# Competition on Constrained Real Parameter Optimization

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# The 28 benchmarked constrained optimization problems can be transformed into the following format:

Minimize: 
$$f(X)$$
,  $X = (x_1, x_2, ..., x_n)$  and  $X \in S$  (1)

Subject to: 
$$g_i(X) \le 0, \qquad i = 1,..., p$$
  
 $h_i(X) = 0, \qquad j = p + 1,..., m$  (2)

Equality constraints are transformed into inequalities of the form

$$|h_j(X)| - \varepsilon \le 0$$
, for  $j = p + 1, ..., m$  (3)

Solution X is regarded as feasible if  $g_i(X) \le 0, i = 1,..., p$  and

$$|h_j(X)| - \varepsilon \le 0$$
, for  $j = p + 1, ..., m$  ( $\varepsilon = 0.0001$ ).

#### Reference:

Guohua Wu, R. Mallipeddi, P. N. Suganthan, "Problem Definitions and Evaluation Criteria for the CEC 2017 Competition and Special Session on Constrained Single Objective Real-Parameter Optimization", Technical Report, Nanyang Technological University, Singapore, December 2016.

#### Problem Properties

| Problem/Search Range | Type of Objective      | Numbe         | er of Constraints      |
|----------------------|------------------------|---------------|------------------------|
| Troblem/Search Range | Type of Objective      | E             | I                      |
| C01                  | Non Separable          | 0             | 1                      |
| $[-100,100]^D$       | •                      |               | Separable              |
| C02                  | Non Separable, Rotated | 0             | 1                      |
| $[-100,100]^D$       | <b>1</b>               |               | Non Separable, Rotated |
| C03                  | Non Separable          | 1             | 1                      |
| $[-100,100]^D$       | rion separasie         | Separable     | Separable              |
| C04                  | Separable              | 0             | 2                      |
| $[-10,10]^D$         | Separasie              |               | Separable              |
| C05                  | Non Separable          | 0             | 2                      |
| $[-10,10]^D$         | 11011 Separasie        | V             | Non Separable, Rotated |
| C06                  | Separable              | 6             | 0                      |
| $[-20,20]^D$         | Separasie              | , v           | Separable              |
| C07                  | Separable              | 2             | 0                      |
| $[-50,50]^{D}$       | Separable              | Separable     | v                      |
| C08                  | Separable              | 2             | 0                      |
| $[-100,100]^D$       | oepui unic             | Non Separable | V                      |
| C09                  | Separable              | 2             | 0                      |
| $[-10,10]^{D}$       | Separable              | Non Separable | V                      |

#### Problem Properties

| Problem/Search Range | Type of Objective | Number        | of Constraints |
|----------------------|-------------------|---------------|----------------|
| Troblem/Scaren Range | Type of Objective | E             | I              |
| C10                  | Separable         | 2             | 0              |
| $[-100,100]^D$       | separasie         | Non Separable | v              |
| C11                  | Separable         | 1             | 1              |
| $[-100,100]^D$       | Separation        | Non Separable | Non Separable  |
| C12                  | Separable         | 0             | 2              |
| $[-100,100]^D$       |                   |               | Separable      |
| C13                  | Non Separable     | 0             | 3              |
| $[-100,100]^D$       |                   |               | Separable      |
| C14                  | Non Separable     | 1             | 1              |
| $[-100,100]^D$       | •                 | Separable     | Separable      |
| C15                  | Separable         | 1             | 1              |
| $[-100,100]^D$       |                   |               |                |
| C16                  | Separable         | 1             | 1              |
| $[-100,100]^D$       | 1                 | Non Separable | Separable      |
| C17                  | Non Separable     | 1             | 1              |
| $[-100,100]^D$       |                   | Non Separable | Separable      |
| C18                  | Separable         | 1             | 2              |
| $[-100,100]^D$       | ~ - F             |               | Non Separable  |

#### Problem Properties

| Problem/Search Range  | Type of Objective |         | <b>Number of Constraints</b> |
|-----------------------|-------------------|---------|------------------------------|
| 1 Toblem/Search Kange | Type of Objective | E       | I                            |
| C19                   | Separable         | 0       | 2                            |
| $[-50,50]^D$          | Беригине          | v       | Non Separable                |
| C20                   | Non Separable     | 0       | 2                            |
| $[-100,100]^D$        |                   |         |                              |
| C21                   | Rotated           | 0       | 2                            |
| $[-100,100]^D$        |                   |         | Rotated                      |
| C22                   | Rotated           | 0       | 3                            |
| $[-100,100]^D$        |                   |         | Rotated                      |
| C23                   | Rotated           | 1       | 1                            |
| $[-100,100]^D$        |                   | Rotated | Rotated                      |
| C24                   | Rotated           | 1       | 1                            |
| $[-100,100]^D$        |                   | Rotated | Rotated                      |
| C25                   | Rotated           | 1       | 1                            |
| $[-100,100]^D$        |                   | Rotated | Rotated                      |
| C26                   | Rotated           | 1       | 1                            |
| $[-100,100]^D$        |                   | Rotated | Rotated                      |
| C27                   | Rotated           | 1       | 2                            |
| $[-100,100]^D$        |                   | Rotated | Rotated                      |
| C28                   | Rotated           | 0       | 2                            |
| $[-50,50]^D$          |                   |         | Rotated                      |

#### **Evaluation Criteria**

#### (1) Rank all algorithms on one problem with multiple runs

- The procedure for ranking algorithms based on mean values:
  - ➤ 1 Rank the algorithms based on feasibility rate;
  - >2 Then rank the algorithms according to the mean violation amounts;
  - ➤ ③ At last, rank the algorithms in terms of mean objective function value.
- The procedure for ranking the algorithms based on the median solutions:
  - 1 A feasible solution is better than an infeasible solution;
  - 2 Rank feasible solutions based on their objective function values;
  - 3 Rank infeasible solutions according to their constraint violation amounts.

#### **Evaluation Criteria**

#### (2) Rank all algorithms on multiple problems

For each problem, algorithm ranks are determined in terms of the mean values and median solutions at the maximum allowed number of evaluations, respectively. The total rank value of each algorithm is calculated as below.

Rank value = 
$$\sum_{i=1}^{28} rank_i$$
 (using mean value) +  $\sum_{i=1}^{28} rank_i$  (using median solution)

# Algorithms

| CAL-SHADE      | Ales Zamuda. Adaptive Constraint Handling and Success History Differential Evolution for CEC 2017 Constrained Real-Parameter Optimization, CEC 2017               |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| LSHADE44 + IDE | Josef Tvrdik and Radka Polakova. A Simple Framework for Constrained Problems with Application of L-SHADE44 and IDE, CEC 2017                                      |
| LSHADE44       | Radka Polakova. L-SHADE with Competing Strategies Applied to Constrained Optimization, CEC 2017                                                                   |
| UDE            | Anupam Trivedi, Krishnendu Sanyal, Pranjal Verma and Dipti Srinivasan. A Unified Differential Evolution Algorithm for Constrained Optimization Problems, CEC 2017 |

Table 1 Ranks based on mean values on the 28 functions of 10 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |    |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| CAL-<br>SHADE     | 3 | 3 | 4 | 4 | 3 | 2 | 4 | 4 | 3 | 3  | 3  | 3  | 3  | 1  | 4  | 4  | 4  | 4  | 1  | 1  | 1  | 3  | 2  | 4  | 4  | 4  | 4  | 1  | 84 |
| LSHADE44<br>+ IDE | 4 | 4 | 3 | 1 | 1 | 4 | 2 | 2 | 2 | 2  | 2  | 2  | 1  | 4  | 1  | 2  | 2  | 1  | 3  | 3  | 3  | 2  | 3  | 2  | 3  | 2  | 1  | 3  | 65 |
| LSHADE44          | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 1  | 1  | 4  | 2  | 3  | 3  | 3  | 1  | 2  | 2  | 2  | 4  | 1  | 4  | 3  | 2  | 1  | 2  | 4  | 61 |
| UDE               | 2 | 2 | 1 | 3 | 4 | 1 | 1 | 3 | 4 | 4  | 4  | 1  | 4  | 2  | 2  | 1  | 3  | 3  | 4  | 4  | 2  | 4  | 1  | 1  | 1  | 3  | 3  | 2  | 70 |

Table 2 Ranks based on median solution on the 28 functions of 10 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | 27 | 28 |    |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|----|----|----|
| CAL-<br>SHADE     | 1 | 1 | 4 | 4 | 1 | 2 | 2 | 4 | 4 | 1  | 2  | 2  | 1  | 2  | 3  | 4  | 4  | 4  | 1  | 3  | 3  | 1  | 3  | 4  | 4  | 4         | 4  | 1  | 74 |
| LSHADE44<br>+ IDE | 1 | 1 | 3 | 1 | 1 | 4 | 3 | 1 | 1 | 2  | 1  | 3  | 1  | 4  | 2  | 3  | 1  | 3  | 3  | 2  | 4  | 1  | 4  | 3  | 3  | 2         | 3  | 2  | 63 |
| LSHADE44          | 1 | 1 | 2 | 2 | 1 | 3 | 4 | 1 | 1 | 2  | 2  | 4  | 1  | 3  | 4  | 2  | 2  | 1  | 2  | 1  | 1  | 1  | 2  | 2  | 2  | 1         | 2  | 4  | 55 |
| UDE               | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 2  | 4  | 1  | 1  | 1  | 1  | 1  | 3  | 2  | 4  | 4  | 1  | 1  | 1  | 1  | 1  | 3         | 1  | 3  | 49 |

Table 3 Ranks of the four methods on problems of 10 dimensions based on mean value and median solution

|                   | CAL-SHADE | LSHADE44 + IDE | LSHADE44 | UDE |
|-------------------|-----------|----------------|----------|-----|
| Total rank values | 158       | 128            | 116      | 119 |
| Rank              | 4         | 3              | 1        | 2   |

Table 4 Ranks based on mean values on the 28 functions of 30 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | 27 | 28 |    |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|----|----|----|
| CAL-<br>SHADE     | 1 | 1 | 4 | 4 | 4 | 2 | 4 | 1 | 4 | 1  | 3  | 3  | 4  | 1  | 4  | 4  | 4  | 4  | 1  | 2  | 1  | 4  | 4  | 4  | 4  | 4         | 3  | 1  | 81 |
| LSHADE44<br>+ IDE | 1 | 1 | 3 | 2 | 1 | 4 | 3 | 4 | 1 | 4  | 2  | 2  | 2  | 4  | 2  | 2  | 2  | 3  | 3  | 3  | 4  | 2  | 3  | 3  | 2  | 2         | 4  | 3  | 72 |
| LSHADE44          | 1 | 1 | 2 | 1 | 1 | 3 | 2 | 2 | 1 | 2  | 1  | 1  | 1  | 3  | 3  | 3  | 1  | 1  | 2  | 1  | 3  | 3  | 2  | 2  | 3  | 1         | 2  | 4  | 53 |
| UDE               | 1 | 1 | 1 | 3 | 3 | 1 | 1 | 3 | 3 | 3  | 4  | 4  | 3  | 2  | 1  | 1  | 3  | 2  | 4  | 4  | 2  | 1  | 1  | 1  | 1  | 3         | 1  | 2  | 60 |

Table 5 Ranks based on median solution on the 28 functions of 30 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | 27 | 28 |           |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|----|----|-----------|
| CAL-<br>SHADE     | 1 | 1 | 4 | 4 | 1 | 2 | 4 | 1 | 1 | 1  | 3  | 3  | 4  | 2  | 4  | 4  | 4  | 3  | 1  | 2  | 1  | 4  | 2  | 4  | 4  | 4         | 2  | 1  | 72        |
| LSHADE44<br>+ IDE | 1 | 1 | 3 | 1 | 1 | 4 | 3 | 4 | 2 | 4  | 2  | 1  | 1  | 4  | 2  | 2  | 2  | 4  | 3  | 3  | 4  | 2  | 4  | 3  | 2  | 1         | 4  | 3  | 71        |
| LSHADE44          | 1 | 1 | 2 | 2 | 1 | 3 | 2 | 2 | 2 | 2  | 1  | 2  | 3  | 3  | 3  | 3  | 1  | 1  | 2  | 1  | 3  | 3  | 3  | 2  | 3  | 2         | 3  | 4  | 61        |
| UDE               | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 3 | 4 | 3  | 4  | 4  | 2  | 1  | 1  | 1  | 3  | 2  | 4  | 4  | 1  | 1  | 1  | 1  | 1  | 3         | 1  | 2  | <b>56</b> |

Table 6 Ranks of the four methods on problems of 30 dimensions based on mean value and median solution

|                   | CAL-SHADE | LSHADE44 + IDE | LSHADE44 | UDE |
|-------------------|-----------|----------------|----------|-----|
| Total rank values | 153       | 143            | 114      | 116 |
| Rank              | 4         | 3              | 1        | 2   |

#### Table 7 Ranks based on mean values on the 28 functions of 50 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | 27 | 28 |    |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|----|----|----|
| CAL-<br>SHADE     | 4 | 4 | 4 | 4 | 3 | 2 | 4 | 4 | 4 | 2  | 3  | 3  | 4  | 1  | 4  | 4  | 4  | 3  | 1  | 2  | 2  | 4  | 4  | 4  | 4  | 4         | 2  | 1  | 89 |
| LSHADE44<br>+ IDE | 3 | 1 | 3 | 2 | 1 | 4 | 3 | 3 | 3 | 4  | 1  | 1  | 2  | 4  | 2  | 2  | 2  | 2  | 3  | 3  | 3  | 1  | 3  | 3  | 3  | 2         | 3  | 4  | 71 |
| LSHADE44          | 1 | 1 | 2 | 1 | 1 | 3 | 2 | 1 | 2 | 1  | 2  | 4  | 1  | 3  | 3  | 3  | 1  | 1  | 2  | 1  | 4  | 2  | 2  | 2  | 2  | 1         | 1  | 3  | 53 |
| UDE               | 1 | 1 | 1 | 3 | 4 | 1 | 1 | 2 | 1 | 3  | 4  | 2  | 3  | 2  | 1  | 1  | 3  | 4  | 4  | 4  | 1  | 3  | 1  | 1  | 1  | 3         | 4  | 2  | 62 |

Table 8 Ranks based on median solution on the 28 functions of 50 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | 27 | 28 |           |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|----|----|-----------|
| CAL-<br>SHADE     | 1 | 1 | 3 | 4 | 1 | 2 | 4 | 4 | 3 | 2  | 3  | 3  | 4  | 1  | 4  | 4  | 4  | 4  | 1  | 2  | 2  | 4  | 2  | 4  | 4  | 4         | 2  | 1  | <b>78</b> |
| LSHADE44<br>+ IDE | 1 | 1 | 4 | 1 | 1 | 4 | 3 | 3 | 4 | 4  | 2  | 1  | 2  | 4  | 2  | 3  | 2  | 3  | 3  | 3  | 3  | 2  | 4  | 3  | 3  | 2         | 4  | 4  | <b>76</b> |
| LSHADE44          | 1 | 1 | 2 | 2 | 1 | 3 | 2 | 1 | 2 | 1  | 1  | 4  | 1  | 3  | 3  | 2  | 1  | 1  | 2  | 1  | 4  | 3  | 3  | 2  | 2  | 1         | 1  | 3  | 54        |
| UDE               | 1 | 1 | 1 | 3 | 4 | 1 | 1 | 2 | 1 | 3  | 4  | 2  | 3  | 2  | 1  | 1  | 3  | 2  | 4  | 4  | 1  | 1  | 1  | 1  | 1  | 3         | 3  | 2  | 57        |

Table 9 Ranks of the four methods on problems of 50 dimensions based on mean value and median solution

|                   | CAL-SHADE | LSHADE44 + IDE | LSHADE44 | UDE |
|-------------------|-----------|----------------|----------|-----|
| Total rank values | 167       | 147            | 107      | 119 |
| Rank              | 4         | 3              | 1        | 2   |

#### Table 10 Ranks based on mean values on the 28 functions of 100 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | <b>27</b> | 28 |    |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|-----------|----|----|
| CAL-<br>SHADE     | 3 | 3 | 4 | 4 | 3 | 1 | 4 | 4 | 3 | 3  | 3  | 3  | 3  | 1  | 4  | 4  | 4  | 4  | 1  | 1  | 2  | 3  | 4  | 4  | 4  | 4         | 2         | 1  | 84 |
| LSHADE44<br>+ IDE | 4 | 4 | 3 | 2 | 2 | 4 | 2 | 2 | 2 | 2  | 1  | 1  | 1  | 4  | 1  | 2  | 2  | 2  | 3  | 3  | 3  | 2  | 2  | 2  | 3  | 2         | 3         | 3  | 67 |
| LSHADE44          | 1 | 1 | 2 | 1 | 1 | 3 | 3 | 1 | 1 | 1  | 2  | 4  | 2  | 3  | 3  | 3  | 1  | 1  | 2  | 2  | 4  | 1  | 3  | 1  | 2  | 1         | 1         | 4  | 55 |
| UDE               | 2 | 2 | 1 | 3 | 4 | 2 | 1 | 3 | 4 | 4  | 4  | 2  | 4  | 2  | 2  | 1  | 3  | 3  | 4  | 4  | 1  | 4  | 1  | 3  | 1  | 3         | 4         | 2  | 74 |

Table 11 Ranks based on median solution on the 28 functions of 100 dimensions

| Problems          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | <b>26</b> | <b>27</b> | 28 |           |
|-------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----------|-----------|----|-----------|
| CAL-<br>SHADE     | 3 | 3 | 4 | 4 | 3 | 2 | 4 | 4 | 3 | 3  | 3  | 3  | 3  | 1  | 4  | 4  | 4  | 4  | 1  | 1  | 1  | 3  | 2  | 4  | 4  | 4         | 4         | 1  | 84        |
| LSHADE44<br>+ IDE | 4 | 4 | 3 | 1 | 1 | 4 | 2 | 2 | 2 | 2  | 2  | 2  | 1  | 4  | 1  | 2  | 2  | 1  | 3  | 3  | 3  | 2  | 3  | 2  | 3  | 2         | 1         | 3  | 65        |
| LSHADE44          | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 1  | 1  | 4  | 2  | 3  | 3  | 3  | 1  | 2  | 2  | 2  | 4  | 1  | 4  | 3  | 2  | 1         | 2         | 4  | 61        |
| UDE               | 2 | 2 | 1 | 3 | 4 | 1 | 1 | 3 | 4 | 4  | 4  | 1  | 4  | 2  | 2  | 1  | 3  | 3  | 4  | 4  | 2  | 4  | 1  | 1  | 1  | 3         | 3         | 2  | <b>70</b> |

Table 12 Ranks of the four methods on problems of 100 dimensions based on mean value and median solution

|                   | CAL-SHADE | LSHADE44 + IDE | LSHADE44 | UDE |
|-------------------|-----------|----------------|----------|-----|
| Total rank values | 168       | 132            | 116      | 144 |
| Rank              | 4         | 2              | 1        | 3   |

Table 13 Ranks of the four methods on problems of all considered dimensions

|                   | CAL-SHADE       | LSHADE44 + IDE  | LSHADE44        | UDE             |
|-------------------|-----------------|-----------------|-----------------|-----------------|
| Total rank values | 646             | 550             | 453             | 498             |
| Rank              | 4 <sup>th</sup> | 3 <sup>rd</sup> | 1 <sup>st</sup> | 2 <sup>nd</sup> |