Exercise 1

1. The Legendre polynomials $P_n(x)$ are defined by the following recurrance relation

$$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$$

with $P_0(x) = 1, P_1(x) = x, P_2(x) = (3x^2 - 1)/2$. Compute the next three Legendre polynomials and plot all 6 over the interval [-1,1].

2. Interpolate Runge function $f(x) = \dfrac{1}{1+x^2}$ with equidistant nodes on the interval $[-5, 5]$, plot polynomials of degree four, eight and sixteen, and compare them with $f(x)$.