1 Online course

The summary of the online course is attached to the end of this report.

2 Exercises

A Cone Sensitivity

To fascilitate the following calculations, interpolation is performed on the cone sensitivity data to obtain values per 0.5 nm. It is done by using the `interp1d` function from the `scipy.interpolate` module in Python with the `cubic` method, to make smooth curves.

The cone sensitivity $f_a(\lambda)$ for each type of cone $a \in \{L, M, S\}$ is shown in Fig. 1.

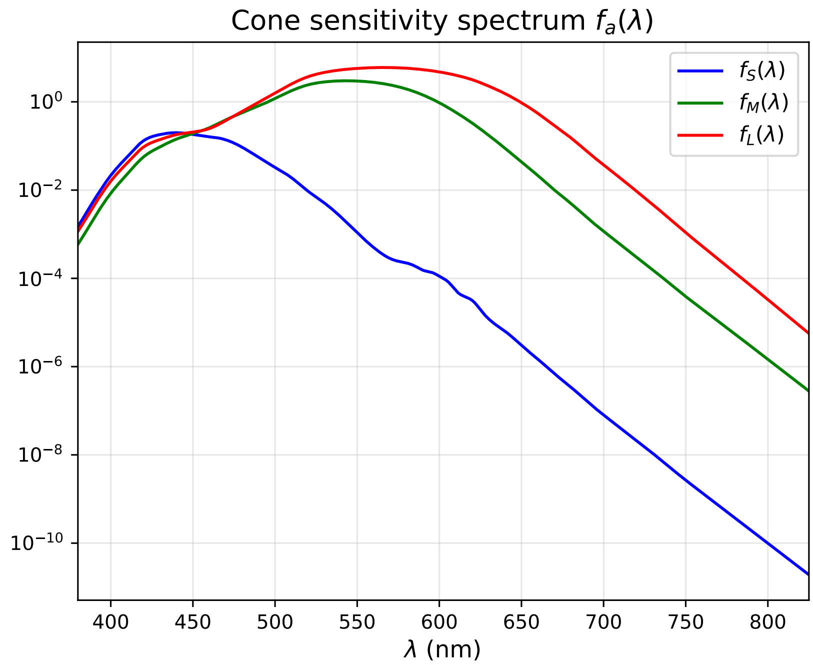

Fig. 1: Cone sensitivity spectrum $f_a(\lambda)$ for each type of cone $a \in \{L, M, S\}$.

B Likelihoods on a given input

Given a input $\mathbf{S}(\lambda, I)$ with $I = 100$ and $\lambda = 570$ nm, the likelihood $P(r_a|\mathbf{S}(\lambda, I))$ and the mean cone absorption rate $\overline{r} = If_a(\lambda)$ for each type of cone $a \in \{L, M, S\}$ is shown in Fig. 2.

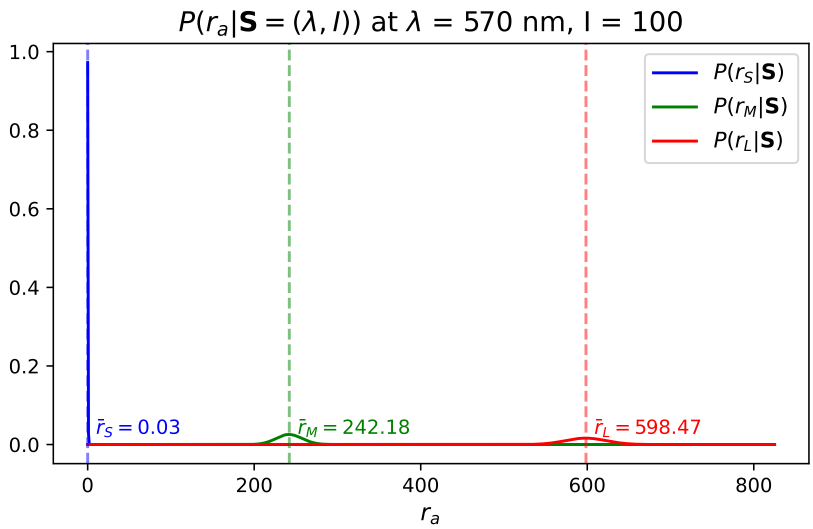

Fig. 2: Likelihood $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$ for each type of cone $a \in \{L, M, S\}$. The dashed lines indicate the average absorption rates $\bar{r}$.

C Samples and 3D Visualization

Here 500 samples are drawn from the Poisson distributions $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$ for each type of cone $a \in \{L, M, S\}$. The 3D scatter plot of the samples is shown in Fig. 3.

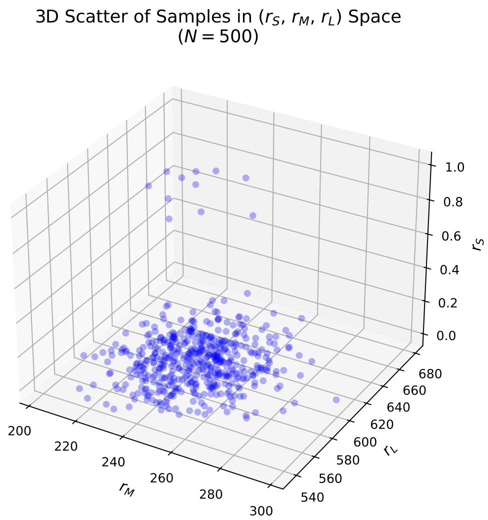

Fig. 3: 3D scatter plot of 1000 samples drawn from the Poisson distributions $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$ for each type of cone $a \in \{L, M, S\}$.

D Another input with different wavelength

Set another input $\mathbf{S}'(\lambda', I)$ with $\lambda' = \lambda + d\lambda$, where different values of $d\lambda$ are tried to find the approximate value that makes the samples drawn from $P(r_a|\mathbf{S}(\lambda, I))$ and

$P(r_a|\mathbf{S}'(\lambda', I))$ have about 30% overlap and 5% overlap in the 3D space.

After several trials, it is estimated (by naked eyes) that $d\lambda \approx 8$ nm gives about 30% overlap and $d\lambda \approx 12$ nm gives about 5% overlap. As is shown in Fig. 4.

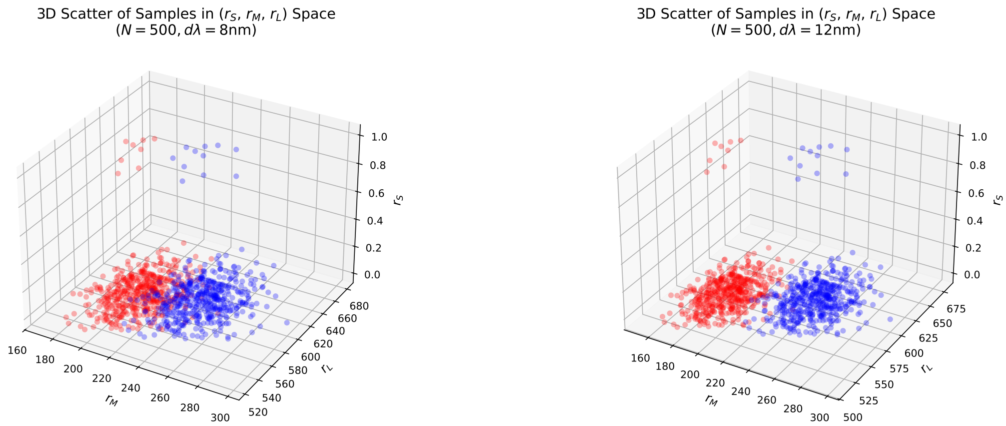

Fig. 4: 3D scatter plot of samples drawn from the Poisson distributions $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm (blue) and $P(r_a|\mathbf{S}'(\lambda', I))$ at $\lambda' = \lambda + d\lambda$ (red) for each type of cone $a \in \{L, M, S\}$. Left: $d\lambda \approx 8$ nm for about 30% overlaps. Right: $d\lambda \approx 12$ nm for about 5% overlaps.

E Decoding the wavelength from a single sample

Using the samples drawn from $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$, the log-likelihood $\log P(r_S, r_M, r_L|\mathbf{S} = (\hat{\lambda}, I))$ is computed for $\hat{\lambda}$ ranging from 400 nm to 700 nm. Here log-likelihood is used instead of likelihood to avoid numerical underflow issues, resulting from multiplying small probabilities. The value of $\hat{\lambda}$ that maximizes the log-likelihood is taken as the estimated wavelength.

The plot of the log-likelihood, with the estimated $\hat{\lambda}$ that maximizes it, is shown in Fig. 5.

Then the decoding error for this single sample can be calculated as $\delta\lambda = \hat{\lambda} - \lambda = 572.5 - 570 = 2.5$ nm.

F Histogram of Decoding Errors

Using multiple samples drawn before, the decoding error $\delta\lambda = \hat{\lambda} - \lambda$ is computed for each sample. The histogram of these decoding errors is shown in Fig. 6.

It can be seen that most of the decoding errors fall in the range of $|\delta\lambda| < 10$ nm, so the width of the histogram is about 10 nm, smilar to the $d\lambda = 12$ nm that gives about 5% overlap in the previous subsection.

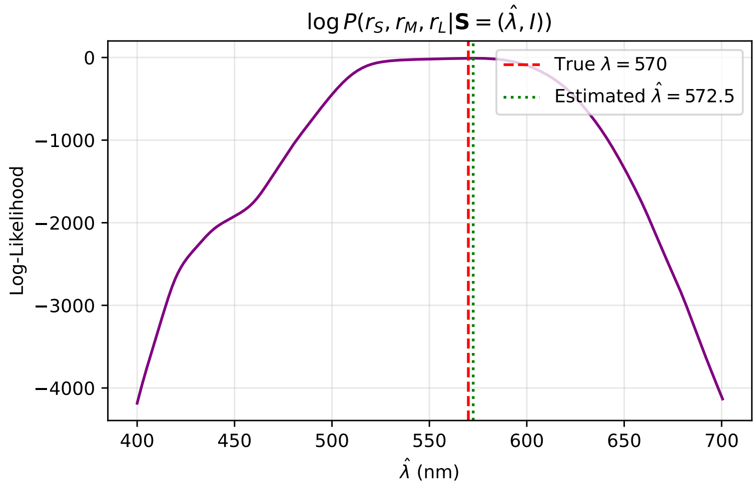

Fig. 5: Log-likelihood $\log P(r_S, r_M, r_L|\mathbf{S} = (\hat{\lambda}, I))$ computed from a single sample drawn from $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$, as a function of $\hat{\lambda}$. The red dashed line indicates the true wavelength $\lambda = 570$ nm, and the green dotted line indicates the estimated wavelength $\hat{\lambda}$ that maximizes the log-likelihood.

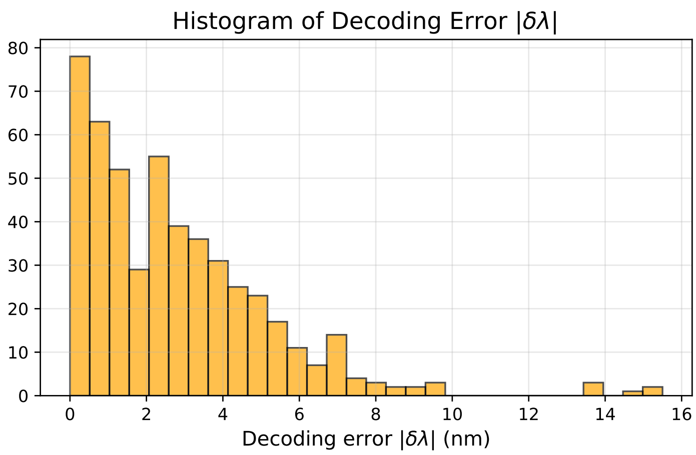

Fig. 6: Histogram of decoding errors $\delta\lambda = \hat{\lambda} - \lambda$ computed from multiple samples drawn from $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 570$ nm and $I = 100$.

G Decoding errors for a different input wavelength

Using another input $\mathbf{S}(\lambda, I)$ with $\lambda = 450$ nm and $I = 100$, the same procedure is performed to compute the decoding errors $\delta\lambda = \hat{\lambda} - \lambda$ for multiple samples drawn from $P(r_a|\mathbf{S}(\lambda, I))$. The histogram of these decoding errors is shown in Fig. 7.

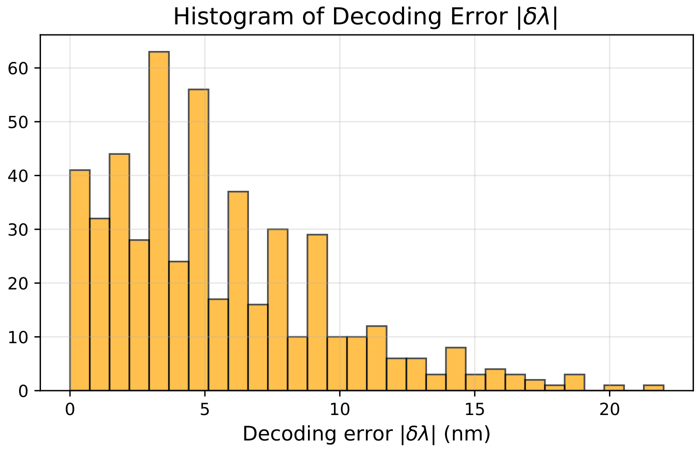

Fig. 7: Histogram of decoding errors $\delta\lambda = \hat{\lambda} - \lambda$ computed from multiple samples drawn from $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 450$ nm and $I = 100$.

Also, it is estimated (by naked eyes) that $d\lambda \approx 13$ nm gives about 30% overlap and $d\lambda \approx 18$ nm gives about 5% overlap. As is shown in Fig. 4.

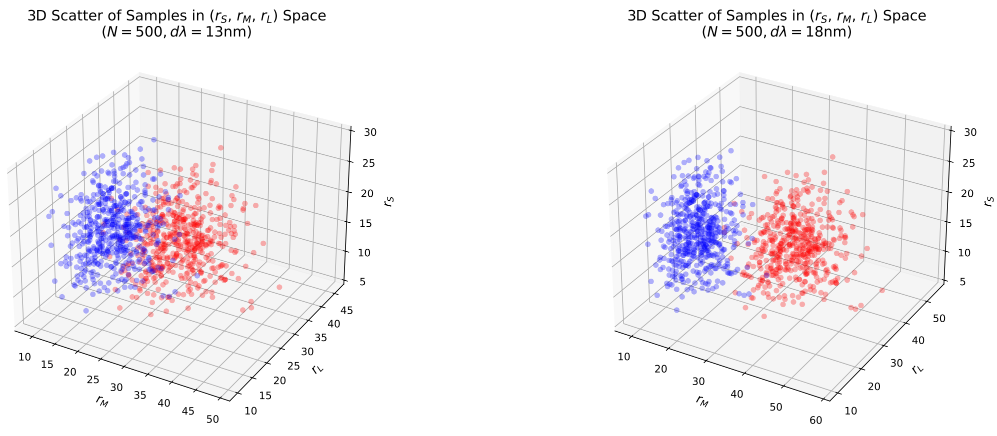

Fig. 8: 3D scatter plot of samples drawn from the Poisson distributions $P(r_a|\mathbf{S}(\lambda, I))$ at $\lambda = 450$ nm (blue) and $P(r_a|\mathbf{S}'(\lambda', I))$ at $\lambda' = \lambda + d\lambda$ (red) for each type of cone $a \in \{L, M, S\}$. Left: $d\lambda \approx 13$ nm for about 30% overlaps. Right: $d\lambda \approx 18$ nm for about 5% overlaps.

It can be seen that most of the decoding errors fall in the range of $|\delta\lambda| < 19$ nm, so the width of the histogram is about 19 nm, smilar to the $d\lambda = 18$ nm that gives about 5% overlap in the previous subsection.