4、刚体对轴的转动惯量

刚体对轴的转动惯量

简单形状物体的转动惯量计算

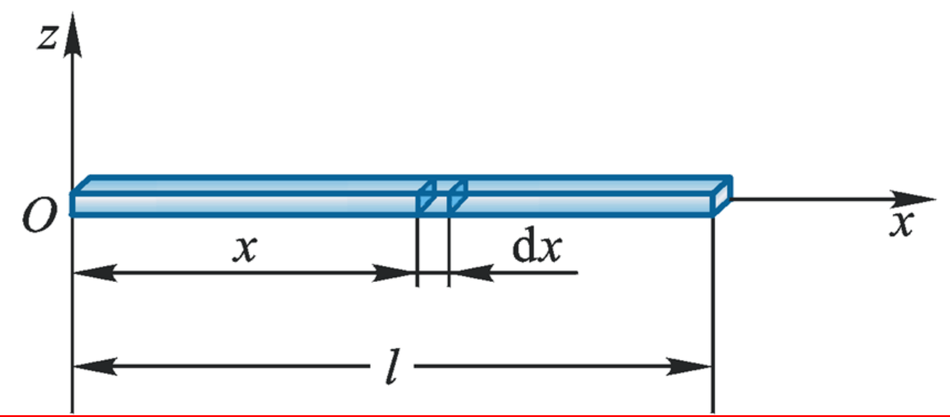
$$J_z = \sum_{i=1}^{n} m_i r_i^2$$

● 均质细直杆对一端的转动惯量

$$J_z = \int_0^l \rho_l x^2 \mathrm{d}x = \frac{\rho_l l^3}{3}$$

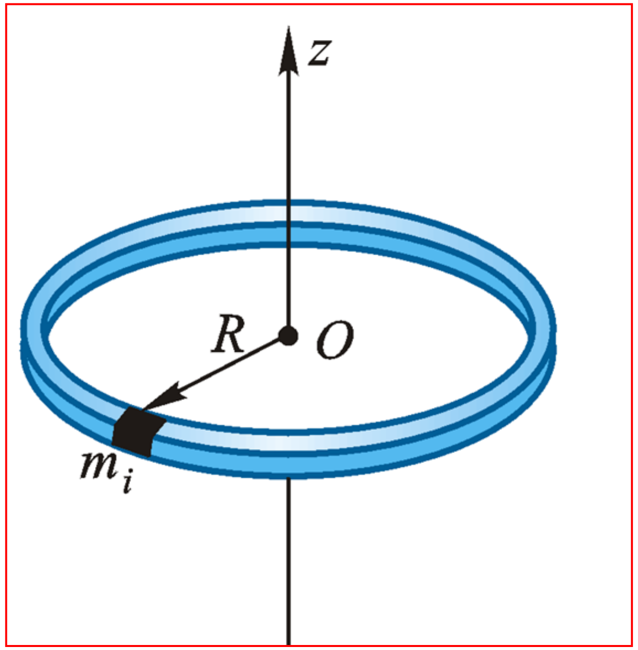
由 $m = \rho_l l$，得

$$J_z = \frac{1}{3} m l^2$$



● 均质薄圆环对中心轴的转动惯量

$$J_z = \sum m_i R^2 = R^2 \sum m_i = mR^2$$



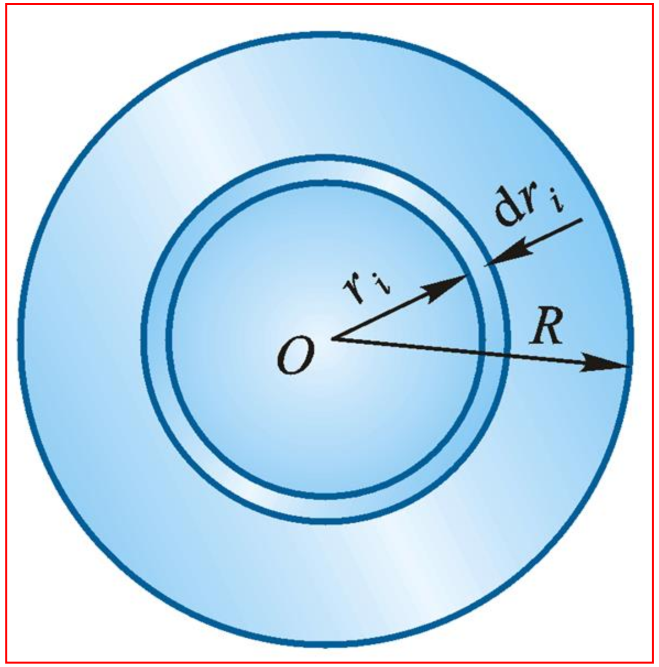
● 均质圆板对中心轴的转动惯量

$$m_i = 2\pi r_i \,\mathrm{d}r_i \cdot \rho_A$$

式中： $\rho_A = \dfrac{m}{\pi R^2}$

$$J_O = \int_0^R (2\pi r \rho_A \mathrm{d}r \cdot r^2) = 2\pi \rho_A \frac{R^4}{4}$$

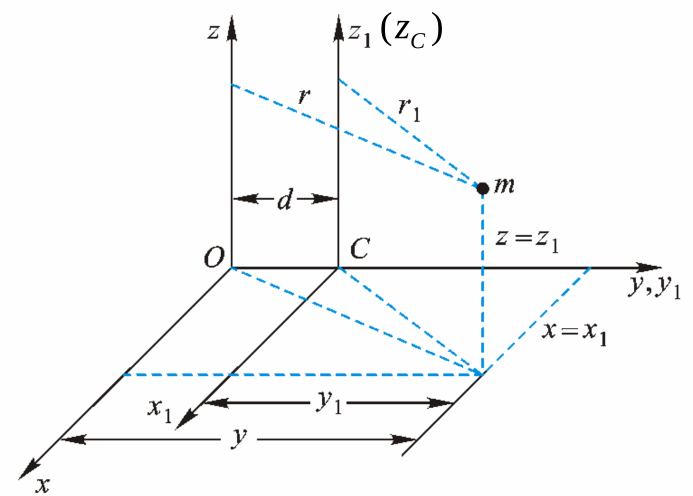
$$J_O = \frac{1}{2} mR^2$$



回转半径（惯性半径）

$$\rho_z = \sqrt{\frac{J_z}{m}}$$ 或 $$J_z = m\rho_z^2$$

平行轴定理



$$J_z = J_{z_C} + md^2$$

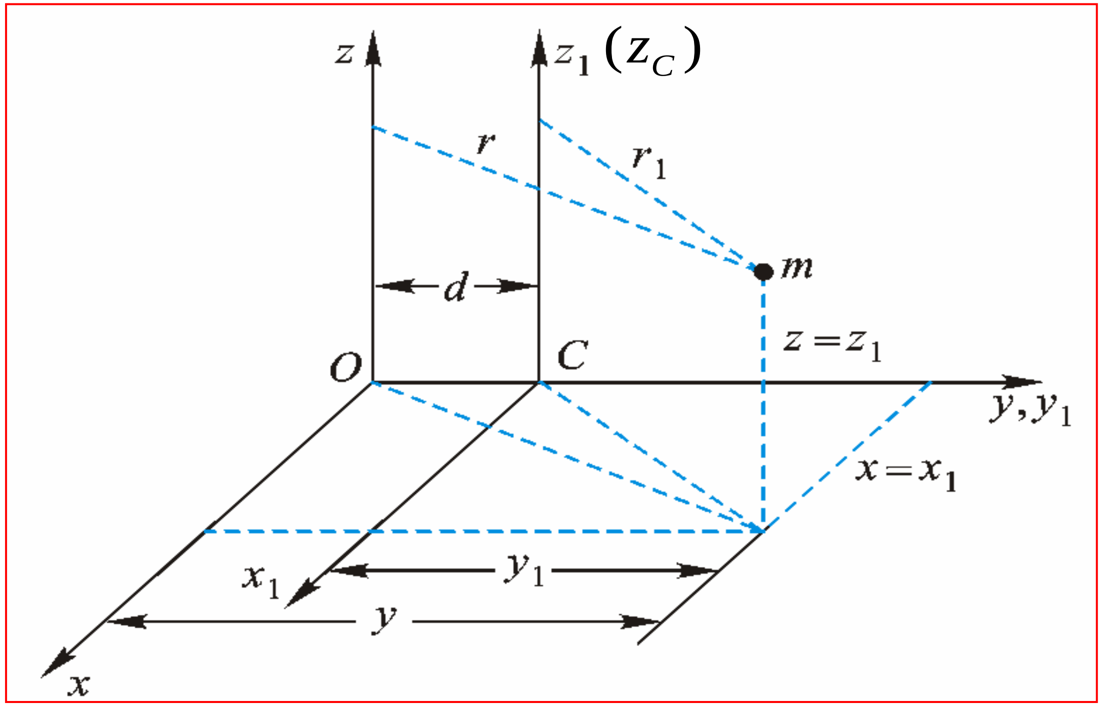
刚体对于任一轴的转动惯量，等于刚体对于通过质心并与该轴平行的轴的转动惯量，加上刚体的质量与两轴间距离平方的乘积.

证明： $$J_{z_C} = \sum m_i (x_1^2 + y_1^2)$$

$$J_z = \sum m_i r^2 = \sum m_i (x^2 + y^2) = \sum m_i [x_1^2 + (y_1 + d)^2]$$

$$= \sum m_i (x_1^2 + y_1^2) + 2d\boxed{0} + d^2 \sum m_i$$

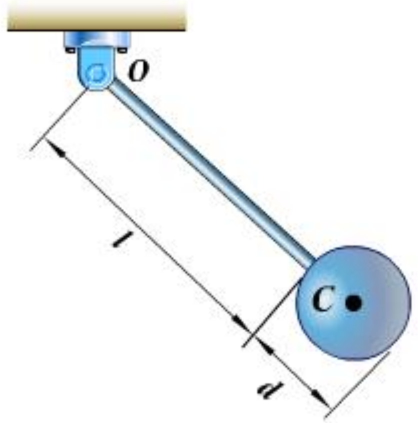
$$J_z = J_{z_C} + md^2$$



组合法

已知：杆长为 l 质量为 m_1，圆盘半径为 d，质量为 m_2 .

求：J_O .



解：

$$J_O = J_{O杆} + J_{O盘}$$

$$J_{O杆} = \frac{1}{3} m l^2$$

$$J_{O盘} = \frac{1}{2} m_2 (\frac{d}{2})^2 + m_2 (l + \frac{d}{2})^2$$

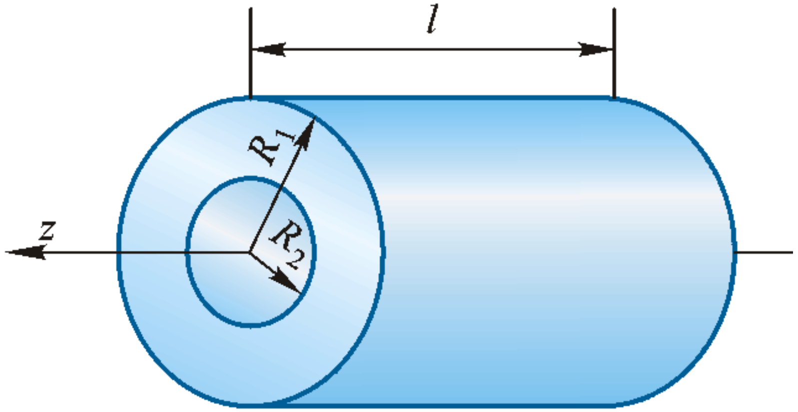
$$= m_2 (\frac{3}{8} d^2 + l^2 + ld)$$

$$J_O = \frac{1}{3} m_1 l^2 + m_2 (\frac{3}{8} d^2 + l^2 + ld)$$

已知：m, R_1, R_2。　　求：J_z.

解： $J_z = J_1 - J_2$

$$= \frac{1}{2}m_1 R_1^2 - \frac{1}{2}m_2 R_2^2$$



其中 $m_1 = \rho\pi R_1^2 l \quad m_2 = \rho\pi R_2^2 l$

$$J_z = \frac{1}{2}\rho\pi l(R_1^4 - R_2^4)$$

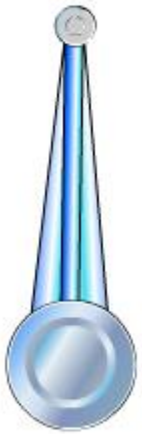
$$= \frac{1}{2}\rho\pi l(R_1^2 - R_2^2)(R_1^2 + R_2^2)$$

由 $\rho\pi l(R_1^2 - R_2^2) = m$，得

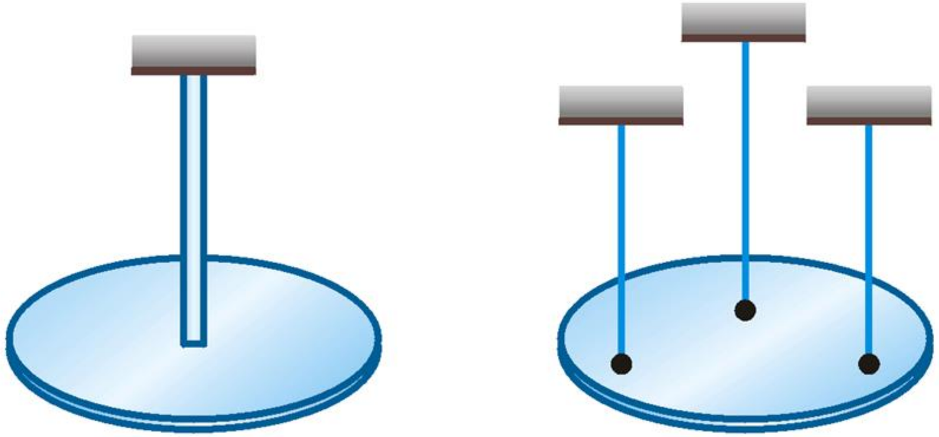
$$J_z = \frac{1}{2}m(R_1^2 + R_2^2)$$

实验法

- 摆振法 $$T=2\pi\sqrt{\frac{J}{mgl}}$$



- 扭振法



- 落体观察法