1. (Synthesis of ECG Signal Using Fourier Series) The electrocardiogram (ECG) is a common physiological signal which can be synthesized by a set of sinusoids. For a normal human at rest, the electrocardiogram signal is roughly a periodic signal with period $T = 0.833$s, corresponding to a heart rate of 72 beats per minute. The fundamental frequency of such an ECG signal is $1.2Hz$. Consider the first three harmonics expressed as

$$e_4(t) = \sum_{k=1}^{4} p_k \cdot \cos(2\pi \cdot f_k \cdot t + \phi_k) \tag{1}$$

where $f_k = k \cdot f_0$ with the fundamental frequency $f_0 = 1.2$Hz and ϕ_k is the phase shift of each sinusoid. The values of p_k and ϕ_k $(k = 1, 2, 3, 4)$ are given in Fig. 2. The sum $e_4(t)$ is shown in Fig. 1

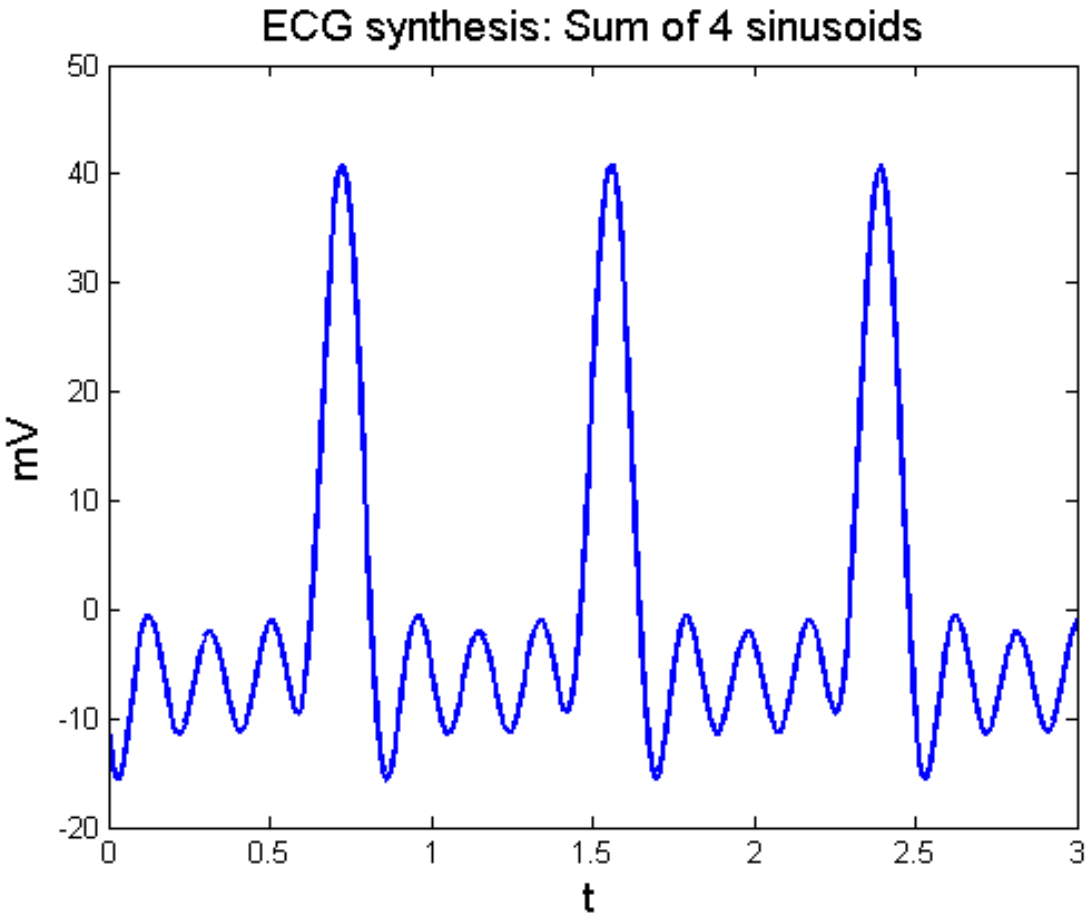

Figure 1: Sum of 4 sinusoids

Develop tour Python codes for the following tasks:

(a) Consider first ten harmonics, i.e., $k = 1, 2, \ldots, 10$.

(b) Consider an DC term and 30 harmonics, i.e., $k = 0, 1, 2, \ldots, 30$.

Sinusoid number k	Frequency $f_k = k \cdot f_o$ (Hz)	Magnitude p_k (mV)	Phase ϕ_k (radians)
0	0.0	8.000	0.000
1	1.2	12.434	0.912
2	2.4	10.775	1.769
3	3.6	9.121	2.513
4	4.8	8.770	3.165
5	6.0	10.053	3.902
6	7.2	12.066	4.800
7	8.4	13.924	5.799
8	9.6	15.069	0.553
9	10.8	15.143	1.593
10	12.0	14.021	2.610
11	13.2	11.911	3.585
12	14.4	9.361	4.489
13	15.6	7.112	5.295
14	16.8	5.732	6.029
15	18.0	5.121	0.531
16	19.2	4.701	1.459
17	20.4	4.115	2.519
18	21.6	3.300	3.670
19	22.8	2.283	4.886
20	24.0	1.129	6.261
21	25.2	0.555	2.713
22	26.4	1.489	4.555
23	27.6	2.250	5.745
24	28.8	2.540	0.545
25	30.0	2.349	1.579
26	31.2	1.815	2.582
27	32.4	1.150	3.573
28	33.6	0.549	4.660
29	34.8	0.250	0.302
30	36.0	0.494	1.979

Figure 2: Sinusoids for ECG synthesis.