EE102A Winter 2021-22 | Stanford University
Signal Processing and Linear Systems I | Joseph M. Kahn

Final Examination Solutions

Thursday, March 17, 2022, 12:15 - 3:15 p.m.

Your exam will be scanned for grading, so please write your answers only on the front side of the paper, and only in the area provided for each part. If you must use additional space for a part of a problem, note that in the area provided for that part, and continue writing on one of the blank pages provided at the end.

This exam is open-book and open-note. You may use texts, notes, handouts, problem sets and solutions, tables of integrals, series and transforms. You may access these via hardcopy or PDF files saved on your laptop or tablet computer. You are permitted to use electronic calculators.

You are not permitted to use software for symbolic or numerical computation or graphing, such as MATLAB, Mathematica, Desmos, etc.

You are not permitted to access the Internet or use any form of electronic communication.

After completing the exam, do not discuss the exam with anyone, including classmates, until the graded exams are returned to you. Please never share the exam or its solutions with anyone in any format.

You will receive full credit if you correctly state answers without showing any work. But it is recommended that you show enough work so that your method can be understood. Partial credit will depend on the clarity of your argument. Clearly demarcate your final answers. If you are asked to sketch any graphs, label all key features (vertical and horizontal axes, etc.). You may use any results derived in class or in the homework without re-deriving them. Any new terms or symbols introduced in your solution must be defined.

In grading your exam, to the extent possible, we will attempt to penalize you only once for each error.

Please write your name and SUNet ID here.

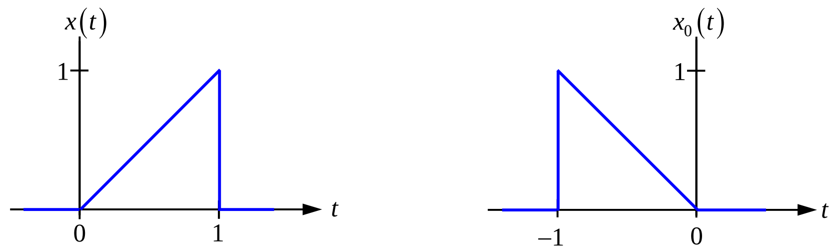
Name *SUNet ID*

Please sign here acknowledging that you accept the Honor Code.

Signature

The total number of points is 180.

1. (30 pts.) *CTFT properties*. Consider a CT signal $x(t)$, which has a CTFT $X(j\omega)$. Also consider a signal $x_0(t)$, which has a CTFT $X_0(j\omega)$.



You can express $x_0(t)$ in terms of $x(t)$ as

$$x_0(t) = x(-t).$$

Then you can express $X_0(j\omega)$ in terms of $X(j\omega)$ by using the CTFT time-reversal property to obtain

$$X_0(j\omega) = X(-j\omega).$$

Since the time signals are real, you could also write

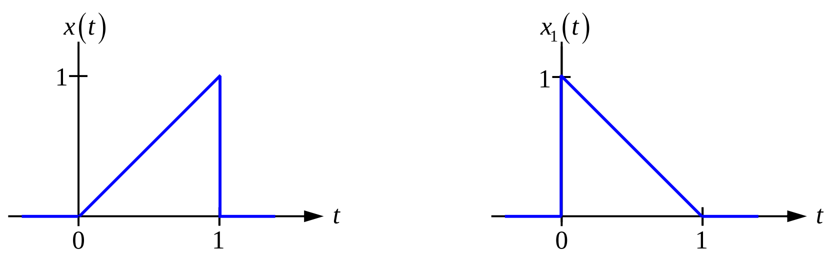
$$X_0(j\omega) = X^*(j\omega).$$

Now you are given four different signals, $x_1(t),\ldots,x_4(t)$, which are obtained from $x(t)$ using operations such as time shifting, time scaling or reversal, integration, differentiation, etc. For each signal, you are asked to:

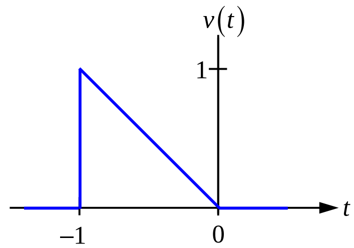
i. Express the signal in terms of $x(t)$, and

ii. Express the signal's CTFT in terms of $X(j\omega)$ using CTFT properties. You need not compute $X(j\omega)$ to do this.

a. (7.5 pts.) Express $x_1(t)$ and its CTFT $X_1(j\omega)$ in terms of $x(t)$ and its CTFT $X(j\omega)$, respectively.



Solution It is helpful to define an intermediate signal $v(t) = x(-t)$, as shown below.



We observe that

$$\begin{aligned} x_1(t) &= v(t-1) \\ &= x(-(t-1)) \ . \\ &= x(-t+1) \end{aligned}$$

Either of the last two is correct. To obtain the CTFT, we first use the time reversal property to write

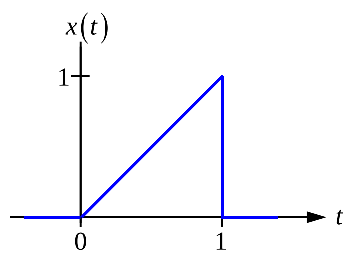
$$V(j\omega) = X(-j\omega) = X^*(j\omega).$$

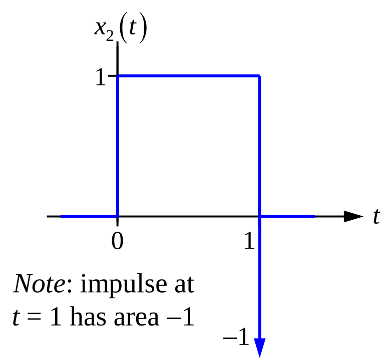
Now we use the time-shifting property to write

$$\begin{aligned} X_1(j\omega) &= V(j\omega)e^{-j\omega} \\ &= X(-j\omega)e^{-j\omega} \ . \\ &= X^*(j\omega)e^{-j\omega} \end{aligned}$$

Either of the last two is correct.

b. (7.5 pts.) Express $x_2(t)$ and its CTFT $X_2(j\omega)$ in terms of $x(t)$ and its CTFT $X(j\omega)$, respectively.





Solution We observe that

$$x_2(t) = \frac{dx}{dt}.$$

Using the CTFT differentiation-in-time property:

$$X_2(j\omega) = j\omega X(j\omega).$$

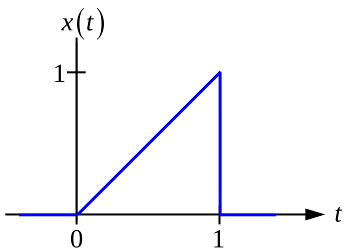
An alternate solution method is to observe that $x_2(t)$ can be expressed as

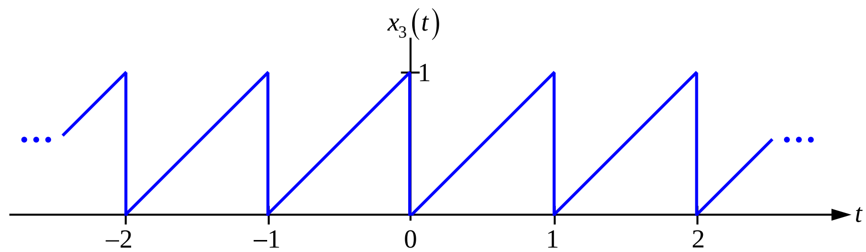
$$\begin{aligned} x_2(t) &= x(t) + x_1(t) - \delta(t-1) \\ &= x(t) + x(-t+1) - \delta(t-1) \end{aligned}.$$

Its CTFT can be expressed as

$$\begin{aligned} X_2(j\omega) &= X(j\omega) + X_1(j\omega) - e^{-j\omega} \\ &= X(j\omega) + X(-j\omega)e^{-j\omega} - e^{-j\omega} \\ &= X(j\omega) + X^*(j\omega)e^{-j\omega} - e^{-j\omega} \end{aligned}.$$

c. (7.5 pts.) Express $x_3(t)$ and its CTFT $X_3(j\omega)$ in terms of $x(t)$ and its CTFT $X(j\omega)$, respectively.





Solution There are two methods of solution, either of which will receive full credit.

Method 1: we can express $x_3(t)$ as a convolution of $x(t)$ with a train of impulses at intervals of 1 second:

$$x_3(t) = x(t) * \sum_{n=-\infty}^{\infty} \delta(t-n).$$

Using the CTFT convolution property, we can express $X_3(j\omega)$ as a product of $X(j\omega)$ with a scaled train of impulses in frequency at intervals of 2π rad/s:

$$\begin{aligned} X_3(j\omega) &= X(j\omega) \cdot 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - k2\pi) \\ &= 2\pi \sum_{k=-\infty}^{\infty} X(jk2\pi)\delta(\omega - k2\pi) \end{aligned}.$$

We have used the sampling property of the impulse in the final step. Either of the two forms given above is considered correct.

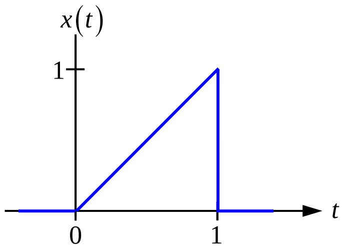
Method 2: we can express $x_3(t)$ as an infinite sum of copies of $x(t)$ shifted by multiples of 1 second:

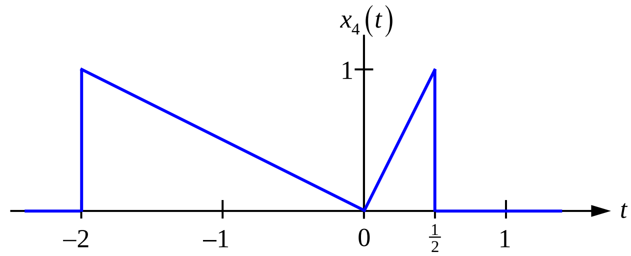
$$x_3(t) = \sum_{n=-\infty}^{\infty} x(t-n).$$

Using the CTFT time-shifting property we can express $X_3(j\omega)$ as

$$
\begin{aligned}
X_3(j\omega) &= \sum_{n=-\infty}^{\infty} X(j\omega) e^{-jn\omega} \\
&= X(j\omega) \sum_{n=-\infty}^{\infty} e^{-jn\omega}
\end{aligned}.
$$

d. (7.5 pts.) Express $x_4(t)$ and its CTFT $X_4(j\omega)$ in terms of $x(t)$ and its CTFT $X(j\omega)$, respectively.





Solution We can express $x_4(t)$ as a sum of two time-scaled copies of $x(t)$:

$$x_4(t) = x(2t) + x\left(-\frac{t}{2}\right).$$

Using the CTFT time-scaling property, we can express $X_4(j\omega)$ as a sum of two frequency-scaled copies of $X(j\omega)$:

$$
\begin{aligned}
X_4(j\omega) &= \frac{1}{2} X\left(j\frac{\omega}{2}\right) + 2X(-j2\omega) \\
&= \frac{1}{2} X\left(j\frac{\omega}{2}\right) + 2X^*(j2\omega)
\end{aligned}.
$$

Either of these two forms is considered correct.

2. (30 pts.) *CT LTI System Analysis.* Consider a CT signal

$$x(t) = 1 + \cos(2\pi t).$$

In each of parts (a)-(d) below, this $x(t)$ is input to a CT LTI system with a specified impulse response $h(t)$, and you are asked to obtain an expression for the output signal $y(t)$. Your output signal $y(t)$ should be purely real. It can include multiplicative factors such as $1/\sqrt{1+\pi^2}$ or phase shifts inside the argument of trigonometric functions, as in $\sin\left(4\pi t + \tan^{-1}(3\pi)\right)$. You may use a time-domain or frequency-domain method to obtain your answer.

a. (7.5 pts.) The signal $x(t) = 1 + \cos(2\pi t)$ is input to a system with impulse response

$$h(t) = \delta(t) - e^{-t}u(t).$$

Find an expression for the output signal $y(t)$.

Solution This system is a first-order highpass filter with time constant τ = 1. The output is found most easily using a frequency-domain method. The filter has a frequency response

$$H(j\omega) = 1 - \frac{1}{1+j\omega} = \frac{j\omega}{1+j\omega},$$

which has magnitude and phase

$$|H(j\omega)| = \frac{|\omega|}{\sqrt{1+\omega^2}} \text{ and } \angle H(j\omega) = \frac{\pi}{2}\operatorname{sgn}(\omega) - \tan^{-1}(\omega).$$

Expressing the input as $x(t) = e^{j0t} + \cos(2\pi t)$ and using the result of HW 5, Problem 1 on the second term, we find the output is

$$\begin{aligned} y(t) &= H(j0)e^{j0t} + |H(j2\pi)|\cos(2\pi t + \angle H(j2\pi)) \\ &= 0e^{j0t} + \frac{2\pi}{\sqrt{1+4\pi^2}}\cos\left(2\pi t + \frac{\pi}{2} - \tan^{-1}(2\pi)\right) \end{aligned}.$$

b. (7.5 pts.) The signal $x(t) = 1 + \cos(2\pi t)$ is input to a system with impulse response

$$h(t) = \frac{1}{2}\left[\delta(t) - \delta\left(t - \frac{1}{2}\right)\right].$$

Find an expression for the output signal $y(t)$.

Solution This is a delay-and-subtract system.

Time-domain solution: the output is

$$\begin{aligned} y(t) &= \frac{1}{2}\left[x(t) - x\left(t-\frac{1}{2}\right)\right] \\ &= \frac{1}{2}\left\{[1+\cos(2\pi t)] - \left[1+\cos\left(2\pi\left(t-\frac{1}{2}\right)\right)\right]\right\} \\ &= \frac{1}{2}[1-1+\cos(2\pi t) - \cos(2\pi t - \pi)] \\ &= \cos(2\pi t) \end{aligned}$$

Frequency-domain solution: the frequency response is

$$H(j\omega) = \frac{1}{2}\left(1 - e^{-j\frac{\omega}{2}}\right) = je^{-j\frac{\omega}{4}}\sin\left(\frac{\omega}{4}\right),$$

which has magnitude and phase

$$\left|H(j\omega)\right| = \left|\sin\left(\frac{\omega}{4}\right)\right| \text{ and } \angle H(j\omega) = \frac{\pi}{2} - \frac{\omega}{4} + \begin{cases} 0 & \sin\left(\frac{\omega}{4}\right) > 0 \\ \pm\pi & \sin\left(\frac{\omega}{4}\right) < 0 \end{cases}.$$

Expressing the input as $x(t) = e^{j0t} + \cos(2\pi t)$ and using the result of HW 5, Problem 1 on the second term, we find the output is

$$\begin{aligned} y(t) &= H(j0)e^{j0t} + \left|H(j2\pi)\right|\cos\left(2\pi t + \angle H(j2\pi)\right) \\ &= 0e^{j0t} + \left|\sin\left(\frac{\pi}{2}\right)\right|\cos\left(2\pi t + \frac{\pi}{2} - \frac{\pi}{2}\right) \\ &= \cos(2\pi t) \end{aligned}.$$

c. (7.5 pts.) The signal $x(t) = 1 + \cos(2\pi t)$ is input to a system with impulse response

$$h(t) = \Pi(t).$$

Find an expression for the output signal $y(t)$.

Solution This is a non-causal finite-time integrator.

Time-domain solution: the output is

$$y(t) = \int_{t-1/2}^{t+1/2} x(t')\,dt' = \int_{t-1/2}^{t+1/2} (1)\,dt' + \int_{t-1/2}^{t+1/2} \cos(2\pi t')\,dt'.$$

The first term is an integral of a constant 1 over a 1-second interval and yields 1. The second term is an integral of a cosine over one cycle and yields 0. Hence the output is

$$y(t) = 1.$$

Frequency-domain solution: the frequency response is

$$H(j\omega) = \operatorname{sinc}\left(\frac{\omega}{2\pi}\right),$$

which has magnitude and phase

$$\left|H(j\omega)\right| = \left|\operatorname{sinc}\left(\frac{\omega}{2\pi}\right)\right| \text{ and } \angle H(j\omega) = \begin{cases} 0 & \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) > 0 \\ \pm\pi & \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) < 0 \end{cases}.$$

Expressing the input as $x(t) = e^{j0t} + \cos(2\pi t)$ and using the result of HW 5, Problem 1 on the second term, we find the output is

$$\begin{aligned} y(t) &= H(j0)e^{j0t} + \left|H(j2\pi)\right|\cos\left(2\pi t + \angle H(j2\pi)\right) \\ &= \operatorname{sinc}\left(\frac{0}{2\pi}\right)e^{j0t} + \left|\operatorname{sinc}\left(\frac{2\pi}{2\pi}\right)\right|\cos(2\pi t + ?) \\ &= 1e^{j0t} + 0\cos(2\pi t + ?) \\ &= 1 \end{aligned}.$$

In the phase of the second term, the “?” indicates that we are free to choose the phase arbitrarily since the magnitude is zero.

d. (7.5 pts.) The signal $x(t) = 1 + \cos(2\pi t)$ is input to a system with impulse response

$$h(t) = \text{sinc}(t)\cos(2\pi t).$$

Find an expression for the output signal $y(t)$.

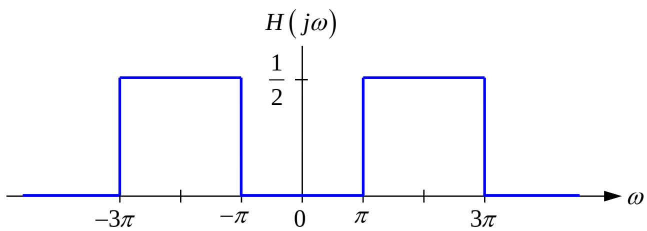
Solution This is a bandpass filter with bandwidth 2π and center frequency $\pm 2\pi$. The output is found most easily using a frequency-domain method. Using the CTFT pair

$$\text{sinc}(t) \overset{F}{\leftrightarrow} \Pi\left(\frac{\omega}{2\pi}\right)$$

and the modulation property of the CTFT, we find the filter has a frequency response

$$H(j\omega) = \frac{1}{2}\left[\Pi\left(\frac{\omega - 2\pi}{2\pi}\right) + \Pi\left(\frac{\omega + 2\pi}{2\pi}\right)\right],$$

which is plotted here.



Expressing the input as $x(t) = e^{j0t} + \cos(2\pi t)$ and using the result of HW 5, Problem 1 on the second term, we find the output is

$$\begin{aligned} y(t) &= H(j0)e^{j0t} + |H(j2\pi)|\cos(2\pi t + \angle H(j2\pi)) \\ &= 0e^{j0t} + \frac{1}{2}\cos(2\pi t) \\ &= \frac{1}{2}\cos(2\pi t) \end{aligned}.$$

3. (30 pts.) *DT LTI system analysis using DTFT*. A stable DT LTI system has an impulse response

$$h[n]=\left[\left(\frac{1}{2}\right)^n-\left(\frac{1}{3}\right)^n\right]u[n].$$

a. (9 pts.) Find an expression for the frequency response $H\left(e^{j\Omega}\right)$.

Solution Using the DFTF pair

$$a^n u[n]\overset{F}{\leftrightarrow}\frac{1}{1-ae^{-j\Omega}}\,,\ |a|<1\,,$$

and the linearity of the DTFT, we find

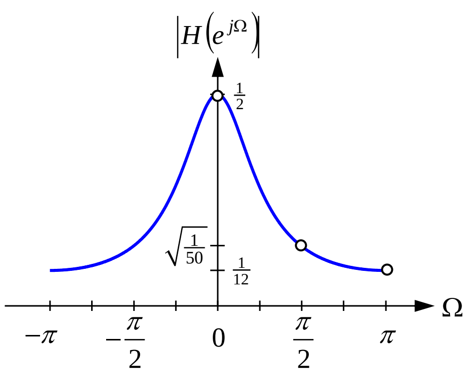
$$\begin{aligned}H\left(e^{j\Omega}\right)&=\frac{1}{1-\frac{1}{2}e^{-j\Omega}}-\frac{1}{1-\frac{1}{3}e^{-j\Omega}}\\&=\frac{\frac{1}{6}e^{-j\Omega}}{1-\frac{5}{6}e^{-j\Omega}+\frac{1}{6}e^{-j2\Omega}}\end{aligned}.$$

b. (9 pts.) Sketch the magnitude $\left|H\left(e^{j\Omega}\right)\right|$ for at least $-\pi\le\Omega\le\pi$. *Hint*: evaluate $\left|H\left(e^{j\Omega}\right)\right|$ at two or three well-chosen values of Ω.

Solution We evaluate $\left|H\left(e^{j\Omega}\right)\right|$ at $\Omega=0$, π/2 and π:

$$\left|H\left(e^{j0}\right)\right|=\left|\frac{\frac{1}{6}}{1-\frac{5}{6}+\frac{1}{6}}\right|=\left|\frac{1}{2}\right|=\frac{1}{2}\qquad\left|H\left(e^{j\pi}\right)\right|=\left|\frac{-\frac{1}{6}}{1+\frac{5}{6}+\frac{1}{6}}\right|=\left|-\frac{1}{12}\right|=\frac{1}{12}\approx 0.083$$

$$\left|H\left(e^{j\frac{\pi}{2}}\right)\right|=\left|\frac{-\frac{1}{6}j}{1+\frac{5}{6}j-\frac{1}{6}}\right|=\left|\frac{j}{5+5j}\right|=\sqrt{\frac{1}{25+25}}=\sqrt{\frac{1}{50}}\approx 0.141$$



You will receive full credit if you only evaluated the magnitude at Ω = 0 and π.

c. (6 pts.) Find the difference equation relating the input $x[n]$ and output $y[n]$.

Solution We can find the difference equation by inspection of $H\left(e^{j\Omega}\right)$, referring to the course reader, pages 222-224 (especially equations (46) and (53)). $H\left(e^{j\Omega}\right)$ is in the rational function form (53) with nonzero coefficients $b_1=\frac{1}{6}$, $a_0=1$, $a_1=-\frac{5}{6}$ and $a_2=\frac{1}{6}$. Hence, the difference equation is in the form (46) with these same nonzero coefficients:

$$\frac{1}{6}x[n-1]=y[n]-\frac{5}{6}y[n-1]+\frac{1}{6}y[n-2].$$

d. (6 pts.) Evaluate the integral $\frac{1}{2\pi}\int_{-\pi}^{\pi}H\left(e^{j\Omega}\right)e^{j2\Omega}d\Omega$ without using the expression for $H\left(e^{j\Omega}\right)$.

Solution By the definition of the inverse DTFT, we have

$$h[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}H\left(e^{j\Omega}\right)e^{jn\Omega}d\Omega.$$

At n = 2, we have

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}H\left(e^{j\Omega}\right)e^{j2\Omega}d\Omega=h[2]=\left(\frac{1}{2}\right)^2-\left(\frac{1}{3}\right)^2=\frac{5}{36}.$$

4. (30 pts.) *DT system frequency response*. Consider a stable DT LTI system having an impulse response:

$$h[n]=\frac{1}{4}\mathrm{sinc}\left(\frac{n}{4}\right)\sin\left(\frac{\pi}{2}n\right).$$

a. (12 pts.) Sketch the real and imaginary parts of the frequency response $H\left(e^{j\Omega}\right)$ for $-\pi<\Omega<\pi$.

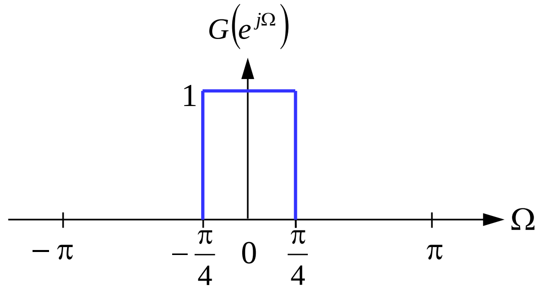
Solution Let us define a signal

$$g[n]=\frac{1}{4}\mathrm{sinc}\left(\frac{n}{4}\right).$$

We use the DTFT pair

$$g[n]=\frac{1}{4}\mathrm{sinc}\left(\frac{n}{4}\right)\overset{F}{\leftrightarrow}G\left(e^{j\Omega}\right)=\sum_{l=-\infty}^{\infty}\Pi\left(\frac{\Omega-l2\pi}{\pi/2}\right).$$

Here is a sketch of $G\left(e^{j\Omega}\right)$ for $-\pi<\Omega<\pi$.



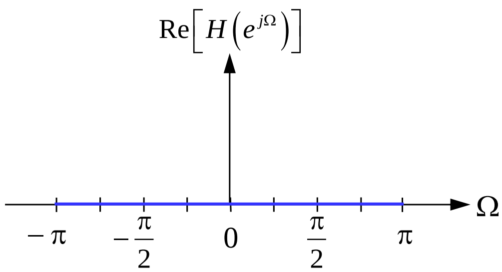
Note that

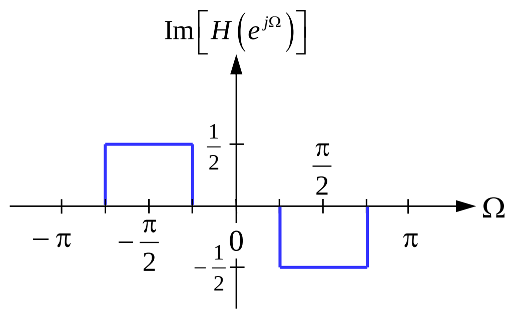
$$\sin\left(\frac{\pi}{2}n\right)=\frac{1}{2j}\left(e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}\right).$$

Using the frequency-shifting property of the DTFT, we obtain the frequency response of the system:

$$h[n]=g[n]\cdot\sin\left(\frac{\pi}{2}n\right)\overset{F}{\leftrightarrow}H\left(e^{j\Omega}\right)=\frac{1}{2j}\left[G\left(e^{j\left(\Omega-\frac{\pi}{2}\right)}\right)-G\left(e^{j\left(\Omega+\frac{\pi}{2}\right)}\right)\right].$$

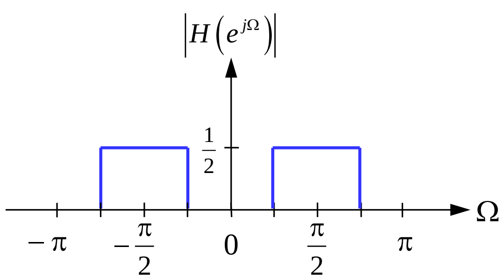
This frequency response is purely imaginary. Here is a sketch of its real and imaginary parts for $-\pi<\Omega<\pi$.

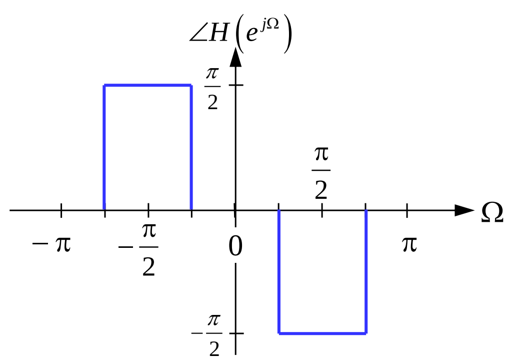




b. (6 pts.) Sketch the magnitude and phase of the frequency response $H\left(e^{j\Omega}\right)$ for $-\pi<\Omega<\pi$.

Solution In sketching the magnitude and phase, we note that $\left|\pm\frac{j}{2}\right|=\frac{1}{2}$ and $\angle\left(\pm\frac{j}{2}\right)=\pm\frac{\pi}{2}$.





c. (12 pts.) The input to the system is

$$x[n]=1+\sin\left(\frac{\pi}{2}n\right).$$

Find a real expression for the output $y[n]$.

Solution We express the input $x[n]$ as a sum of imaginary exponentials:

$$x[n]=1\cdot e^{j0n}+\frac{1}{2j}\left(e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}\right).$$

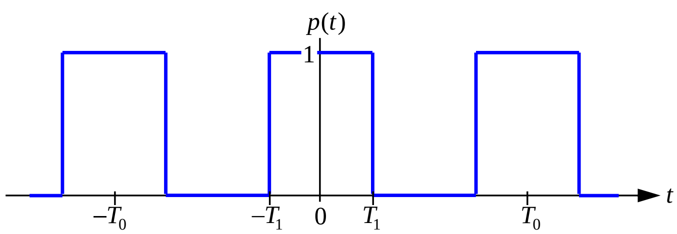
We can find the output by scaling each one by the corresponding value of the frequency response:

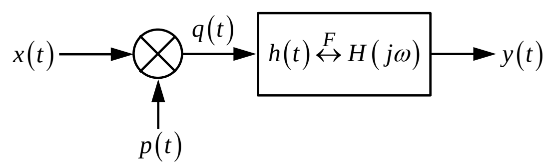
$$y[n]=1\cdot H\left(e^{j0}\right)\cdot e^{j0n}+\frac{1}{2j}\left(H\left(e^{j\frac{\pi}{2}}\right)\cdot e^{j\frac{\pi}{2}n}-H\left(e^{-j\frac{\pi}{2}}\right)\cdot e^{-j\frac{\pi}{2}n}\right).$$

Noting that $H\left(e^{j0}\right)=0$, $H\left(e^{j\frac{\pi}{2}}\right)=\frac{1}{2j}$ and $H\left(e^{-j\frac{\pi}{2}}\right)=-\frac{1}{2j}$, the output is

$$\begin{aligned}y[n]&=0\cdot e^{j0n}+\frac{1}{2j}\left(\frac{1}{2j}\cdot e^{j\frac{\pi}{2}n}+\frac{1}{2j}\cdot e^{-j\frac{\pi}{2}n}\right)\\&=-\frac{1}{4}\left(e^{j\frac{\pi}{2}n}+e^{-j\frac{\pi}{2}n}\right)\\&=-\frac{1}{2}\cos\left(\frac{\pi}{2}n\right)\end{aligned}.$$

5. (30 pts.) *Alternate DSB-AM modulator.* Here we consider performing double-sideband amplitude modulation by multiplying a message signal $x(t)$ by a rectangular pulse train $p(t)$, then filtering the resulting signal $q(t)$.





a. (8 pts.) Find an expression for $P(j\omega)$, the CTFT of the rectangular pulse train $p(t)$. The variables T_1 and T_0 (or $\omega_0=2\pi/T_0$) may appear in your answer.

Solution Using the Fourier series coefficients of the rectangular pulse train

$$p_k=\frac{2T_1}{T_0}\operatorname{sinc}\left(k\frac{2T_1}{T_0}\right)=\frac{\omega_0 T_1}{\pi}\operatorname{sinc}\left(k\frac{\omega_0 T_1}{\pi}\right)$$

and the formula for the Fourier transform of a periodic signal

$$P(j\omega)=2\pi\sum_{k=-\infty}^{\infty}p_k\delta\left(\omega-k\frac{2\pi}{T_0}\right)=2\pi\sum_{k=-\infty}^{\infty}p_k\delta\left(\omega-k\omega_0\right),$$

we obtain

$$P(j\omega)=\frac{4\pi T_1}{T_0}\sum_{k=-\infty}^{\infty}\operatorname{sinc}\left(k\frac{2T_1}{T_0}\right)\delta\left(\omega-k\frac{2\pi}{T_0}\right)=2\omega_0 T_1\sum_{k=-\infty}^{\infty}\operatorname{sinc}\left(k\frac{\omega_0 T_1}{\pi}\right)\delta\left(\omega-k\omega_0\right).$$

b. (7 pts.) Find an expression for $Q(j\omega)$, the CTFT of the multiplier output signal $q(t)$, in terms of $X(j\omega)$, the CTFT of the message signal $x(t)$. The variables T_1 and T_0 (or $\omega_0 = 2\pi / T_0$) may appear in your answer.

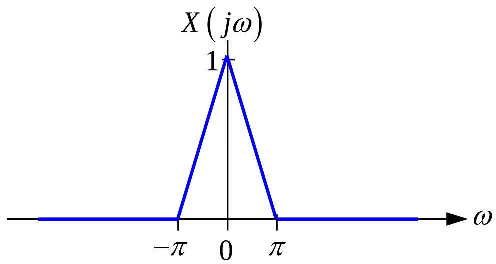
Solution Using the CTFT multiplication property and the expression for $P(j\omega)$ found in part (a), we have

$$q(t) = x(t) \cdot p(t) \overset{F}{\leftrightarrow} Q(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$= \frac{2T_1}{T_0} \sum_{k=-\infty}^{\infty} \text{sinc}\left(k\frac{2T_1}{T_0}\right) X\left(j\left(\omega - k\frac{2\pi}{T_0}\right)\right).$$

$$= \frac{\omega_0 T_1}{\pi} \sum_{k=-\infty}^{\infty} \text{sinc}\left(k\frac{\omega_0 T_1}{\pi}\right) X\left(j(\omega - k\omega_0)\right)$$

c. (8 pts.) For the remainder of the problem, assume $T_0 = 1$, $T_1 = 1/4$ and let the message signal have the CTFT $X(j\omega)$ indicated.



Sketch $Q(j\omega)$ (real or imaginary part, whichever is nonzero) for at least $-8\pi \le \omega \le 8\pi$.

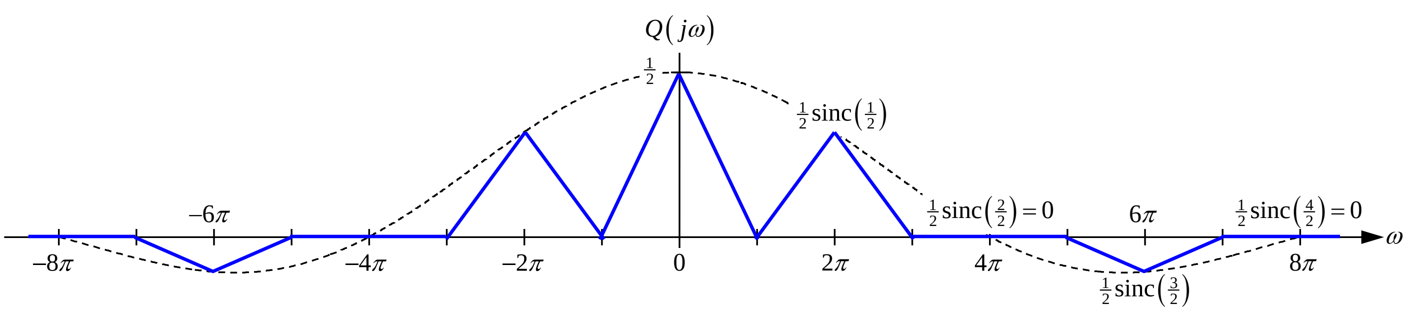
Solution Given the assumed parameter values, we have

$$\omega_0 = \frac{2\pi}{T_0} = 2\pi \text{ and } \frac{2T_1}{T_0} = \frac{\omega_0 T_1}{\pi} = \frac{1}{2},$$

so

$$Q(j\omega) = \frac{1}{2} \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k}{2}\right) X\left(j(\omega - k2\pi)\right),$$

which is shown below.

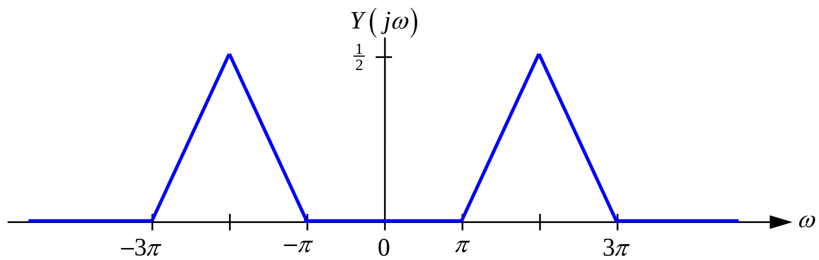


d. (7 pts.) Assume the same parameters as in part (c). We want the filter output to be $y(t) = x(t)\cos(2\pi t)$. Sketch the required filter frequency response $H(j\omega)$ (real or imaginary part, whichever is nonzero). Specify the filter's passband gain.

Solution We want the filter output and its CTFT to be

$$y(t) = x(t)\cos(2\pi t) \leftrightarrow Y(j\omega) = \tfrac{1}{2}\left[X\left(j(\omega-2\pi)\right) + X\left(j(\omega+2\pi)\right)\right],$$

as shown.



$H(j\omega)$ should be as shown. The passband gain should be $\dfrac{\frac{1}{2}}{\frac{1}{2}\text{sinc}\left(\frac{1}{2}\right)} = \dfrac{1}{\text{sinc}\left(\frac{1}{2}\right)}$.