HW1: (1b) $y' - 2y = t$

$$\text{I.F.} = e^{-2t}$$

$$e^{-2t}y' - 2e^{-2t}y = e^{-2t}t$$

$$(e^{-2t}y)' = e^{-2t}t$$

$$e^{-2t}y = \int e^{-2t}t \, dt$$

$$= -\frac{1}{2}te^{-2t} + \frac{1}{2}\int e^{-2t}dt$$

$$= -\frac{1}{2}te^{-2t} - \frac{1}{4}e^{-2t} + C$$

$$\Rightarrow y = -\frac{1}{2}t - \frac{1}{4} + Ce^{2t}$$

3a) $\frac{dq}{dr} + 2q = 4, \; q(0) = -1$

$$\text{I.F.} = e^{2r}$$

$$e^{2r}q' + 2q = 4e^{2r} \Rightarrow (e^{2r}q)' = 4e^{2r}$$

$$e^{2r}q = 4\int e^{2r}dr = 2e^{2r} + C$$

$$\Rightarrow q = Ce^{-2r} + 2$$

$$q(0) = C + 2 = -1 \Rightarrow C = -3, \; q(r) = -3e^{-2r} + 2$$

3b) $\frac{dp}{dr} + 4p = -r$, $p(0) = 0$

I.F. $= e^{4r}$ $\quad e^{4r}p' + e^{4r}p = -re^{4r}$

$(e^{4r}p)' = -e^{4r}r$

Integrate 0 to r $\Rightarrow$ $e^{4r}p(r) - e^{4\cdot 0}p(0) = -\int_0^r e^{4s} s\,ds$

$$= -\frac{1}{4}e^{4s}s\Big|_0^r + \frac{1}{4}\int_0^r e^{4s}\,ds$$

$$= -\frac{1}{4}re^{4r} + \frac{1}{16}e^{4r} - \frac{1}{16}$$

$$\Rightarrow p(r) = e^{-4r}p(0) - \frac{1}{4}r + \frac{1}{16} - \frac{1}{16}e^{-4r}$$

$$= \frac{1}{16} - \frac{1}{4}r - \frac{1}{16}e^{-4r}$$

HW2)

2.1 1b) $y' = y^3 e^{-t}$

① $y = 0$ is a solution

② $\frac{dy}{y^3} = e^{-t}dt \Rightarrow -\frac{1}{2y^2} = -e^{-t} + C$

$$y^2 = \frac{1}{2(e^{-t} - C)} \Rightarrow y = \pm\sqrt{\frac{1}{2(e^{-t} - C)}}$$

$$y = \pm\sqrt{\frac{1}{2(e^{-t} - C)}} \text{ or } y = 0$$

1h) $y' = -2^{y}$

$$\frac{dy}{2^y} = -dt \Rightarrow \frac{dy}{e^{\ln 2\, y}} = -dt$$

$$e^{-\ln 2\, y}\, dy = -dt \Rightarrow -\frac{1}{\ln 2} e^{-\ln 2 y} = -t + C$$

$$e^{-\ln 2 y} = \ln 2 (t - C)$$

$$-\ln 2\, y = \ln(\ln 2 (t - C))$$

$$y = -\frac{\ln(\ln 2 (t - C))}{\ln 2}$$

4a) $y' = 1 + \frac{1}{y},\ y(0) = 1$

$$\frac{dy}{dx} = \frac{1+y}{y}$$

$$\frac{y\,dy}{1+y} = dx \Rightarrow \left(1 - \frac{1}{1+y}\right) dy = dx$$

r

$$y - \ln|1+y| = x + C$$

$$y(0) = 1 \Rightarrow 1 - \ln 2 = C$$

$$y - \ln|1+y| = x - \ln 2 + 1$$

4d) $y' = \frac{e^y}{1+e^y},\ y(0) = 2$

$$\Rightarrow dy \frac{(1+e^y)}{e^y} = dx \qquad (1 + e^{-y})\, dy = dx$$

$$\Rightarrow y - e^{-y} = x + C,\ y(0) = 2 \Rightarrow C = 2 - e^{-2} \Rightarrow y - e^{-y} = x + 2 - e^{-2}$$

2e) $y' = te^{-y}$, $y(0) = -1$

$e^y\,dy = t\,dt$ $\quad$ $\cancel{e^y = \frac{1}{2}t^2}$

Integrate from 0 to t $\quad$ $e^y - e^{y(0)} = \frac{1}{2}t^2$

$\Rightarrow y = \ln(\frac{1}{2}t^2 + 1)$