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Comment

Comment on "A simpler proof of regular polygon solutions of the N body problem" by Zhifu Xie and Shiqing Zhang [Phys. Lett. A 277 (2000) 156–158] ☆

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In the above mentioned paper, the authors present what they claim to be a very short proof of a classical result of Perko–Walter [1] and Elmabsout [2] which states that, for any $n \geqslant 4$, the regular n-gon is a central configuration if and only if all masses are equal. Unfortunately, their claim is unfounded.

More precisely, they set rightly—and even better than Perko and Walter who write them in a somewhat awkward fashion—the equations of the problem, but they make the following mistake: the spectral decomposition of the circulant matrix C is correctly stated, but it is assumed by the authors that the only eigenvector with 0 eigenvalue is $X_1 = (1, 1, \ldots, 1)$. This is simply false: the kernel of C is generated by X_1 and $X_2 = (z, z^2, \ldots, z^n)$ (where $z = \exp(2\pi i/n)$) if n is even and by X_1, X_2 and $X_3 = (z^{(p+1)}, z^{2(p+1)}, \ldots, z^{n(p+1)})$ if $n = 2p + 1$ is odd. These assertions are equivalent to Lemmas 10–12 of Perko and Walter's paper. The end of the proof can be obtained in the following way (compare to Lemmas 6 and 7 of Perko and Walter's paper):

- When $n > 2$ is even, the complex subspace of $\mathbb{C}^n$ generated by X_1 and X_2 contains no *real* vectors other than the multiples of $(1, 1, \ldots, 1)$ (if $n = 2$, these 2 vectors generate $\mathbb{C}^2$).
- When $n > 3$ is odd, the complex subspace of $\mathbb{C}^n$ generated by X_1, X_2 and X_3 contains no *real* vectors other than the multiples of $(1, 1, \ldots, 1)$ (if $n = 3$, these 3 vectors generate $\mathbb{C}^3$).

To summarize, the authors rightly notice that if $n > 4$ the matrix C is not identically zero but they do not touch to the main problem which is controlling the *real* vectors in its kernel.

Nota bene If true, the proposed "proof" would have implied that n-choreographies must have equal masses. At the moment, this is proved only if $n \leqslant 5$ [3].

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