$$\Delta y_1 = log_2(4n) - log_2(n) = 2$$

$$\Delta y_2 = \sqrt{4n} - \sqrt{n} = \sqrt{n}$$

$$\Delta y_3 = 4n - n = 3n$$

$$\Delta y_4 = 16n^2 - n^2 = 15n^2$$

$$\Delta y_5 = 64n^3 - n^3 = 63n^3$$

$$\Delta y_6 = 2^{4n} - 2^n = 16^n - 2^n$$

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(1)

$\because t(n) \in O(g(n))$

$\therefore \exists N_0 \in R$ 对于 $\forall n > N_0$ 有 $t(n) \le cg(n)$

$\therefore \frac{1}{c}t(n) \le g(n)$

不妨令 $\frac{1}{c} = k(k > 0)$ $\therefore$ 有 $g(n) \ge kt(n)$

$\therefore g(n) \in \Omega(t(n))$

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(2)

$\because \Theta(\alpha g(n)) \quad \alpha > 0$

由 $\Theta(\alpha g(n))$ 可知 $\exists c_1, c_2 > 0$

$c_1 \alpha g(n) \le t(n) \le c_2 \alpha g(n)$

令 $c_1\alpha = k_1, c_2\alpha = k2$

$\therefore k_1 g(n) \le t(n) \le k_2 g(n)$

$\therefore \Theta(\alpha g(n)) \subseteq \Theta(g(n))$

同理可得 由 $\Theta(g(n))$ 可知 $\exists c_1, c_2 > 0$

$c_1 g(n) \le t(n) \le c_2 g(n)$

令 $c_1 = k_1\alpha, c_2 = k2\alpha$

$\therefore k_1(\alpha g(n)) \le t(n) \le k_2(\alpha g(n))$

$\therefore \Theta(\alpha g(n)) \supseteq \Theta(g(n))$

$\therefore \Theta(\alpha g(n)) = \Theta(g(n))$

---

(3)

由 $\Theta(g(n))$ 可知 $\exists c_1, c_2 > 0$

$c_1 g(n) \le t(n) \le c_2 g(n)$

$\therefore \exists t(n) \le c_2 g(n) \cap t(n) \ge c_1 g(n)$

$\therefore O(g(n)) \cap \Omega(g(n)) \supseteq \Theta(g(n))$

不妨令 $O(g(n)) \cap \Omega(g(n)) = t(n)$ 有 $n > N1, t(n) \le c_1 g(n); n > N2, t(n) \ge c_2 g(n)$

$\therefore$ 对于 $\forall n \ge max\{N_1, N_2\}$ 有 $t(n) \le c_1 g(n) \cap t(n) \ge c_2 g(n)$

$\therefore O(g(n)) \cap \Omega(g(n)) \subseteq \Theta(g(n))$

$\therefore O(g(n)) \cap \Omega(g(n)) = \Theta(g(n))$

$$(4)$$

对于周期震荡函数来说，不存在与之同阶的函数，也不存在$O(f(n))$和$\Omega(f(n))$

考察函数$f(n) = sin(n) + 1 \quad g(n) = 1/n$

找不到任意一个N_0，对于$\forall n > N_0$, 使得$f(n) \le c_1 g(n)$或$f(n) \ge c_2 g(n)$或$c_3 g(n) \le f(n) \le c_4 g(n)$恒成立

$$\therefore \lim_{n \to +\infty} \frac{si(n) + 1}{1/n} \quad \text{不存在}$$

$\therefore$ 命题不成立

$$(1)$$
$$quesetion : x(n) = 3x(n-1) \quad for \, n > 1, x(1) = 4$$
$$x(n) = 3x(n-1) = 3 * 3x(n-2) = 3^{n-1} * x(1)$$
$$= 4 * 3^{n-1}$$
$$(2)$$
$$quesetion : x(n) = x(n-1) + n \quad n > 0, x(0) = 0$$
$$x(n) = x(n-1) + n = x(n-2) + (n-1) + n = x(0) + 1 + 2 + \ldots + n$$
$$= \frac{n(n+1)}{2}$$
$$(3)$$
$$question : x(n) = x(\frac{n}{2}) + n \quad n > 1, x(1) = 1$$
$$let \quad n = 2^k$$
$$x(2^k) = x(2^{k-1}) + 2^k = x(2^{k-2}) + 2^{k-1} + 2^k = x(1) + 2 + 2^2 + \ldots + 2^k$$
$$= x(1) + 2^{k+1} - 2 = n^{k+1} - 1$$
$$\therefore x(n) = 2n - 1$$