$\log|\mathcal{S}(k)|$