① $2^a 3^b = 2^c 3^d$

Assume: $a \neq c$, $b \neq d$

$a > c$, $b < d$ without loss of generality.

$\Rightarrow 2^{(a-c)} = 3^{(d-b)}$

$\Downarrow$ even

odd?

Lemma

odd · odd = odd

$(2k+1)\cdot(2p+1)$

$= 4kp + 2p + 2k + 1$

$= 2\underbrace{(2kp + p + k)}_{k'} + 1$

$=$ odd

$\Rightarrow 3^{(d-b)}$ is odd $\Rightarrow$ LHS $\neq$ RHS

$\Uparrow$

assumption is incorrect

$\Downarrow$

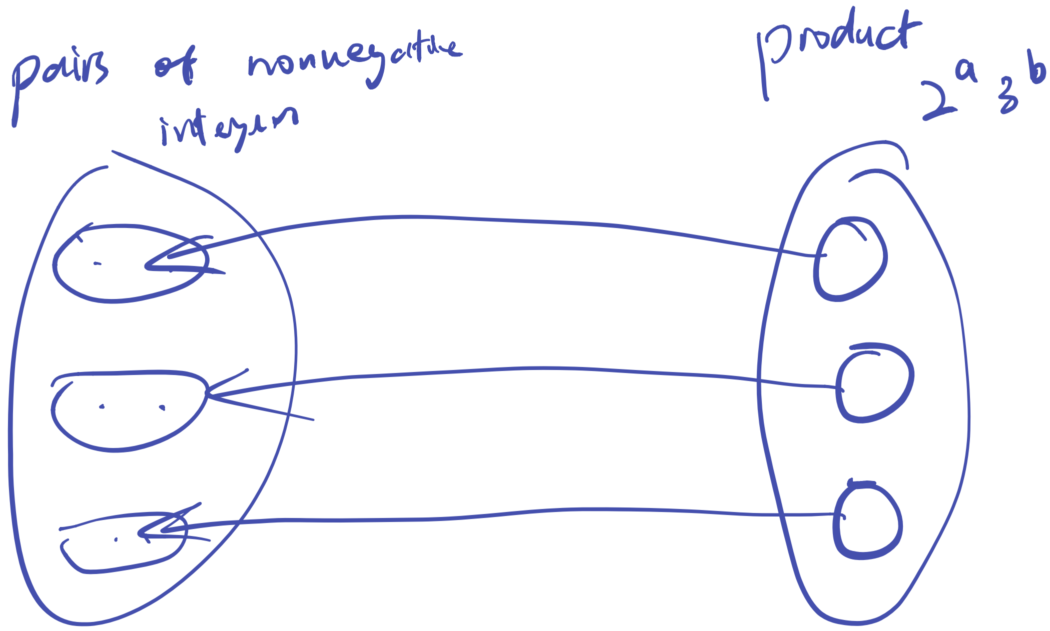
$a = c$, $b = d$

injectivity is proved.

② Surjectivity?

for every pair of nonnegative integers p and q we can always construct $2^p \cdot 3^q$

$\Rightarrow$ Surjectivity is proved.



bijective !!

$\Rightarrow$ therefore, as long as we have can extract a, b from $2^a 3^b$, this representation is valid.

③ odd × even = even

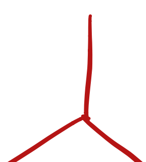
$\Leftarrow$ $(2k+1)(2m) = 4km + 2m$

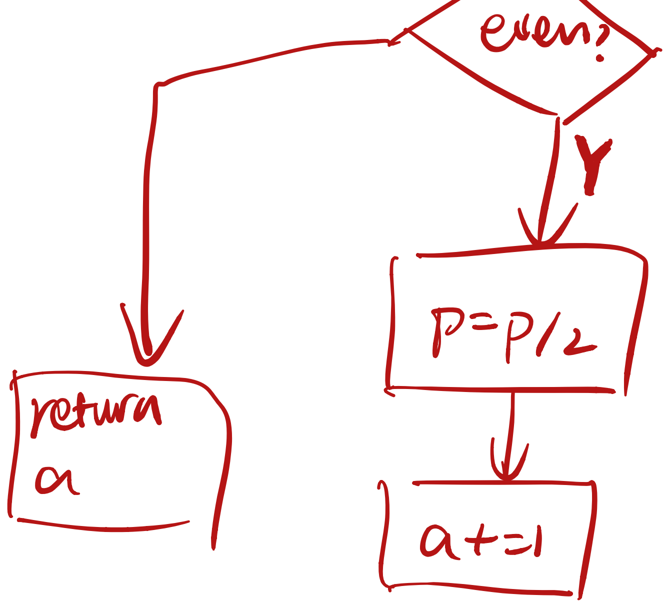
$$= 2(\underbrace{2km + m}_{k'})$$

$$= \text{even}$$

extraction of a and b ($a_0 = 0$, $b_0 = 0$)

a





b

lemma

$$\boxed{2^k \mid 3 \neq 0}$$

$2 \mid 3 \neq 0$

$2^k \mid 3 \neq 0$

$2^{k+1} \mid 3$

$2^k \mid 3 \neq 0$

$2^k = 3\cdot m+1$ ①

$= 3m+2$ ②

① $(3m+1)\times 2$
$= 6m+2$

$(6m+2) \mid 3$
$= 2.$

② $(3m+2)\times 2$
$= 6m+4$

$(6m+4) \mid 3$
$= 1$

$\neq 0$