Homework 1

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Problem 1:

For Katz centrality

$$\mathbf{c}_k = \beta(\mathbf{I} - \alpha\mathbf{A})^{-1}\vec{1}$$

the selection of α should be appropriate: α should not be very tiny or the Katz centrality will be meaningless; α should not be too large as well or the matrix $\mathbf{I} - \alpha\mathbf{A}$ could become ill-conditioned. If the factor

$$\alpha < \frac{1}{\lambda_{max}}$$

then

$$\det(\mathbf{I} - \alpha\mathbf{A}) \neq 0 \quad \text{for all the conditions}$$

The convergence of the centrality could be guaranteed, where λ_{max} is the maximum of all the eigenvalues.

Problem 2:

Considering the nodes v_i and v_j in the graph $G = (V, E)$, it's obvious that all the common neighbors between them are located on the walks with length 2. Therefore,

$$|N(v_i) \cap N(v_j)| = N_{ij}^{(2)} = \sum_{k=1}^{n} A_{ik} A_{kj} = [A^2]_{ij}$$

where A is the adjacency matrix.

Problem 3:

A. The Jaccard's similarity matrix is given by

$$S_{ij}^{Jaccard} = \frac{|N(v_i) \cap N(v_j)|}{|N(v_i) \cup N(v_j)|} = \frac{[A^2]_{ij}}{d_j + d_j - [A^2]_{ij}}$$

where A is the adjacency matrix and d_i represents the degree centrality of node i.

B. As shown in Fig. 1, only two families have similarities with Ginori family, which are Guadagni and Medici.

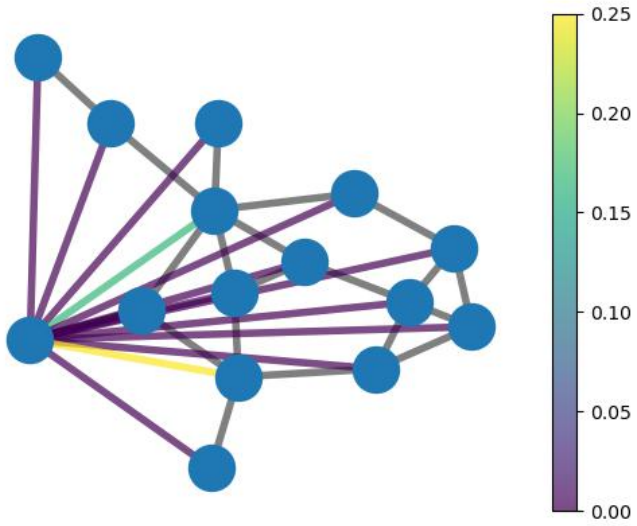

Figure 1: The similarity between "Ginori" family and other families in the Florentine Families graph.