_0)\right)$$

$$= 2A\eta \frac{\sin(\eta\pi)}{\eta\pi} \cos\left(\frac{2\pi}{T_0}(t-t_0)\right)$$

$$= \frac{2A}{\pi} \sin(\eta\pi) \cos\left(\frac{2\pi}{T_0} \cdot (t-t_0)\right)$$

$\eta = \frac{1}{2}$ maximize the peak to peak value.

max peak-peak value is $\frac{4A}{\pi}$.

3. $x(t) = \cos(w_0 t) = \frac{1}{2}\left(e^{jw_0 t} + e^{-jw_0 t}\right)$, $a_{-1} = \frac{1}{2}$, $a_1 = \frac{1}{2}$.

$$y(t) = a_{-1} H(-jw_0) e^{-jw_0 t} + a_1 H(jw_0) e^{jw_0 t}$$

(because $H(-jw_0) = H^*(jw) = |H(jw_0)|(-\angle H(jw_0))$).

$$= \frac{1}{2}\left(|H(jw_0)|(-\angle H(jw_0)) e^{-jw_0 t} + |H(jw_0)|(\angle H(jw_0)) e^{jw_0 t}\right)$$

$$= \frac{1}{2}|H(jw_0)|\left(e^{-j\angle H(jw_0)} e^{-jw_0 t} + e^{j\angle H(jw_0)} e^{jw_0 t}\right)$$

$$= \frac{1}{2}|H(jw_0)|\left(e^{-j(w_0 t + \angle H(jw_0))} + e^{j(w_0 t + \angle H(jw_0))}\right)$$

$$= |H(jw_0)| \cos(w_0 t + \angle H(jw_0))$$

4. $-x(t) = x(t - \frac{T_0}{2})$

thus $-a_k = e^{-j\frac{2\pi}{T_0} \cdot \frac{T_0}{2} k} a_k$

$-a_k = e^{-j\pi k} a_k$

$-a_k = (-1)^k a_k$

$0 = [(-1)^k + 1] a_k$.

when k even.

$2a_k = 0 \Rightarrow a_k = 0$.

5.

Signal	DTFS	Explanation
1	g_k	$x_1[n]$, simillar to signal in 4, is half period odd symmetry. g_k are the DTFS coefficients that are 0 for even k.
2	e_k	$x_2[n]$: negative periodic impulse train centered at 0. DTFS coefficients of $x_2[n]$ equals $-1/N$. (magnitude $\frac{1}{N}$, phase π)
3	a_k	rectangular pulse train: width $2N_1+1=5$. centered at $n=0$. real and even $\Rightarrow$ DTFS coefficient real & even. phase 0 & π; mean: $\frac{5}{16}$.
4	b_k	real & odd, $\Rightarrow$ DTFS coefficients imaginary and odd. $\Rightarrow$ DTFS magnitude even, phase odd. ($\pm\frac{\pi}{2}$)
5	c_k	real & even $\Rightarrow$ DTFS real & even $\Rightarrow$ magnitude even, phase odd phase $=0$ or π.
6	f_k	delayed version of $x_5[n]$, thus magnitude is same as in x_5, phase have negative slope.
7	h_k	rectangular pulse train: $2N_1+1=5$, shifted left, mean value is 5/16, which equals CTFS at $k=0$. magnitude same as x_3, phase positive slope.
8	d_k	positive impulse train, center $n=0$. all DTFS coefficients equals $1/N$, zero phase, $1/N$ magnitude.

6. $S=\sum_{n=0}^{N-1} e^{j(k-l)\left(\frac{2\pi}{N}\right)n}$

$$=\sum_{n=0}^{N-1}\left[e^{j(k-l)\left(\frac{2\pi}{N}\right)}\right]^n$$

$$=\frac{1\times\left(1-e^{j(k-l)\left(\frac{2\pi}{N}\right)(N-1+1)}\right)}{1-e^{j(k-l)\left(\frac{2\pi}{N}\right)}}=\frac{1-e^{j(k-l)(2\pi)}}{1-e^{j(k-l)\left(\frac{2\pi}{N}\right)}}$$

since $e^{j(k-l)(2\pi)}=1$

thus: $S=\dfrac{1-1}{1-e^{j(k-l)\left(\frac{2\pi}{N}\right)}}=0.$

7. $P=\frac{1}{N}\sum_{n=\langle N\rangle}|x[n]|^2=\sum_{k=\langle N\rangle}|a_k|^2$

$=\frac{2N_1+1}{N^2}$

8\. a. analysis equation:

$a_k=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\Omega_0 n}$

$=\frac{1}{N}x[0]e^{-jk\Omega_0\cdot 0}$

$=\frac{1}{4},\ k=-1,0,1,2.$

therefore

$x[n]=\sum_{k=-1}^{2}\frac{1}{4}e^{jk\frac{\pi}{2}n}$

b. let $x[n]=\sum_{k=-1}^{2}\frac{1}{4}e^{jk\frac{\pi}{2}n}$

they $y[n]=\sum_{k=-1}^{2}a_k H(e^{jk\Omega_0})e^{jk\Omega_0 n}$

$H(e^{-j\Omega_0})=je^{j\frac{\pi}{2}}\sin(-\frac{\pi}{2})=j\cdot j\cdot(-1)=1$

$H(e^{j0\cdot\Omega_0})=je^{-j0}\sin(0)=0.$

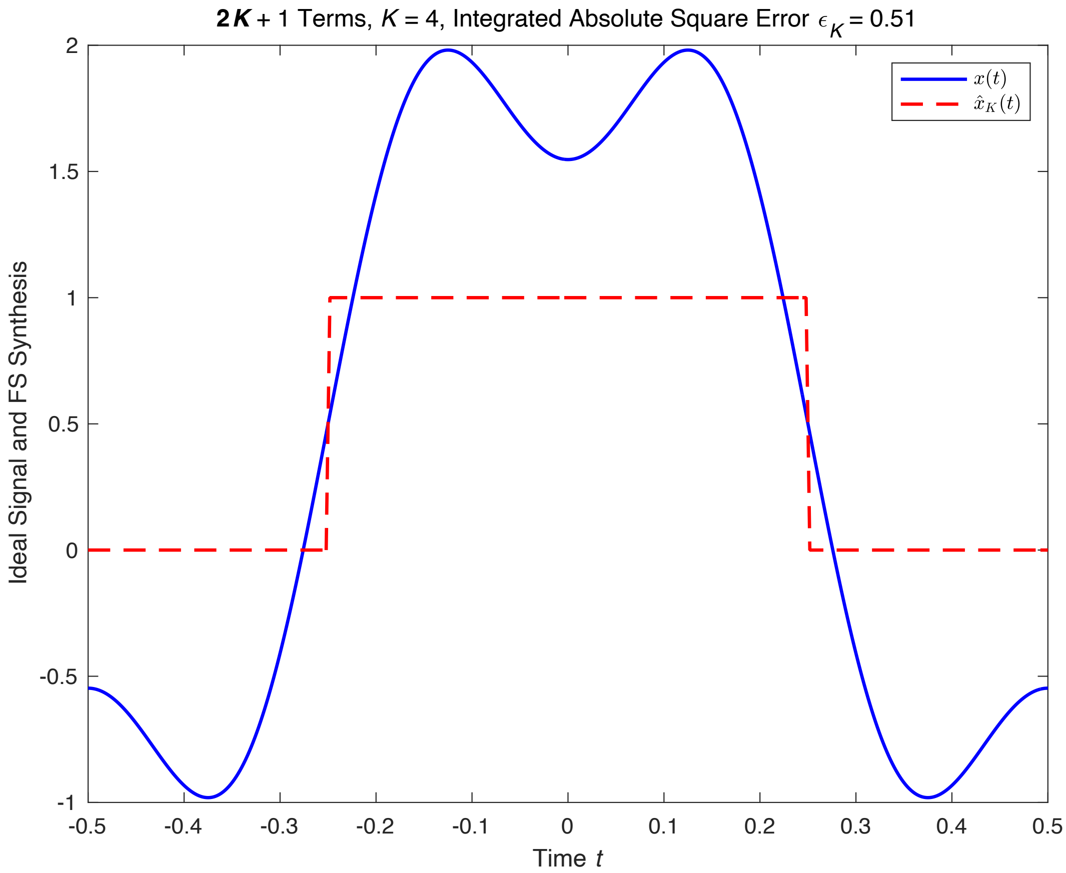
$H(e^{j\Omega_0})=je^{-j\frac{\pi}{2}}\sin(\frac{\pi}{2})=j\cdot -j\cdot 1=1$

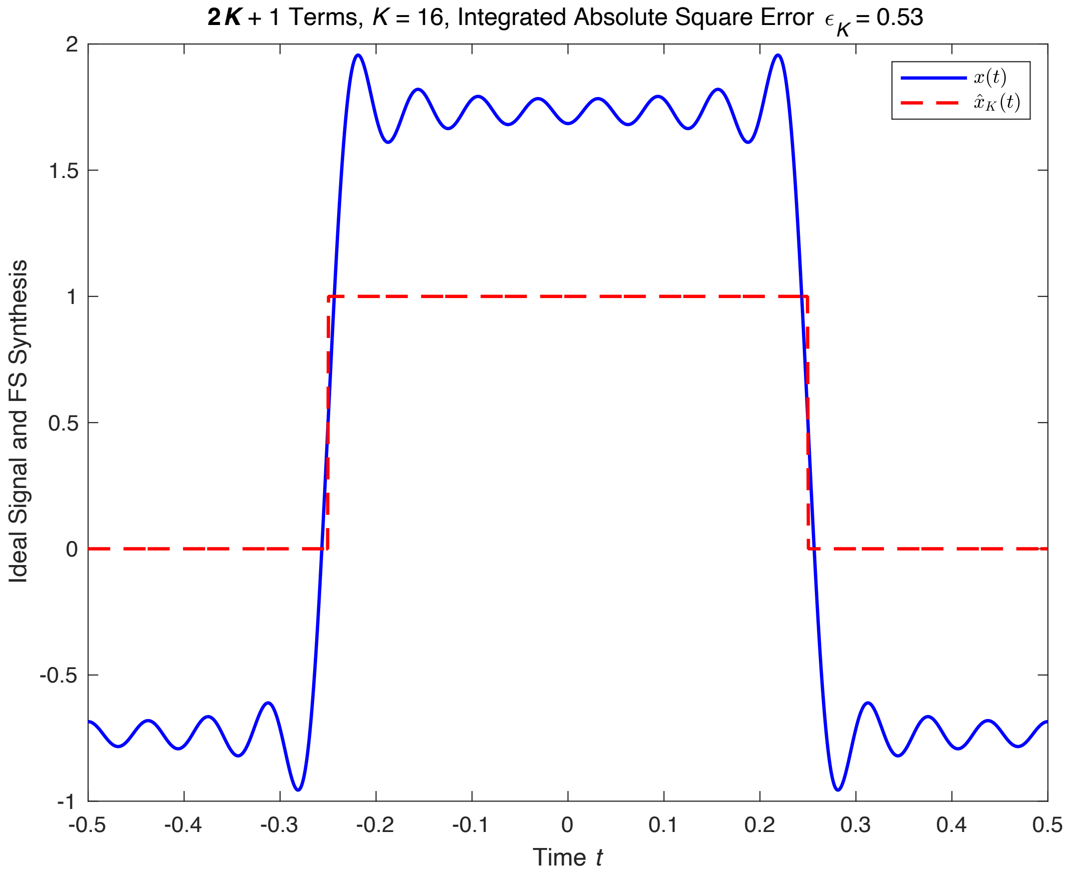
$H(e^{j2\Omega_0})=je^{-j\pi}\sin(\pi)=0.$

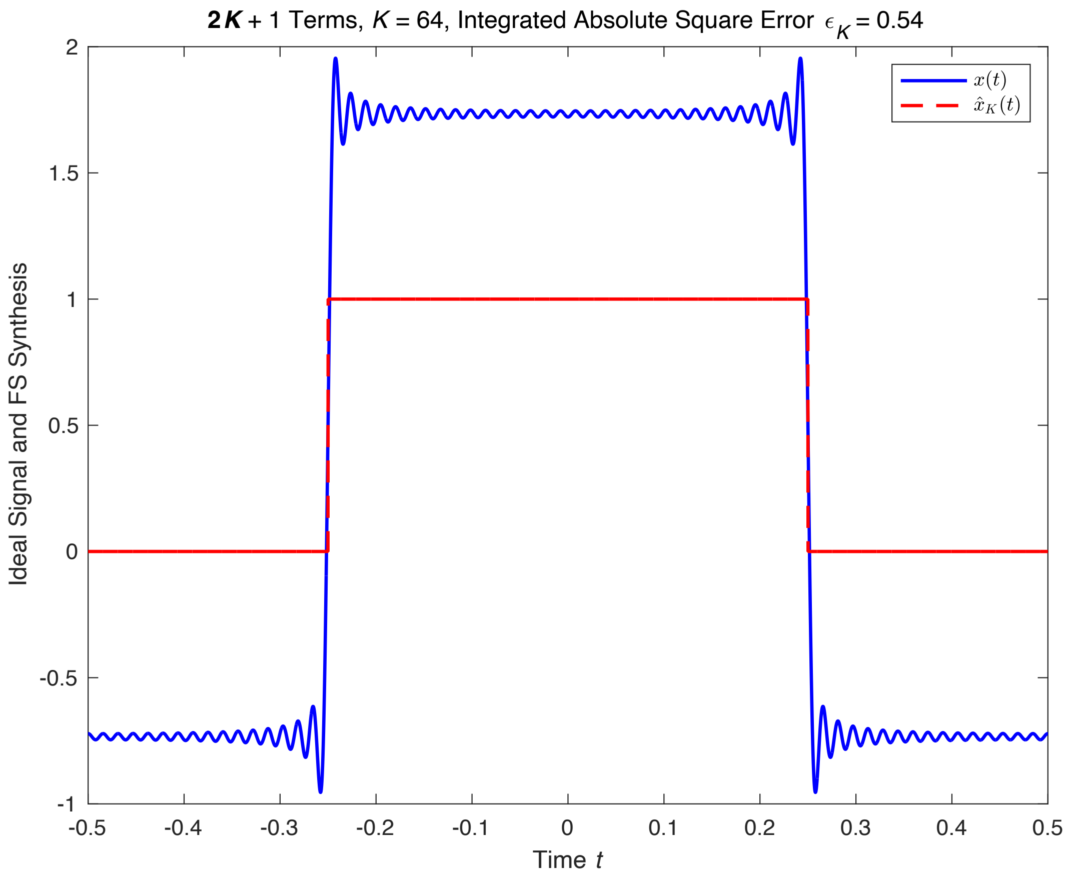
therefore $y[n]=\frac{1}{4}(1\cdot e^{-j\frac{\pi}{2}n}+1\cdot e^{j\frac{\pi}{2}n})$

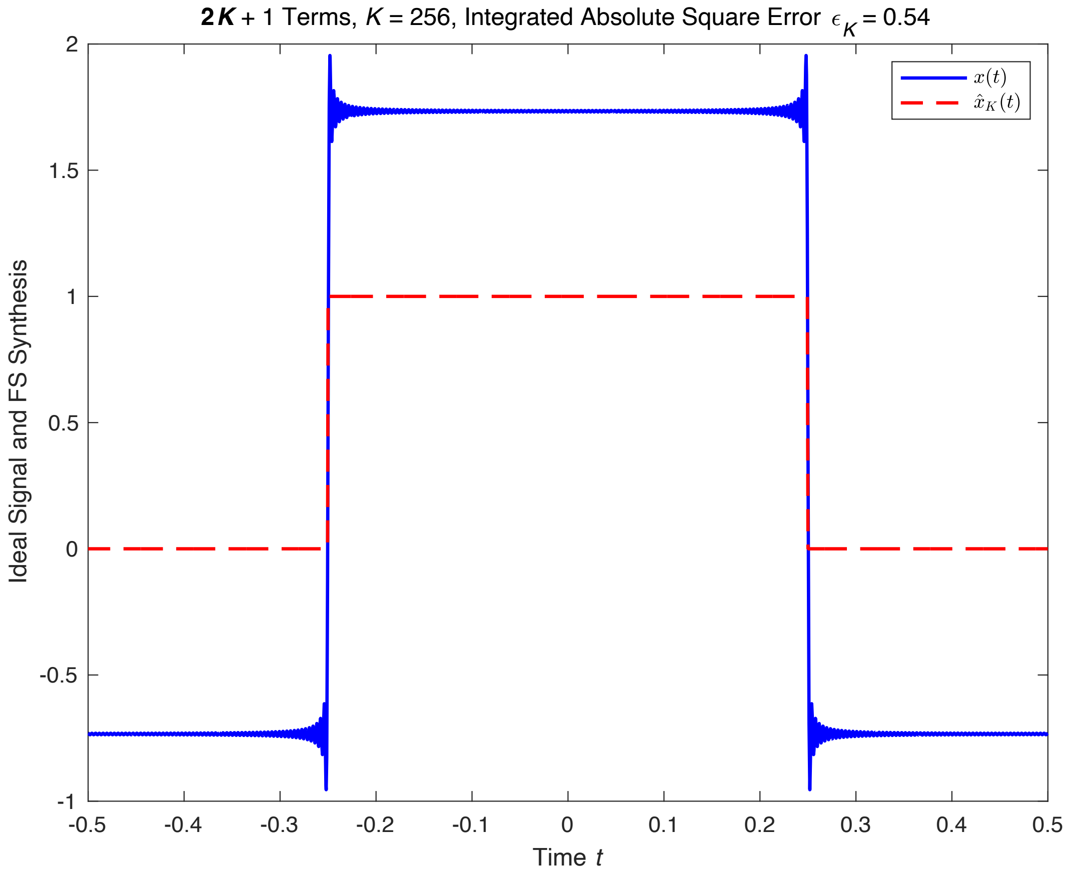
$=\frac{1}{2}\cos(\frac{\pi}{2}n)$

```
function ak = a_rectpulsetrain(k,omega0,T1)
if k == 0
    ak = omega0*T1/pi;
else
    ak = omega0*T1/pi*sin(k*omega0*T1)/k*omega0*T1;
end
```



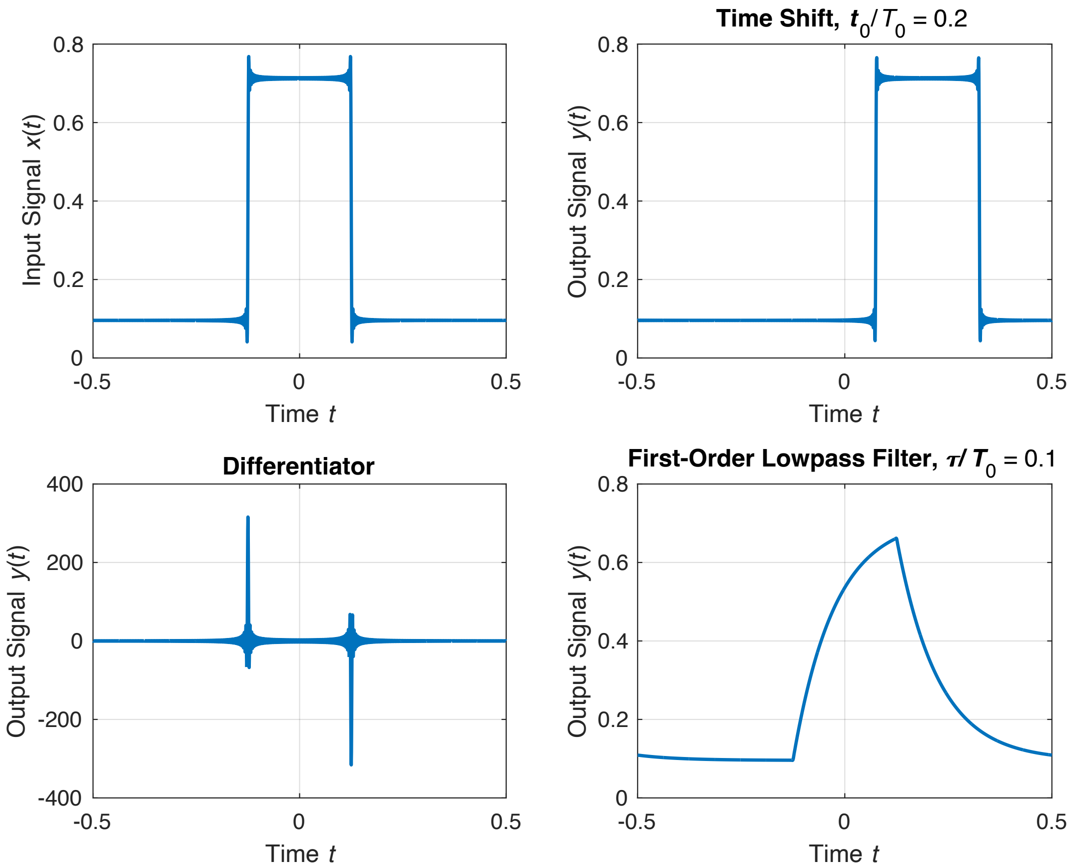

2K + 1 Terms, K = 16, Integrated Absolute Square Error ϵ_K = 0.53
Ideal Signal and FS Synthesis
Time t
$x(t)$
$\hat{x}_K(t)$
2
1.5
1
0.5
0
-0.5
-1
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


2K + 1 Terms, K = 64, Integrated Absolute Square Error ϵ_K = 0.54
Ideal Signal and FS Synthesis
Time t
$x(t)$
$\hat{x}_K(t)$
2
1.5
1
0.5
0
-0.5
-1
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

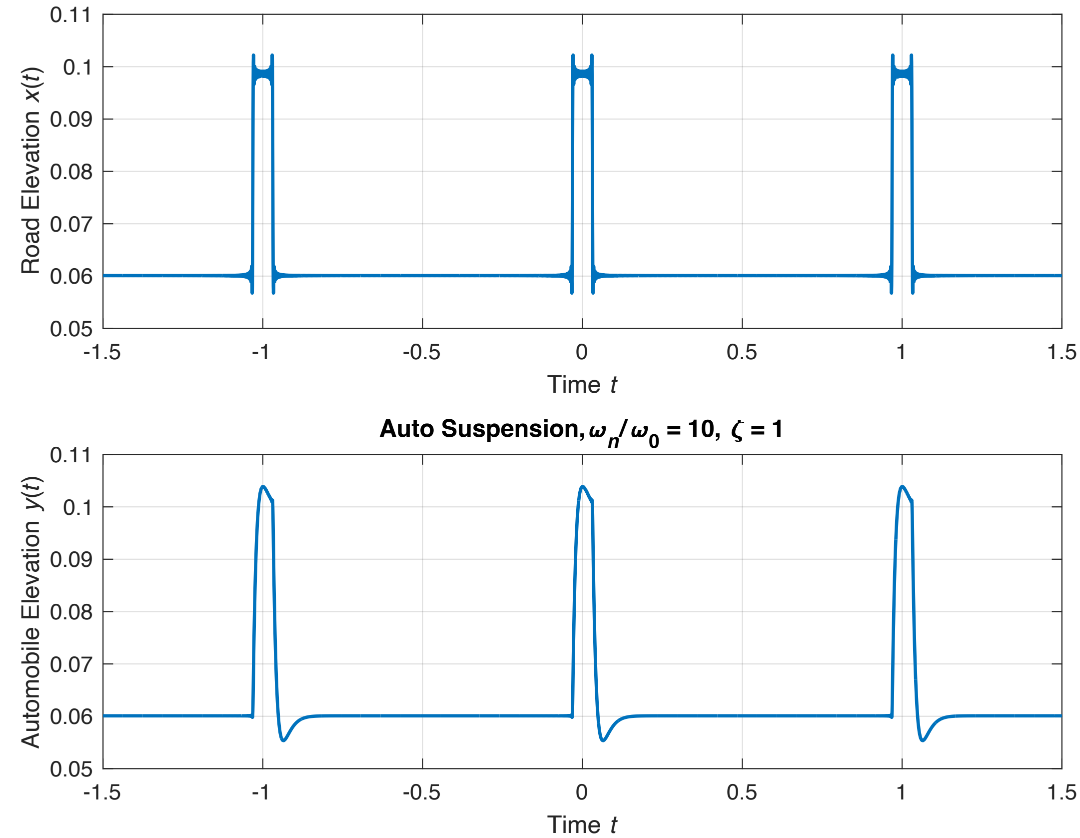


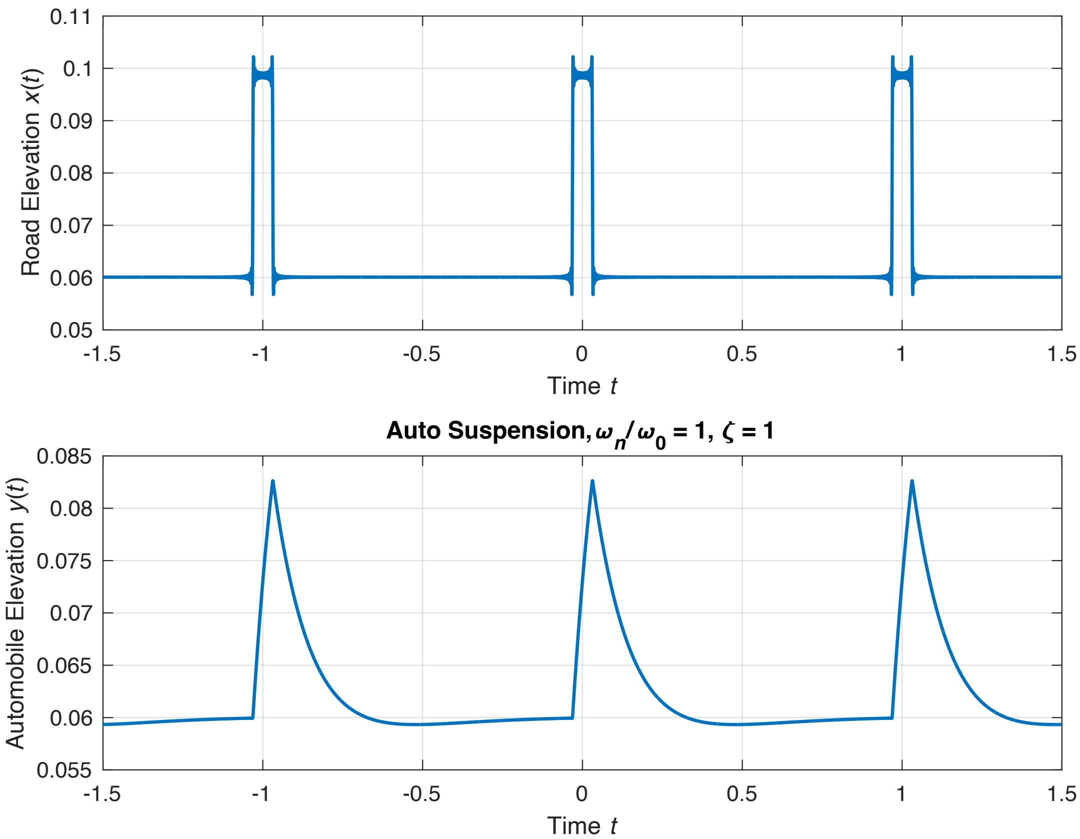
As k becomes bigger and bigger, the reconstruction error in Fourier series became smaller and smaller, and the reconstructed signal becomes closer and closer to the original signal.

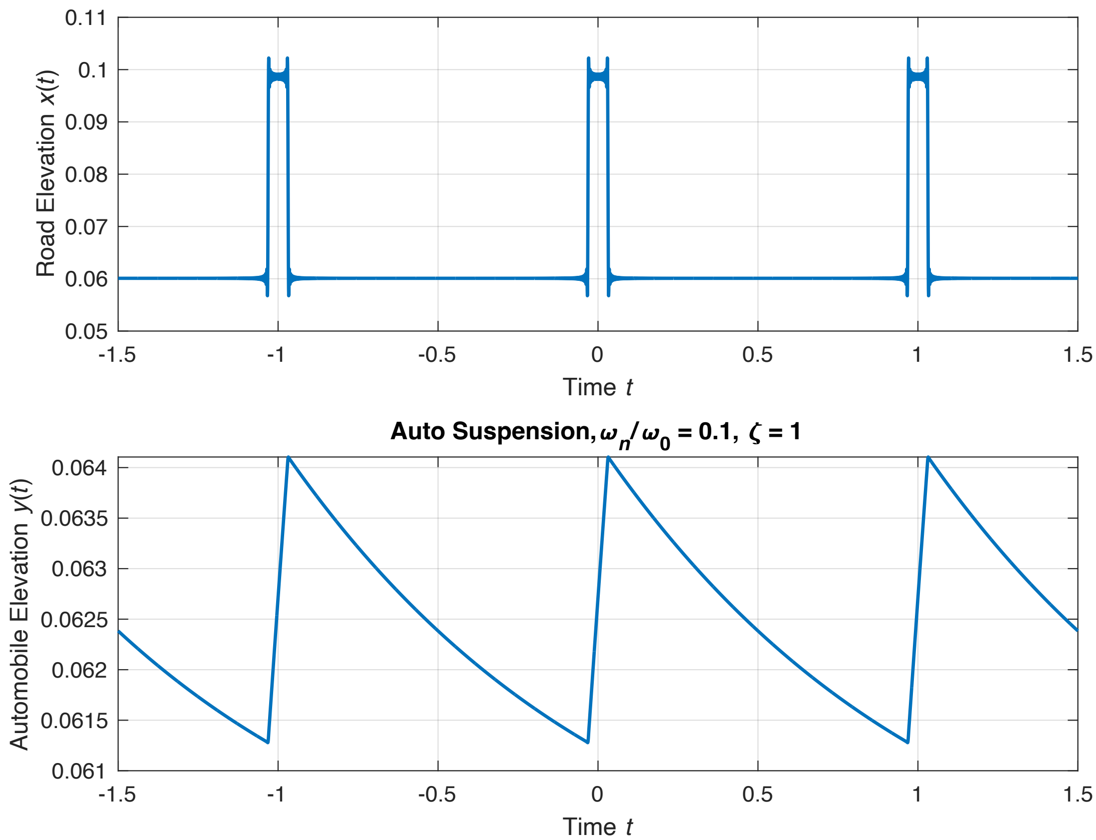
```
function H = Hdiff(omega)
H = sqrt(-1)*omega;
end

function H = Hfolpf(omega,tau)
H = 1/(1+sqrt(-1)*omega*tau);
end
```




```
function H = Hautosusp(omega,omegan,zeta)
H = (omegan*omegan+2*zeta*omegan*(sqrt(-1)*omega))/((sqrt(-
1)*omega)*(sqrt(-1)*omega)+2*zeta*omegan*(sqrt(-1)*omega)+omegan*omegan);
end
```


Road Elevation x(t)
Time t
Auto Suspension, $\omega_n/\omega_0 = 10$, $\zeta = 1$
Automobile Elevation y(t)
Time t


Road Elevation x(t)
Time t
Auto Suspension, $\omega_n/\omega_0 = 1$, $\zeta = 1$
Automobile Elevation y(t)
Time t



As omega N / omega 0 ratio became smaller and smaller, the car became more and more sluggish and smoother.