

$$3a) y'' + y' - 2y = 0$$

$$r^2 + r - 2 = 0$$

$$(r+2)(r-1) = 0$$

$$r_1 = -2, r_2 = 1$$

$$y = C_1 e^{-2x} + C_2 e^x$$

$$3i) y'' - 2y' + 2y = 0$$

$$r^2 - 2r + 2 = 0$$

$$r_{1,2} = \frac{2 \pm \sqrt{4 - 2 \cdot 4}}{2} = 1 \pm i$$

$$y = e^x (C_1 \sin x + C_2 \cos x)$$

$$4g) y'' + 2y' + y = 0, y(0) = 1, y'(0) = 0$$

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0, r_1 = r_2 = -1$$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$y' = -C_1 e^{-x} - C_2 x e^{-x} + C_2 e^{-x}$$

$$y(0) = 1 = C_1$$

$$y'(0) = -C_1 + C_2 = 0 \Rightarrow C_2 = 1$$

$$y = -e^{-x} - x e^{-x}$$

$$4i) y'' + 2y' + 5y = 0$$

$$y(0) = -1, y'(0) = 1$$

$$r^2 + 2r + 5 = 0$$

$$r_{1,2} = \frac{-2 \pm \sqrt{4 - 5 \cdot 4}}{2}$$

$$= -1 \pm 2i$$

$$y = C_1 e^x \sin 2x$$

$$+ C_2 e^x \cos 2x$$

$$y' = e^x [2C_1 \cos 2x - 2C_2 \sin 2x]$$

$$= e^x [C_1 \sin 2x + C_2 \cos 2x]$$

$$y(0) = C_2 = -1$$

$$y'(0) = 2C_1 - C_2 = 1 \Rightarrow C_1 = 0$$

$$y = -\cos 2x e^{-x}$$