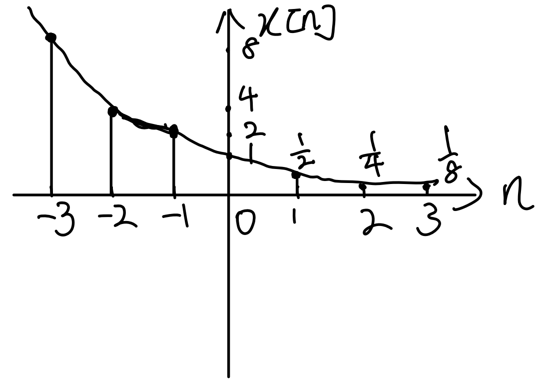
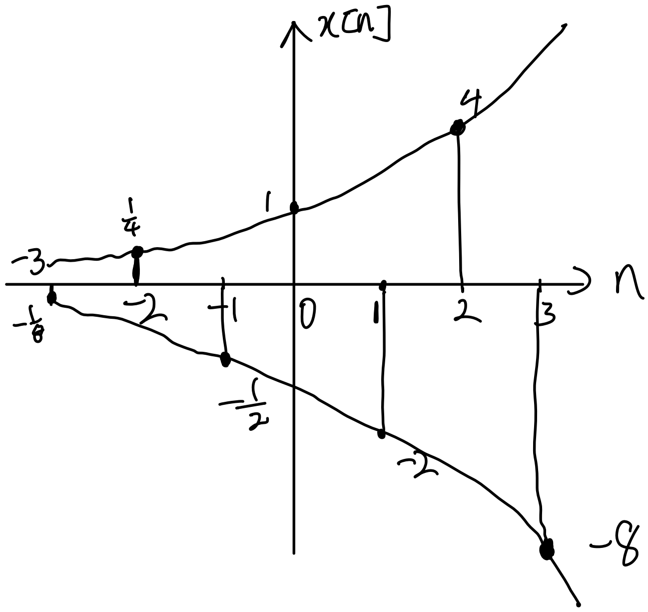
Homework 1

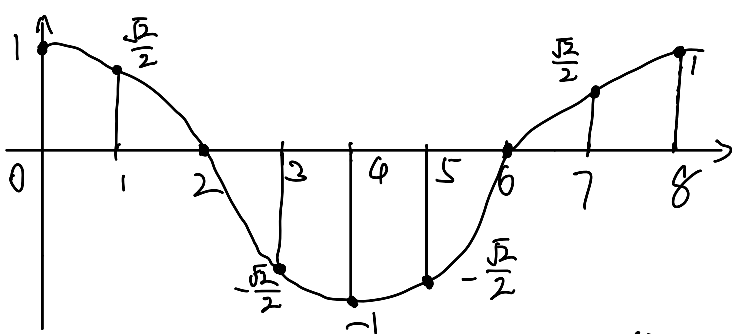
1. (a)



(b)

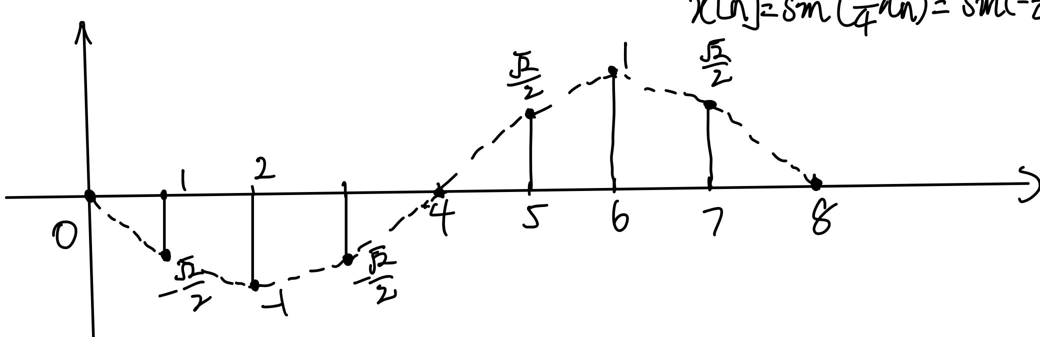


2. (a)



$x[n] = \sin(\frac{5}{4}\pi n) = \sin(-\frac{\pi}{4}n) = -\sin(\frac{\pi}{4}n)$

(b)



3. (a) $x(t) = \cos(2\pi t)$ periodic: $T = \frac{2\pi}{2\pi} = 1$.

(b). $x(t) = \cos^2(2\pi t) = 1 + \frac{1}{2}\cos(4\pi t)$ periodic: $T = \frac{2\pi}{4\pi} = \frac{1}{2}$.

(c) $x(t) = \cos(2\pi t) + \sin(2t)$ not periodic.

define $f(t) = \cos(2\pi t)$, $g(t) = \sin(2t)$.

$T_f = \frac{2\pi}{2\pi} = 1$; $T_g = \frac{2\pi}{2} = \pi$.

suppose T_x is the least common multiple of T_f & T_g.

then ① $T_x = aT_f$ for some $a \in \mathbb{Z}$, $T_f \in \mathbb{Q}$, $\Rightarrow T_x = \frac{m}{n}$ for some $m \in \mathbb{Z}$, $n \in \mathbb{Z}$;

② $T_x = b\,T_g$ for some $b \in \mathbb{Z}$, $T_g \in \mathbb{R} \setminus \mathbb{Q}$, $\Rightarrow T_x \in \mathbb{R} \setminus \mathbb{Q}$

① & ② are contradiction. therefore T_x not defined.

(d) not periodic. as $w(t) = 2\pi t$ increases as t goes by.

4. $E = \int_{-\infty}^{\infty} x^2(t)\,dt$. $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt$.

(a) $x(t) = e^{-2|t|}$, $-\infty < t < \infty$.

$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} (e^{-2|t|})^2 dt = \int_{-\infty}^{0} (e^{2t})^2 dt + \int_0^{\infty} (e^{-2t}) dt$

$= \int_{-\infty}^{0} e^{4t} dt + \int_0^{\infty} e^{-4t} dt = \frac{1}{4} e^{4t} \Big|_{-\infty}^{0} - \frac{1}{4} e^{-4t} \Big|_{0}^{\infty}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

$P = \lim_{T\to\infty} \frac{E}{2T} = 0$. therefore it's an Energy signal.

(b). $x(t) = e^{-t}$, $-\infty < t < \infty$.

$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} e^{2t} dt = \lim_{t\to\infty} \frac{1}{2} e^{2t} - \lim_{t\to\infty} \frac{1}{2} e^{2t} = \infty$

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} e^{2t} dt = \lim_{T\to\infty} \frac{\frac{1}{2} e^{2T} - \frac{1}{2} e^{-2T}}{2T} = \infty$.

This is neither an energy signal, nor a power signal.

(c). $x(t)=\cos^2(2\pi t)$, $-\infty<t<\infty$.

$E=\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} \cos^4(4\pi t)\,dt=\infty$.

$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^4(4\pi t)dt=\frac{3}{8}$.

it is a power signal.

(d) $x[n]=(\frac{1}{2})^{|n|}$, $-\infty<n<\infty$.

$E=\sum_{n=-\infty}^{\infty}(\frac{1}{2})^{|n|} = \sum_{n=-\infty}^{0}(\frac{1}{2})^{-n}+\sum_{n=0}^{\infty}(\frac{1}{2})^{n}-1$

$=2\left(\frac{1\,(1-(\frac{1}{2})^{\infty})}{1-\frac{1}{2}}\right)-1=3.$

$P=\lim_{N\to\infty}\frac{E}{2N}=0.$ therefore it's an energy signal.

5. proof:

$|x(t)|^2 = |x_e(t)+x_o(t)|^2$

$= |x_e(t)|^2+|x_o(t)|^2+[x_e(t)x_o^*(t)+x_e^*(t)x_o(t)]$

Since complex conjugation of a signal preserves its eveness or oddness,

$x_e(t)x_o^*(t)$ is odd signal, so is $x_e^*(t)x_o(t)$,

thus: $\int_{-\infty}^{\infty}[x_e^*(t)x_o(t)+x_e(t)x_o^*(t)]\,dt=0$

thus: $E=\int_{-\infty}^{\infty}|x(t)|^2dt=\int_{-\infty}^{\infty}|x_e(t)+x_o(t)|^2dt$

$=\int_{-\infty}^{\infty}|x_e(t)|^2dt+\int_{-\infty}^{\infty}|x_o(t)|^2dt+\int_{-\infty}^{\infty}[x_e(t)x_o^*(t)+x_e^*(t)x_o(t)]dt$

$=\int_{-\infty}^{\infty}|x_e(t)|^2dt+\int_{-\infty}^{\infty}|x_o(t)|^2dt.$

HW1

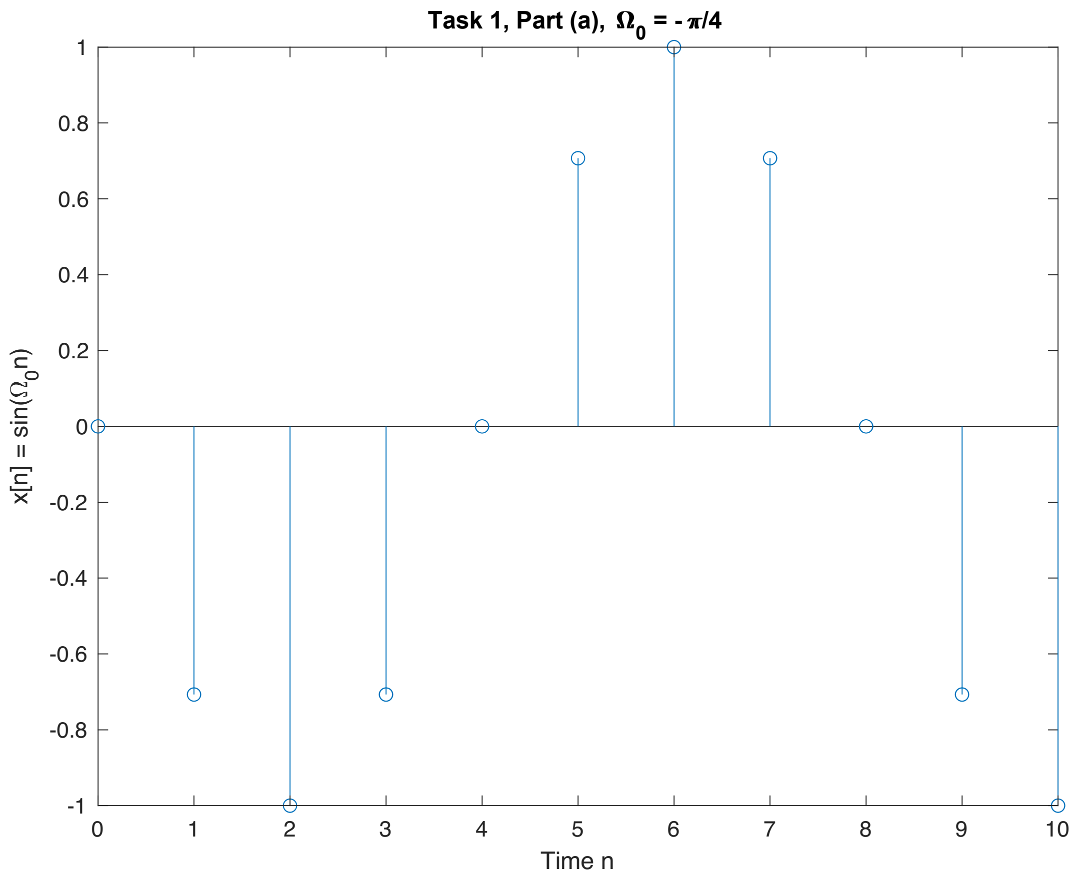
Part A code:

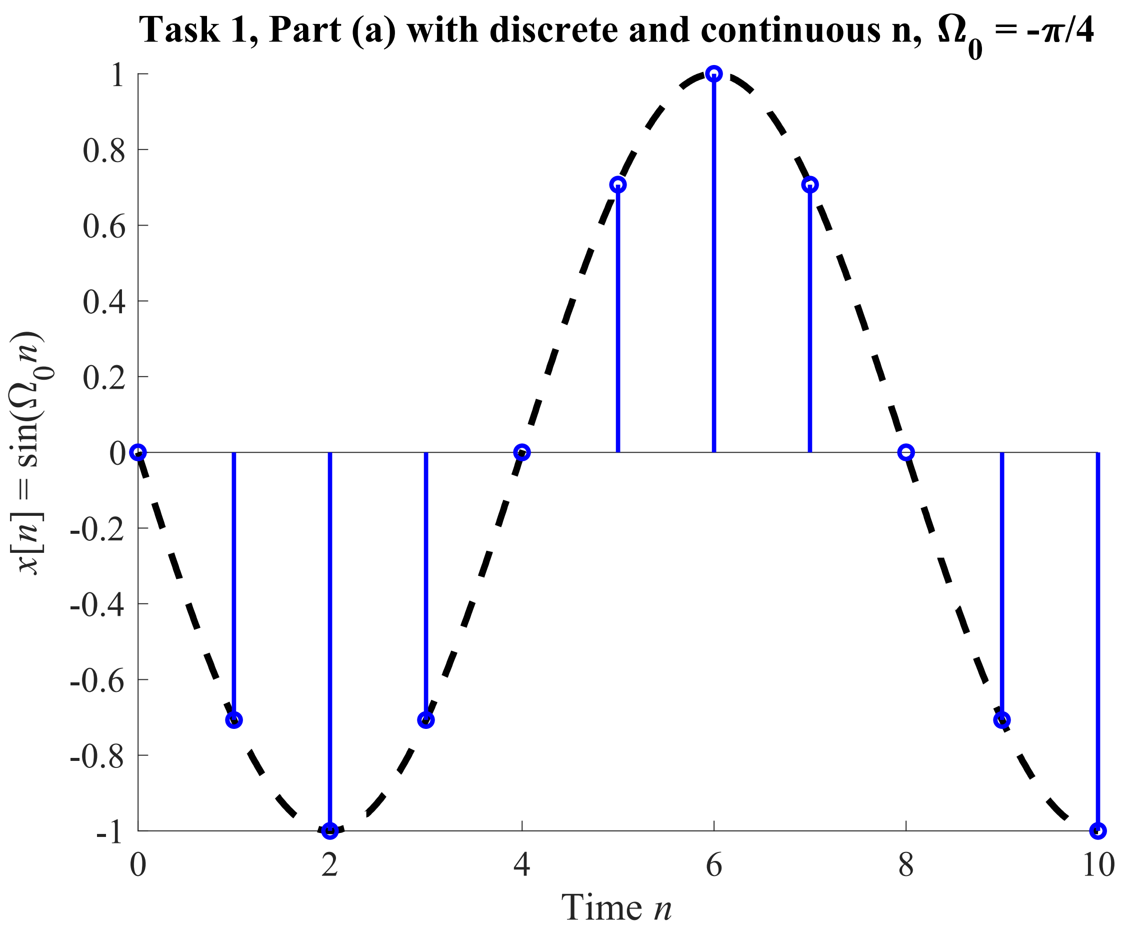
```
% Zhiwei Wen

% EE102A HW1

% Problem 1

clear all; close all;

%% 1a

n = 0:10;

Omega0 = -pi/4;

xn = sin(Omega0*n);

figure;

stem (n,xn);

xlabel('Time n');

ylabel('x[n] = sin(\Omega_{0}n)');

title('Task 1, Part (a), \Omega_{0} = -\pi/4');

t = 0:.01:10;

xn_ct = sin(Omega0 * t);

figure;

hold on;

set(gca,'FontName','times','FontSize',16);

plot(t, xn_ct,'k--', 'LineWidth', 3);

stem(n, xn, 'b', 'LineWidth', 2);

xlabel('Time \itn');

ylabel('\itx\rm[\itn\rm] = sin(\Omega_{0}\itn\rm)');

title('Task 1, Part (a) with discrete and continuous n, \Omega_{0} = -\pi/4')

grid on;

%% 1b

Omega1 = 7*pi/4;

yn = sin(Omega1*n);

yn_ct = sin(Omega1 * t);

figure; hold on;

set(gca,'FontName','times','FontSize',16);

plot(t, xn_ct,'r--', 'LineWidth', 2.5, 'DisplayName', '\Omega = -\pi/4');

plot(t, yn_ct,'k--', 'LineWidth', 2.5, 'DisplayName', '\Omega =7\pi/4');

stem(n, xn, 'r', 'LineWidth', 4, 'DisplayName', '\Omega = -\pi/4');
```

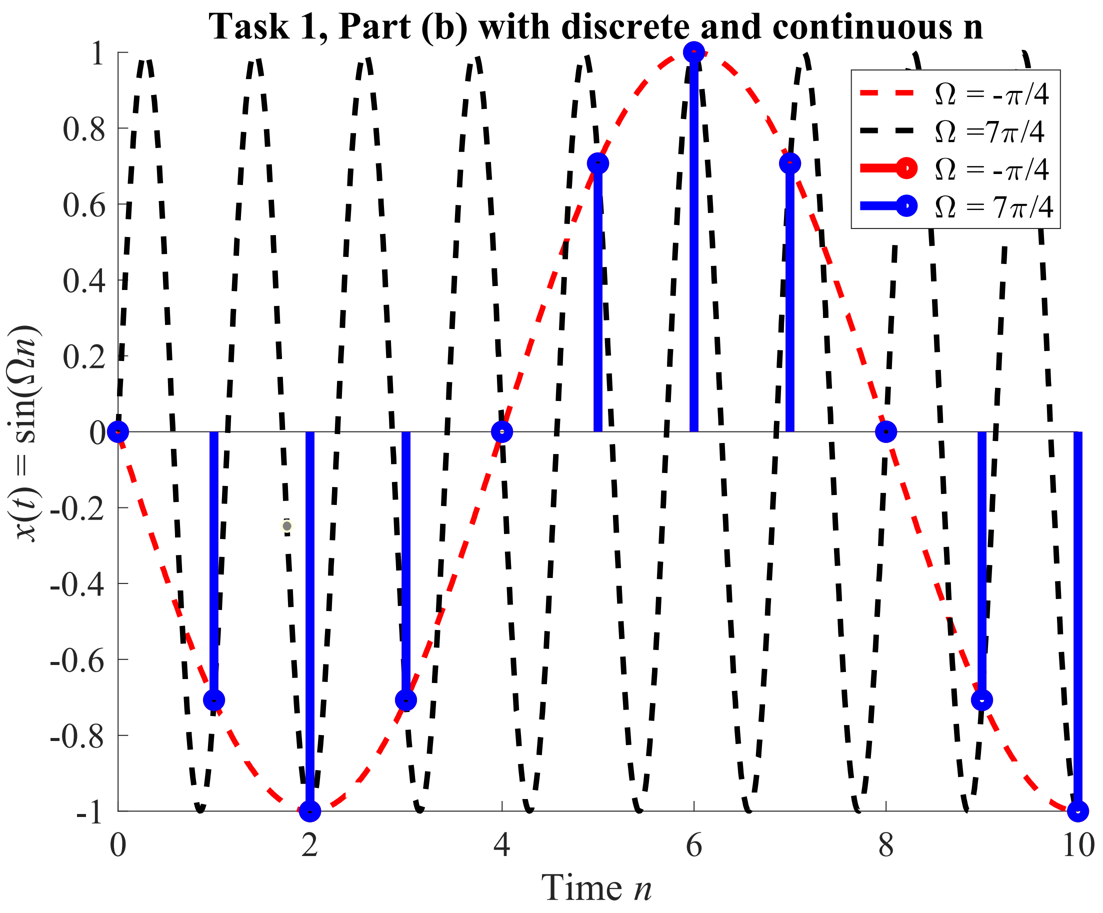
```
stem(n, yn, 'b', 'LineWidth', 4, 'DisplayName', '\Omega = 7\pi/4');
xlabel('Time \itn');
ylabel('\itx\rm(\itt\rm) = sin(\Omega\itn\rm)');
title('Task 1, Part (b) with discrete and continuous n')
legend('show');
grid on;
%% 1c
wn = xn.*xn;
figure;
set(gca, 'FontName', 'times','FontSize',16);
stem(n, wn, 'LineWidth', 3);
xlabel('Time n');
ylabel('w[n]= x^{2}[n]');
grid on;
%periodic signal: T = 4.
%% 1d
t = 0:.005:5;
zt = exp(-t+1j*2*pi*t);
re_zt = real(zt);
im_zt = imag(zt);
envel1 = exp(-1*t);
envel2 = -exp(-1*t);
figure;
hold on;
set(gca,'FontName','times','FontSize',16);
plot(t,re_zt,'r','LineWidth',2.5,'DisplayName','Real Part');
plot(t,im_zt,'k','LineWidth',2.5,'DisplayName','Imaginary Part');
plot(t,envel1,'b--','LineWidth',1.5,'DisplayName','Envelop e^{-t}');
plot(t,envel2,'g--','LineWidth',1.5,'DisplayName','Envelop -e^{-t}');
xlabel('Time t');
title('Real and Imaginary parts of e^{-t+j2\pit}');
legend('show');
grid on;
% as can be seen in the figure, the real part and imaginary part oscillate
% within the envelop defined by +-e^(-t). They are decaying sinusoids.
figure; hold on;
```

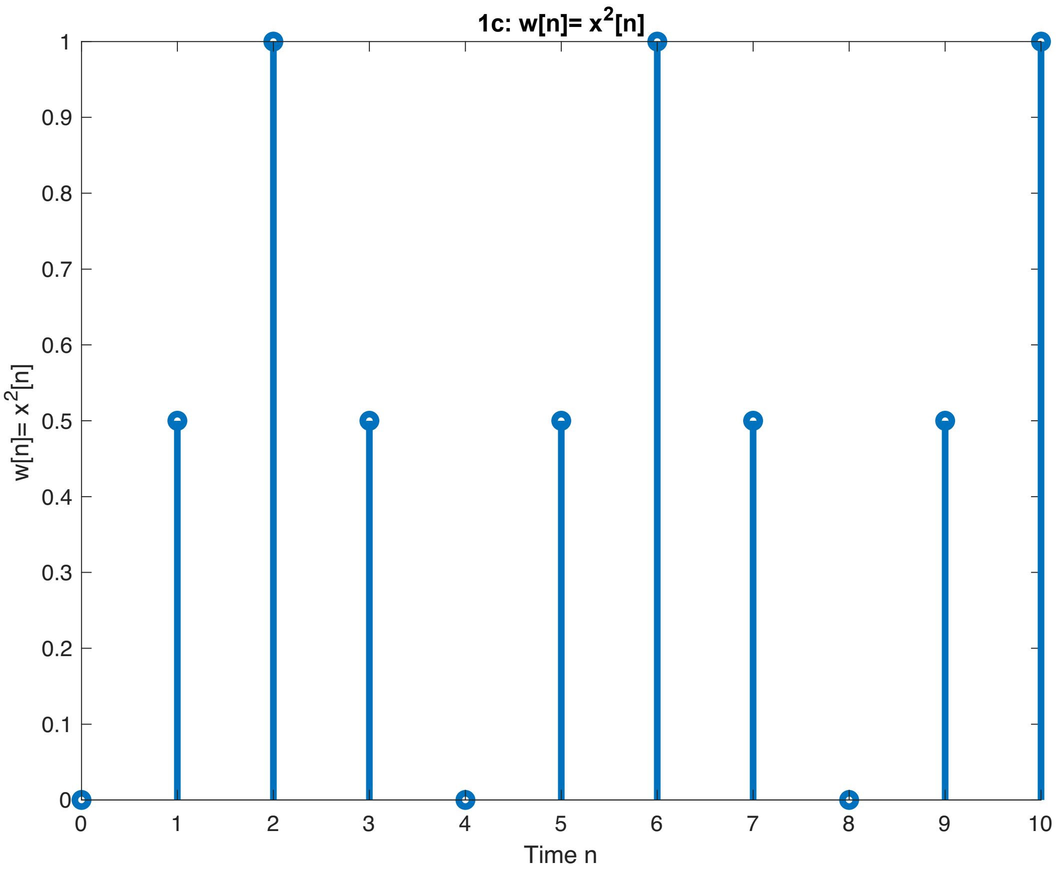
```
set(gca,'FontName','times','FontSize',16);
plot(re_zt, im_zt,'b', 'LineWidth', 4);
xlabel('Re(\itz\rm(\itt\rm))');
ylabel('Im(\itz\rm(\itt\rm))');
title('Task 1, Part (d), \itz\rm(\itt\rm) = exp(-\itt + j2\pit\rm)');
grid on;
% e^{-t+1j*2*pi*t}=e^{-t}*e^{1j*2*pi*t};
% the real part and imaginary part oscillate
% within the envelop defined by +-e^(-t).
% They are decaying sinusoids.
```

Part A figures:

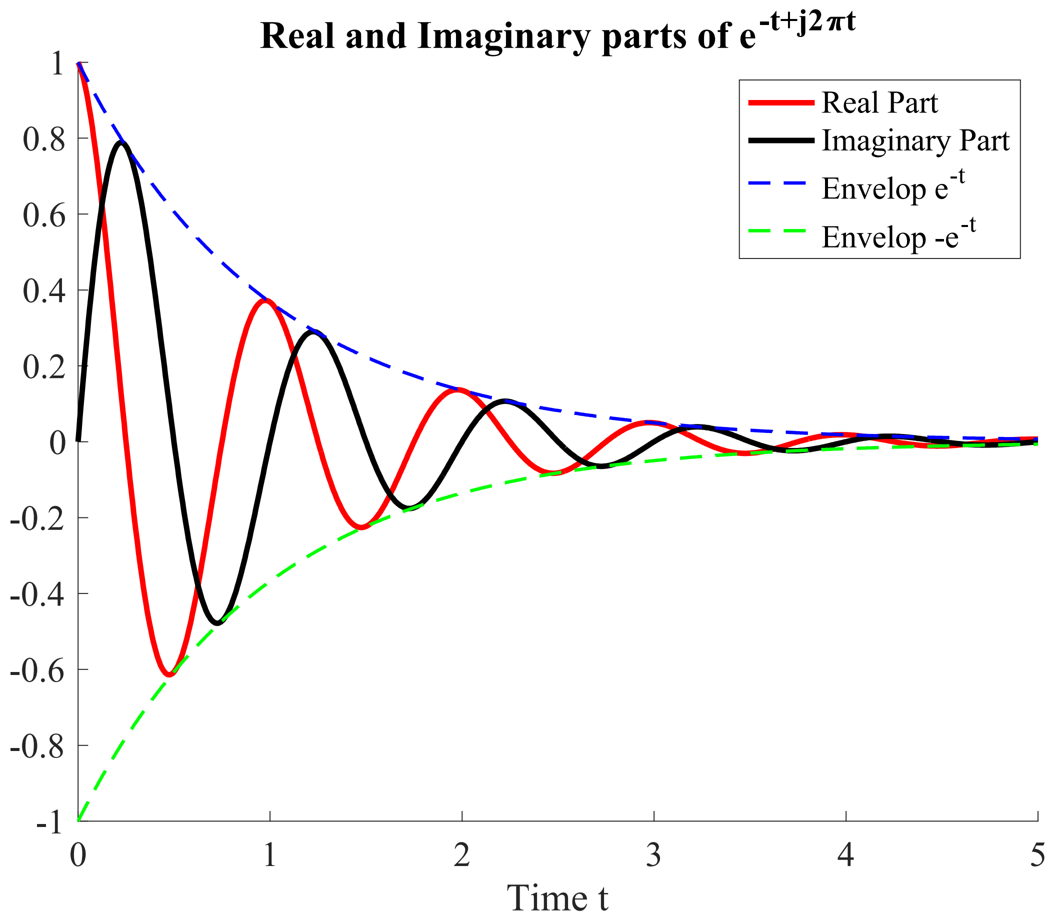



Task 1, Part (a) with discrete and continuous n, $\Omega_0 = -\pi/4$
$x[n] = \sin(\Omega_0 n)$
Time n

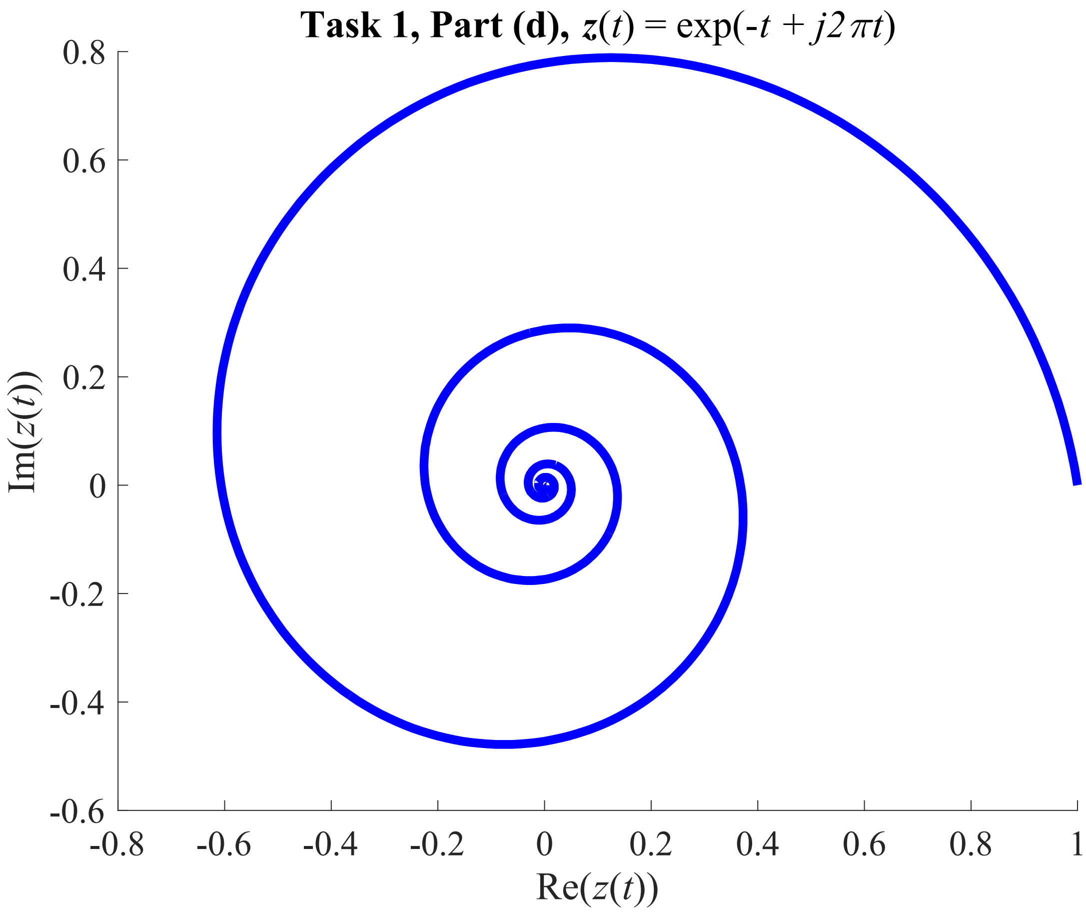

Task 1, Part (b) with discrete and continuous n
$\Omega = -\pi/4$
$\Omega = 7\pi/4$
$\Omega = -\pi/4$
$\Omega = 7\pi/4$
$x(t) = \sin(\Omega n)$
Time n



1c: w[n]= x^2[n]
w[n]= x^2[n]
Time n



Real and Imaginary parts of e^{-t+j2πt}
Real Part
Imaginary Part
Envelop e^{-t}
Envelop -e^{-t}
Time t



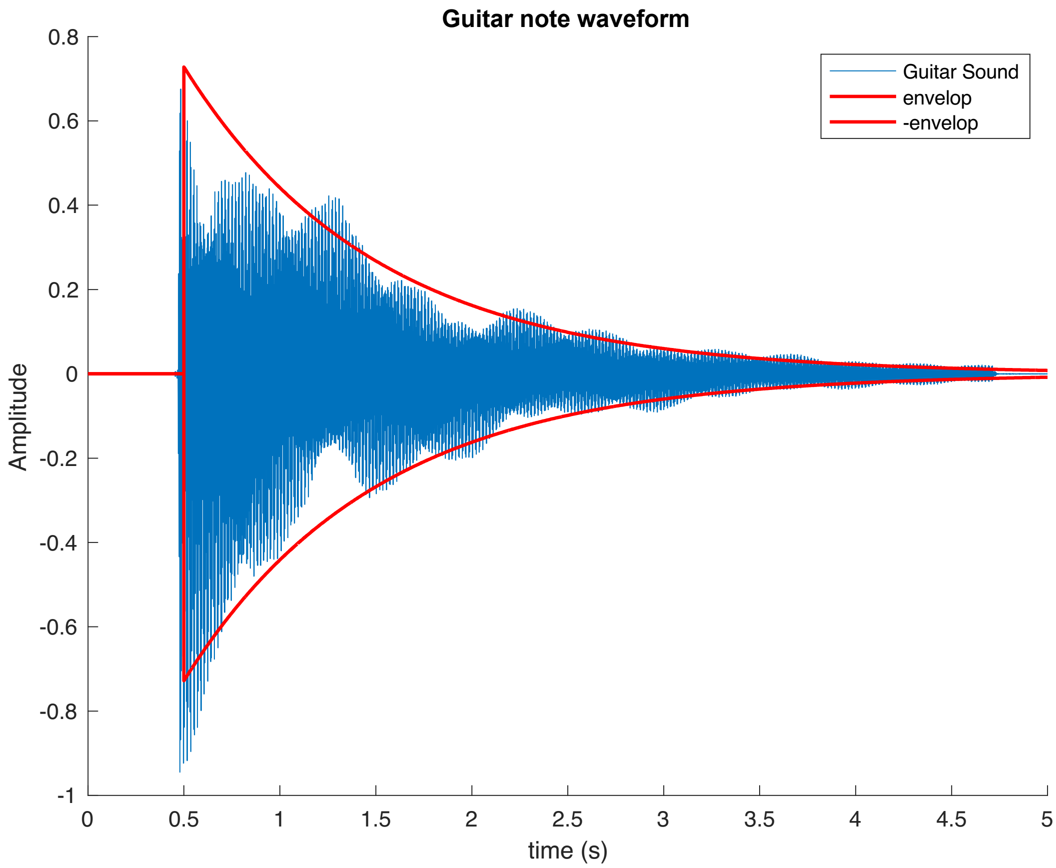
As can be seen in the figure of real and imaginary part, the real part and imaginary part oscillate within the envelop defined by +-e^(-t). They are decaying sinusoids. This is because in e^{-t+1j*2*pi*t} = e^{-t}*e^{1j*2*pi*t}, the coefficient of t in e^{-t+1} is -1. This shows that the signal is decaying its module. the real part and imaginary part oscillate within the envelop defined by +-e^(-t). They are decaying sinusoids.

Part B codes:

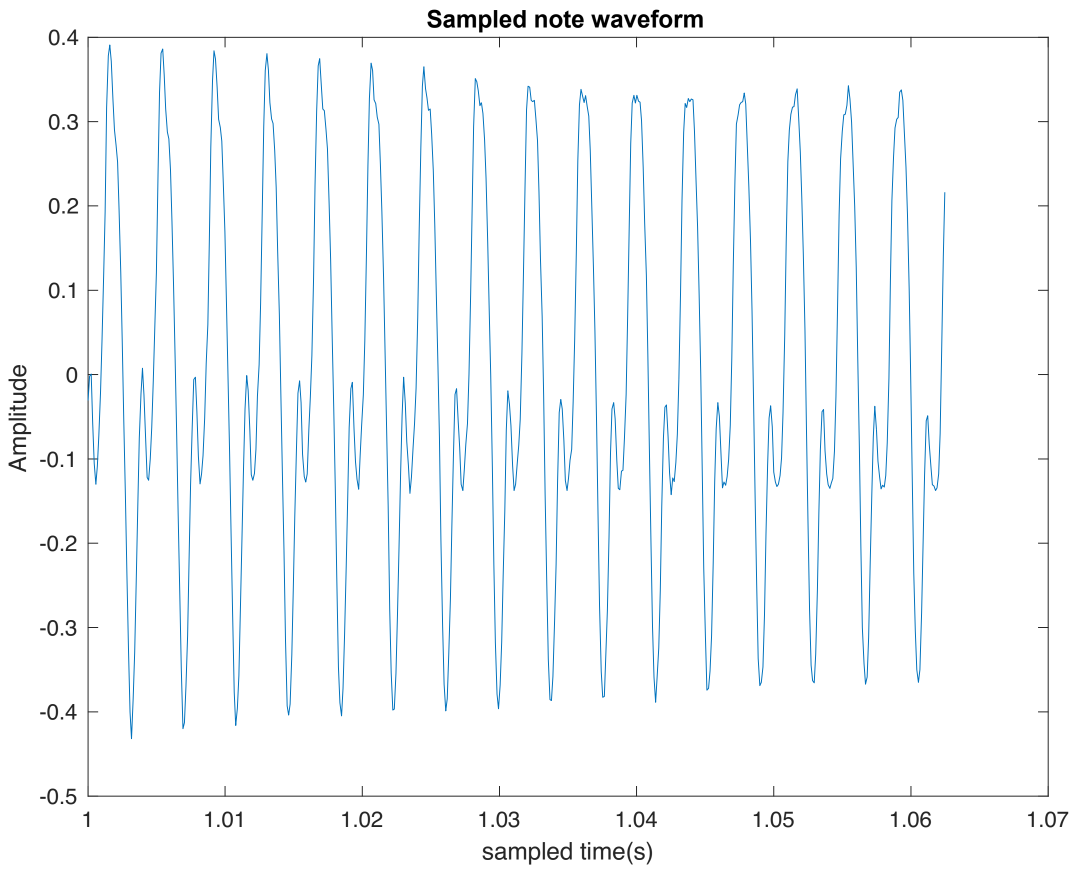
```
% Zhiwei Wen

% EE102A HW1

% Problem 2

clear all; close all;

load guitar_note.mat;

t = (1:length(note))*dt;

figure; hold on;

plot(t(1:8:end),note(1:8:end),'DisplayName','Guitar Sound');

xlabel('time (s)');

ylabel('Amplitude');

title('Guitar note waveform');

alpha = 1.2;
```

```
sigma = -1;
t0 = 0.5;
% alpha is set to 1.2, sigma -1, t0 to 0.5.
envelope = alpha*exp(sigma*t).*double(t>=t0);
plot(t,envelope,'r','LineWidth',1.5,'DisplayName','envelop');
plot(t,-envelope,'r','LineWidth',1.5,'DisplayName','-envelop');
legend('show');grid on;
index_sample = 1*fs:round((1+1/16)*fs);
t_sample = t(index_sample);
note_sample = note(index_sample);
figure;
plot(t_sample,note_sample);
xlabel('sampled time(s)');
ylabel('Amplitude');
title('Sampled note waveform');
grid on;
% the cycle should be approximately 16.2 rounds. So the frequency
% should be 16.2 x 16 = 259.2 hz, which is very close to the real
% frequency of middle C : 261.62 hz.
f_note = 259.2;
simulated_note = envelope.*sin(2*pi*f_note*t);
figure;hold on;
plot(t(1:16:end),simulated_note(1:16:end),'DisplayName','Simulated Sound');
xlabel('time (s)');
ylabel('Amplitude');
title('Simulated note waveform');
envelope = alpha*exp(sigma*t).*double(t>=t0);
plot(t,envelope,'r','LineWidth',1.5,'DisplayName','envelop');
plot(t,-envelope,'r','LineWidth',1.5,'DisplayName','-envelop');
legend('show');grid on;
% By playing this sound with(simulated_note,fs) command in the command
% line, it is found that they are the same note with same fading rate.
```

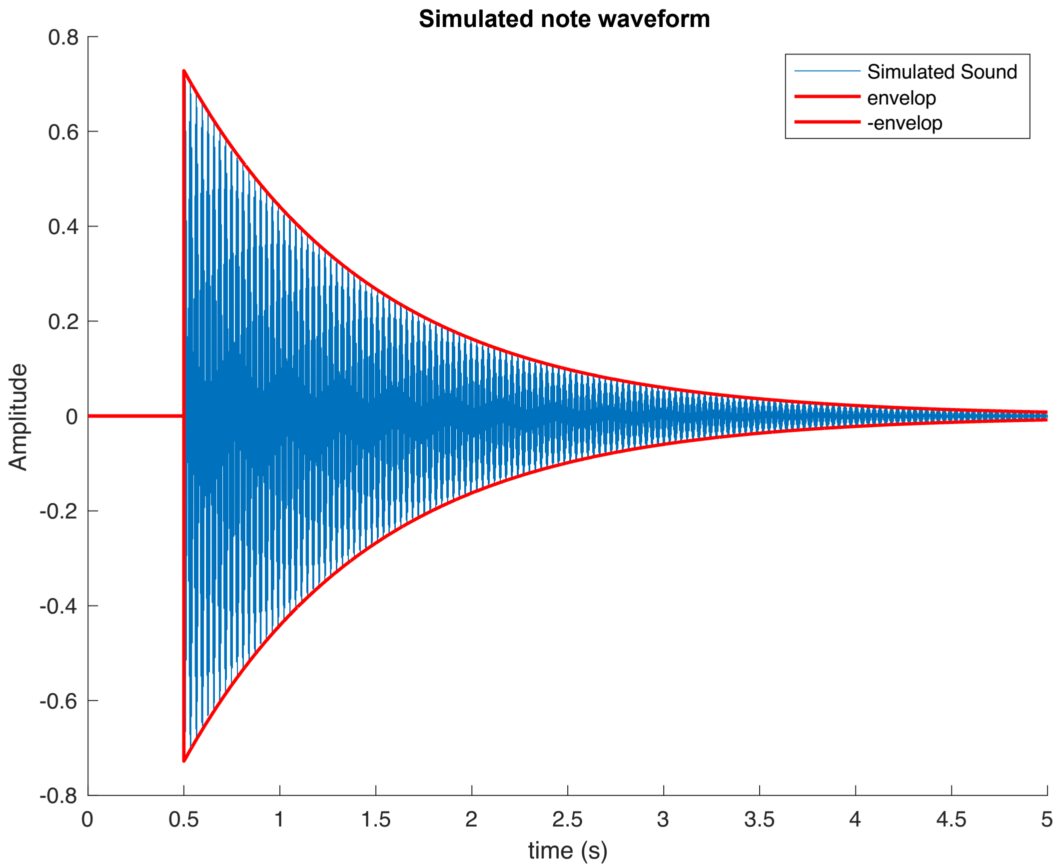
Part B Figures:



Guitar note waveform
Guitar Sound
envelop
-envelop
Amplitude
time (s)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Sampled note waveform
Amplitude
sampled time(s)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
1
1.01
1.02
1.03
1.04
1.05
1.06
1.07


Simulated note waveform
Simulated Sound
envelop
-envelop
Amplitude
time (s)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5