Homework 1, Please submit your completed homework by a single pdf file

1. you must have completed the online videos and quizzes in the online course "A very brief introduction of what is known about vision experimentally". Please send me a record, such as a screen shot at the online course, showing your progress status to indicate either that you have already completed all the lessons, or which lessons you have completed.

2. Write a 2-5 page summary of the materials about "A very brief introduction of what is known about vision experimentally", try to add some of your own reflections on this material.

3. Practice exercises using some programming. Please do the following:

   - (A): Take a photo of your own choice, or use the one I gave you, plot out what it looks like.

   - (B): Filter this photo by the center-surround receptive fields of the retinal ganglion cells. You can start with the practice of the toy models of the center-surround receptive fields that we used in the tutorial. These toy models have receptive field's shape described by $K(x, y)$ as a function of horizontal and vertical displacement x and y from the center of the receptive field.

$$K(x,y) = \begin{cases} K(x,y) = 1 & \text{when}|x| < L/2, |y| < L/2, \\ & \text{i.e., } (x,y) \text{ is within a central square of side length L.} \\ K(x,y) = -v & \text{when}|x| \geqslant L/2, \quad \text{or} \quad |y| \geqslant L/2. \\ & \text{i.e., } (x,y) \text{ is outside this central square.} \end{cases} \tag{1}$$

     give a couple of examples using different parameters L and v. Plot out the responses of the ganglion cells as an image to show the outcomes.

   - (C): Now repeat (B) using a difference-of-gaussian $K(x, y)$ as

$$K(x,y) \quad = \quad \frac{w_c}{\sigma_c^2} \exp\left(-\frac{x^2+y^2}{2\sigma_c^2}\right) - \frac{w_s}{\sigma_s^2} \exp\left(-\frac{x^2+y^2}{2\sigma_s^2}\right), \tag{2}$$

     please play with parameters w_c, w_s, σ_c and σ_s and understand how $K(x, y)$ change with each these parameters, and thus the meaning of these parameters. Filter the original image using this $K(x, y)$ and see the outcome as the retinal ganglion cells population responses. For a retinal ganglion cell, you may try $w_c = 1.1w_s$ and $\sigma_s = 5\sigma_c$. See section 2.2.1 of the textbook if you want some review.

   - (D): Please repeat (C), but use a receptive field filter shape $K(x, y)$ that models a V1's simple cell's receptive field. Use an orientation selective neuron (tuning to vertical orientation), with

$$K(x,y) = \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \cos(\hat{k}x + \phi), \tag{3}$$

vary parameters σ_x, σ_y, $\hat{k}$, and ϕ and plot out $K(x,y)$ using different set of parameters to see how these parameters control the shape of $K(x,y)$. For a V1 cell model, try for example $\sigma_y = 1.5 \cdot \sigma_x$, and $\hat{k} = 2\pi/(3\sigma_x)$ (so that the receptive field contains roughly one or no more than two wavelength of the sinusoidal wave, and filter the original image using this $K(x,y)$ and see the outcome as the population responses of V1 neurons preferring this orientation.

- (E): please repeat (D), but using a $K(x,y)$ that is tuned to horizontal orientation, i.e.,

$$K(x,y) = \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)\cos(\hat{k}y + \phi), \tag{4}$$

  Plot out $K(x,y)$ from equations 3 and 4 side by side to compare and see they how differ from each other. Plot out the neural population responses from these two filters side by side and see how they differ from each other, and comment on these differences.

- (F): Using the retinal ganglion cell's $K(x,y)$ from equation 2 and try to get its contrast sensitivity function $g(k)$ by calculating

$$g_c(k) = \sum_{x,y} K(x,y)\cos(kx) \tag{5}$$

  and

$$g_s(k) = \sum_{x,y} K(x,y)\sin(kx). \tag{6}$$

  However, in order to avoid calculation artifacts caused by boundary conditions, it is important that, for getting the $g_c(k)$ and $g_s(k)$ above, use $K'(x,y)$ function instead of $K(x,y)$ function defined as follows. If your $K(x,y)$ function have $x_{min} \leqslant x \leqslant x_{max}$, and $L \equiv x_{max} - x_{min}$, then your $K'(x,y)$ should have its x range doubled, such that $x_{min} \leqslant x \leqslant x_{max} + L$, so that

$$K'(x,y) = \begin{cases} K(x,y), & \text{when } x \leqslant x_{max}, \\ K(x',y), & \text{when } x > x_{max}, \text{ here } x' = x_{max} - (x - x_{max}) \end{cases} \tag{7}$$

  You can see that $K'(x,y)$ is like doubling $K(x,y)$ in the range of x and its value in the second range of x is like a mirror image of its values in the first range. Hence, you actually get

$$g_c(k) = \sum_{x_{min} \leqslant x \leqslant x_{max}+L,\, y} K'(x,y)\cos(kx) \tag{8}$$

  and

$$g_s(k) = \sum_{x_{min} \leqslant x \leqslant x_{max}+L,\, y} K'(x,y)\sin(kx). \tag{9}$$

  Then you get from $g_c(k)$ and $g_s(k)$

$$g(k) = \sqrt{[g_s(k)]^2 + [g_c(k)]^2} \tag{10}$$

for all kinds of values of k, so that you get $g(k)$ as a function of k, and plot out $g(k)$ versus k. Do you see that $g(k)$ peaks as a particular k? Please see section 2.2.2 of the textbook if you want more information.

Important step to avoid artifacts: In the calculation for $g(k)$, you need to choose k values that are suitable and not cause any artifacts. With your original image of length L pixels, you have now expanded it to $2L$ pixels. Your k values should take $(L+1)$ discrete values

$$k = n \cdot 2\pi/(2L), \text{using } (L+1) \text{ integer values } n = 0, 1, 2, ..., L \tag{11}$$

This k will then be in the units of radian/pixel, so that kx in your $\cos(kx)$ and $\sin(kx)$ will have the unit of radian for kx. Using these units for k, you can also compare different function $g(k)$ functions of k from different images on the same plot of $g(k)$ versus k, so that you can compare them with each other by the same k units in $radian/pixel$.

- (G): repeat (F) with V1's receptive field, i.e., use $K(x, y)$ in equation 3, and get $g(k)$. Which k value is this $g(k)$ peaking at? Is it around $k = \hat{k}$? Please compare this $g(k)$ with the one you have in (F), and see how they differ from each other.
- (H): repeat (G) by replacing $K(x, y)$ by your own original image/photo, note that for most photos, the artifact removal step in equation 7 is quite important. Let us denote your original image as $S(x, y)$, as a function of x and y, after you include the artifact removal step in equation 7, you get $S'(x, y)$, which has its x range doubled, then you calculate

$$\mathcal{S}_c(k) = \sum_{x \text{ in double range}, y} S'(x, y) \cos(kx). \tag{12}$$

and

$$\mathcal{S}_s(k) = \sum_{x \text{ in double range}, y} S'(x, y) \sin(kx). \tag{13}$$

From $\mathcal{S}_c(k)$ and $\mathcal{S}_s(k)$, calculate

$$|\mathcal{S}(k)|^2 = [\mathcal{S}_c(k)]^2 + [\mathcal{S}_s(k)]^2. \tag{14}$$

This $|\mathcal{S}(k)|^2$ as a function of k is called the power spectrum of the image. Plot out this function, and note a general trend of how $|\mathcal{S}(k)|^2$ changes with k. To see this trend clearly, it is better that you do this with multiple photos (make sure that your photos have the same size in terms of number of pixels horizontally and vertically, in order to avoid artifacts related to the mention by equation 11), and then average your $|\mathcal{S}(k)|^2$ over these photos. In anticipation of what we will learn from chapter 3 of the textbook, you can compare your $|\mathcal{S}(k)|^2$ with Figure 3.18C of the textbook "Understanding vision: theory, models, and data".