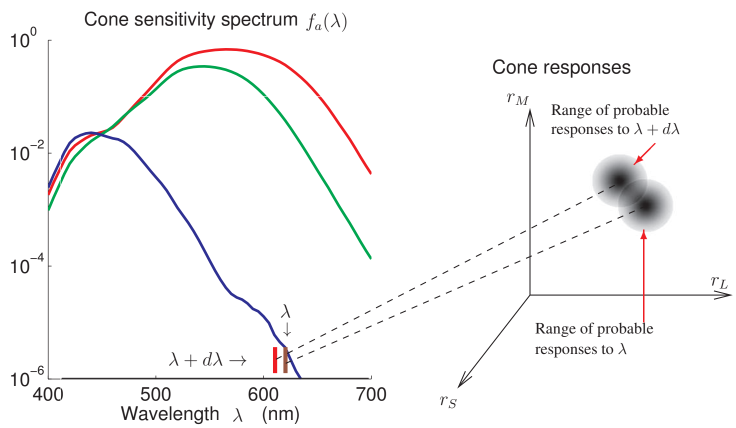

Figure 1: This is from Fig. 6.13 in the textbook. Schematic of input sampling and the likelihoods of photoreceptor responses to monochromatic input. The left-hand graph shows the cone spectral sensitivities $f_a(\lambda)$, where, for cone $a = S, M, L$ (i.e., cones tuned to short, medium, and long wavelength, roughly, blue, green, red cones), after approximately taking into account the effects of cone densities and pre-receptor light transmision factor, with a normalization of $\max_\lambda \sum_a f_a(\lambda) = 1$. A monochromatic input $\mathbf{S} = (\lambda, I)$ of wavelength λ and intensity I can evoke a range of probable responses $\mathbf{r} = (r_L, r_M, r_S)$ from the three cones, with a likelihood $P(\mathbf{r}|\mathbf{S} = (\lambda, I))$. If this range of probable responses overlaps substantially with that evoked by another monochromatic input $\mathbf{S}' = (\lambda + d\lambda, I')$ of a similar wavelength, it will be difficult to distinguish between $\mathbf{S}$ and $\mathbf{S}'$ perceptually. Adapted from Zhaoping, L., Geisler, W. S., and May, K. A., Human wavelength discrimination of monochromatic light explained by optimal wavelength decoding of light of unknown intensity. *PLoS One*, 6(5): e19248, Fig. 1,

Homework 4, Please submit your completed homework by a single pdf file

1. you must have completed the online videos and quizzes in the online course "Visual decoding or recognition" Please send me a record, such as a screen shot at the online course, to indicate either that you have already completed all the lessons, or which lessons you have completed.

2. This is to make up a missing component in homework 2: please go to online course for efficient coding https://zhaoping.thinkific.com/courses/the-efficient-coding-principle and complete chapter 7: "Efficient coding by the receptive fields of the retinal ganglion cells". Give an indication of your completion of this part by a screen shot or similar.

3. Practice exercises for consolidation of the learning on decoding. It will be from a modification of the figure 6.13 in the textbook, as shown in Figure 1.

   - A. Copy $f_a(\lambda)$ from the data file

     https://www.lizhaoping.org/zhaoping/Dir_ForExerciseDataFiles_forVisionCourses/ConeSensitivity_Function_ForExercise2024.xls

     Plot out $f_a(\lambda)$ versus λ for each a on a single plot, using red, green, blue curves to denote different cones.

- B. For a given input $\mathbf{S} = (\lambda, I)$ that is a monochromatic light at wavelength λ at intensity I, the mean cone absorption is

$$\bar{r}_a = I f_a(\lambda). \tag{1}$$

However, the probability distribution of r is Cone absorption is stochastic, following a Poisson distribution. Hence, the likelihoods are

$$P(r_a|\mathbf{S} = (\lambda, I)) = \frac{(\bar{r}_a)^{r_a}}{r_a!}\exp(-\bar{r}_a) = \frac{[If_a(\lambda)]^{r_a}}{r_a!}\exp[-If_a(\lambda)]\,. \tag{2}$$

Now, let $I = 100$, $\lambda = 570$ nm, in $\mathbf{S} = (\lambda, I)$, plot $P(r_a|\mathbf{S} = (\lambda, I))$ vs r_a for each a, and mark the mean value $\bar{r}_a$ in each $P(r_a|\mathbf{S} = (\lambda, I))$. Note that each r_a sample is an integer.

- C. Given the visual input above, give a 3-D plot of random samples of (r_S, r_M, r_L) (from the distributions $P(r_a|\mathbf{S} = (\lambda, I))$), each as a three dimensional vector, so this will be a plot in 3D space. Try to get at least 100 samples of (r_S, r_M, r_L) to get a clear picture.

- D. Now try to find another input $\mathbf{S}' = (\lambda', I)$, with $\lambda' = \lambda + d\lambda$ (for positive $d\lambda$), with the same I, such that the distribution of responses (r_S, r_M, r_L) in the 3D space overlaps with that for input $\mathbf{S} = (\lambda I)$ by about 30%. Try to plot the two sample distributions in a 3D plot, such that the response samples (r_S, r_M, r_L) from the two inputs are marked by different colors, so that you can clearly visualize the overlap of the two distributions of the cone responses.

- E. What is $d\lambda$ value you find to satisfy such a 30% overlap?

- F. What is $d\lambda$ value needed to get an overlap of about 5% overlap?

- G. Pick one of your sample cone responses (r_S, r_M, r_L) to input $\mathbf{S} = (\lambda I)$ above, try to decode to get $\hat{\lambda}$ (assuming you already know I's value (so that you are only decoding for λ and not for I), such that the likelihood

$$P(r_S, r_M, r_L|\mathbf{S} = (\hat{\lambda}, I)) = P(r_S|\mathbf{S} = (\hat{\lambda}, I)) \cdot P(r_M|\mathbf{S} = (\hat{\lambda}, I)) \cdot P(r_L|\mathbf{S} = (\hat{\lambda}, I)) \tag{3}$$

is maximized. You can do this by plotting $P(r_S, r_M, r_L|\mathbf{S} = (\hat{\lambda}, I))$ as a function of $\hat{\lambda}$ (you can scan $\hat{\lambda}$ from 400 to 700 ms in 0.5 nm steps to find this peak), so that you find the peak of this function as the value for $\hat{\lambda}$.

You should give a decoding error $\delta\lambda = \hat{\lambda} - \lambda$.

- H. Repeat G for all your cone response samples (r_S, r_M, r_L) you used for B above, so that you get a collection of decoding errors $\delta\lambda$. Plot a histogram of $\delta\lambda$, and see whether the distribution of $\delta\lambda$ has a width similar to $d\lambda$ in F.

- I. Repeat B -H above, for another $\lambda = 450$. What is $d\lambda$, and plot the distribution of $\delta\lambda$.