Project 5: Inverting the heat equation

Brin Summer School on SciML

Consider the 1D heat equation:

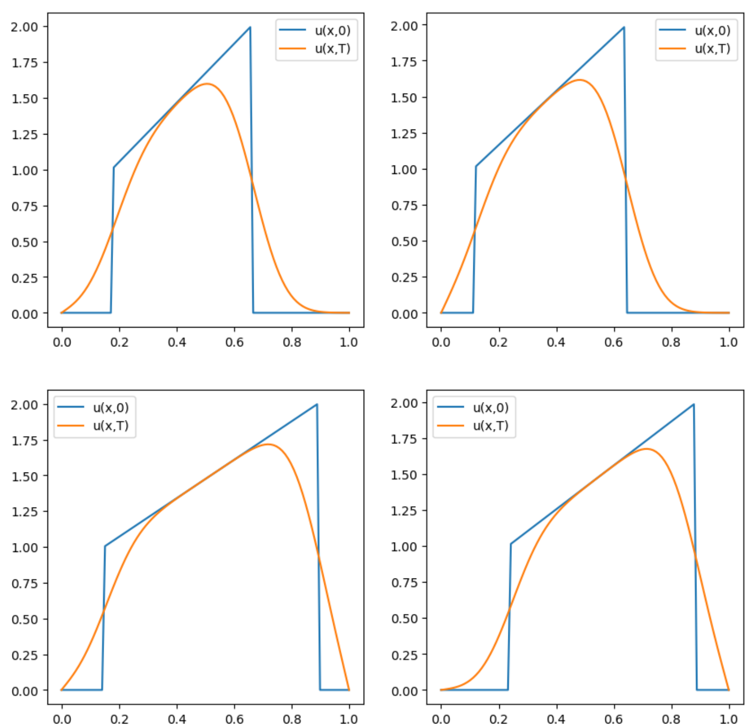
$$\frac{\partial}{\partial t}u(x,t) = \nu \frac{\partial^2}{\partial x^2}u(x,t) \qquad \forall\ x \in (0,1) \quad t \in (0,0.4) \tag{1}$$

$$u(x,0) = u_0(x) \qquad \forall\ x \in [0,1] \tag{2}$$

$$u(0,t) = u(1,t) = 0 \qquad \forall\ t \in [0,0.4] \tag{3}$$

where $\nu > 0$ is the viscosity coefficient. We choose $\nu = 0.01$.

In this project, the goal is to determine the possible (discrete) initial condition u_0, i.e., the inferred field X, given a noisy measurement of the (discrete) final temperature field at $t = 0.4$, i.e., the measured field Y. You have been provided with training samples corresponding to 2000 samples of u_0 at N_x uniformly spaced nodes in $[0,1]$ and the corresponding (clean) final solutions at same nodes at time $t = 0.4$. In addition, you have 100 test samples. We assume that u_0 is given by random linear inclusions as shown below.



;

Now train a conditional generative model on each of the following data:

- Add 5% uncorrelated Gaussian noise to the clean final temperature fields
- Add 10% uncorrelated Gaussian noise to the clean final temperature fields

Once trained, use the final generator for each type of data to predict the possible X profiles for the (similarly) noisy test Y. For a few test samples, plot the mean X (with 1-SD regions) and compare with the true inferred field.

Bonus task: Try to figure out the noise level beyond which the recovery of X becomes challenging.