Untitled

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1 Geometric Brownian Motion

$$S(t) = S(0)e^{(\mu - \frac{\sigma 2}{2} + \sigma W(t))} \tag{1}$$

Thus, we can say

$$S(1) = S(0)e^{(\mu - \frac{\sigma 2}{2} + \sigma W(1))}$$
$$S(2) = S(0)e^{(\mu - \frac{\sigma 2}{2} + \sigma W(2))}$$

If I have two spending at time 0 and time 1, say d_0 and d_1, then the future value at time 2 of these two cash flows are:

$$\begin{aligned} d_0(2) &= d_0 * e^{(\mu - \frac{\sigma 2}{2} + \sigma W(2))} \\ d_1(2) &= d_1 * \frac{S(2)}{S(1)} \\ &= d_1 * e^{(\sigma W(2) - \sigma W(1))} \\ &= d_1 * e^{\sigma * N(0,1)} \end{aligned}$$

The last step is given by the brownian motion property: $W(t) - W(s) = N(0, t-s)$

Amazingly, $d_1(2) \neq d_1 e^{(\mu - \frac{\sigma 2}{2} + \sigma W(1))}$ Double check $d_0(2)$:

$$\begin{aligned} d_0(2) &= d_0 * \frac{S(1)}{S(0)} * \frac{S(2)}{S(1)} \\ &= d_0 * e^{(\mu - \frac{\sigma 2}{2} + \sigma W(1))} * e^{(\sigma W(2) - \sigma W(1))} \\ &= d_0 * e^{(\mu - \frac{\sigma 2}{2} + \sigma W(2))} \end{aligned}$$

It always has the term $\mu - \frac{\sigma^2}{2}$. However, this term is missing for later cashflows.

If the above calculation is correct, then the way we generate random returns and the way we calculate future values are wrong.

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In [ ]:
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