Homework 2

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1.

$$\begin{aligned}
E(X) &= 1P(X=1) + 0P(X=0) \\
&= 1p \\
&= p \\
Var(X) &= E((X - E(X))^2) \\
&= E(X^2 - 2XE(X) + E(X)^2) \\
&= E(X^2) - 2E(X)^2 + E(X)^2 \\
&= E(X^2) - E(X)^2 \\
&= 1^2P(X=1) + 0^2P(X=0) - p^2 \\
&= p - p^2
\end{aligned}$$

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2.

$$\begin{aligned}
E(X) &= \int_0^1 x dx \\
&= \frac{1}{2}x^2\Big|_0^1 \\
&= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
Var(X) &= E(X^2) - E(X)^2 \\
&= \int_0^1 x^2 dx - \frac{1}{4} \\
&= \frac{x^3}{3}\Big|_0^1 - \frac{1}{4} \\
&= \frac{1}{12}
\end{aligned}$$

$$\begin{aligned}
Std(X) &= \sqrt{Var(X)} \\
&= \frac{1}{\sqrt{12}}
\end{aligned}$$

3.
When X and Y are discrete:

$$\begin{aligned}
E(aX + bY) &= \sum_x \left[aP(X=x)x + \sum_y bP(Y=y)y \right] \\
&= \sum_x \left[aP(X=x)x + bE(Y) \right] \\
&= aE(X) + bE(Y)
\end{aligned}$$

When X and Y are continuous:

$$\begin{aligned}
E(aX + bY) &= \int \left[aP(X=x)dx + \int bP(Y=y)dy \right] \\
&= \int \left[aP(X=x)dx + bE(Y) \right] \\
&= aE(X) + bE(Y)
\end{aligned}$$

4.

$$\begin{aligned}
&E\left[[X - E(X)]\,[Y - E(Y)] \right] \\
=&E\left[XY - XE(Y) - YE(X) + E(X)E(Y) \right] \\
=&E(XY) - E(X)E(Y)
\end{aligned}$$

Because X and Y are independent, so $E(XY) = E(X)E(Y)$, and thus

$$E(XY) - E(X)E(Y) = 0$$

When X and Y are bound to the same constant, thought they are not independent, their covariance is 0.